Exercises

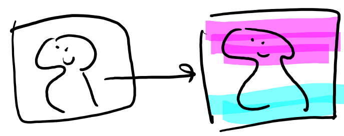
Thursday, December 5, 2019 2:16 PM

Exercise 5 An online social network contains $N \approx 10^5$ users and has a degree distribution $p_k(k)$ as follows:

k	3	5	7	9	11	13	15
p_k	0.6	0.2	0.1	0.05	0.02	0.007	0.003

(all the values of k not reported in the table have $p_k = 0$)

Trend spreading over it



You keep your photo for an exp. distrib. time

avg = 1 week

after which you restore the original photo

and you never put the flag again

If (i,j) is a pair of neighbors, i has flag

j does not $\Rightarrow$ j adopts it with prob. $b = 2\%$

The process is SIR $\mu = 1/7\ \text{days}^{-1}$

β depends on the degree distrib.

If avg degree approx. $\beta = b\langle k\rangle$

(1) homogeneous mixing $\equiv$ avg degree approx

$$\frac{dx}{dt} = \beta s x - \mu x \qquad \frac{ds}{dt} = -\beta s x$$
$$\frac{dr}{dt} = +\mu x$$

(2) does the trend catch? Yes if $\beta/\mu > 1$

$R_0 > 1 \qquad R_0 = \beta/\mu$

$$\langle k \rangle = \sum_{k=0}^{\infty} k p_k = 4.306$$

$R_0 = \beta/\mu = 0.02 \times 4.306 \times 7 = 0.60284$

$R_0 < 1 \Rightarrow$ no spreading

NOTE: Network of contacts is Erdős-Rényi with $\langle k \rangle = 4.306$

Network of infecteds is a sub-network

the avg degree of this subnetwork is R_0

(3) consider block degree approx

It implies that

$$\frac{dx}{dt} = \beta s x - \mu x = b\langle k\rangle s x - \mu x$$

replaced by k

$\alpha = b/\mu$ (BIOLOGICAL PARAM.)

Over a network with block deg. approx

SIR has spreading if $\alpha > \dfrac{\langle k\rangle}{(\langle k^2\rangle - \langle k\rangle)}$

$$\langle k^2\rangle = \sum_{k=0}^{\infty} k^2 p_k = 23.628$$

$\alpha = 0.14$ $\qquad \dfrac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle} = \dfrac{4.306}{19.322} = 0.2229$

STILL NO SPREADING (it's closer, though)

Exercise 6 In a plantation, trees are distributed according to a Poisson point process, so the number of trees in a radius of 100 meters is Poisson distributed with average 5 trees. A parasitic pest is spreading over the plantation and they can move from a tree to another. The spreading radius of the disease from a tree to another is 100 meters (note: this means that the average tree has 4 neighbors able to infect it). For each pair of trees in close proximity, the event that one tree infected by parasites infect the other one is drawn each day with probability $\beta = 1\%$. A fraction q of the trees have flowers. Trees with flowers attract more parasites: they can reach the tree from a distance that is 3 times higher. Parasites infect a tree for ten days of duration, on average. A tree that is free from parasites, is susceptible again.

Normal trees have 5 neighbors

Supershedders have 45 neighbors

(are a fraction q)

(1) Without considering the network this is an SIS spreading

$\tilde{\beta} = \beta \langle k \rangle$ with $\beta = 0.01$ $\mu = 0.1$ days^{-1}

$$\frac{dx}{dt} = \tilde{\beta} s x - \mu x$$

$$\frac{ds}{dt} = \mu x - \tilde{\beta} s x$$

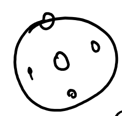
(2) block degree approx?

$\Downarrow$

SIS model with risk structure

so same kind of equations but with

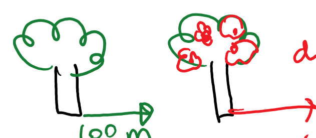
WAIFW $\quad \beta = \begin{bmatrix} \beta_{HH} & \beta_{HL} \\ \beta_{LH} & \beta_{LL} \end{bmatrix}$ high risk class



5 trees in the circle around (including the one in the center)

$\beta_{HH} = \beta_{HL} = \beta \cdot 44$

$\beta_{LH} = \beta_{LL} = \beta \cdot 4$



distance = 3x
area = 9x
so 44 neighbors

(3) What is the lowest q for the disease to spread

Note: if we only had "regular" trees with $\langle k \rangle = 4$ $\Rightarrow$ $\tilde{\beta} = 0.04$

so no spreading $\qquad \mu = 0.1$

WRONG SOLUTION : $\tilde{\beta} \geq 0.1$

$\Rightarrow \quad \langle k \rangle \geq 10$

$$\langle k \rangle = q \cdot 44 + (1-q) \cdot 4$$

<u>MISTAKE</u> : $\tilde{\beta} \neq \beta \langle k \rangle$

SIS model has spreading
with block degree approx if

$$\alpha > \frac{\langle k \rangle}{\langle k^2 \rangle}$$

$\alpha = \frac{1\%}{10\%} = 0.1$ $\quad \langle k \rangle = 40q + 4$

$\langle k^2 \rangle = 44^2 q + 16(1-q)$ $\quad \langle k^2 \rangle = 1920q + 16$

$$0.1 > \frac{40q+4}{1920q+16} = \frac{10q+1}{480q+4}$$

$$480q + 4 > (10q+1)\cdot 10$$

$$q > 3/190 \simeq 0.0156$$

Note: with Supershredders

$$\begin{bmatrix} F\beta & \beta \\ F\beta & \beta \end{bmatrix}$$

Superspreaders

$$\begin{bmatrix} F^2\beta & F\beta \\ F\beta & \beta \end{bmatrix}$$

Our case here:

$$\begin{bmatrix} F\beta & F\beta \\ \beta & \beta \end{bmatrix}$$

side note: for our model

$$\frac{dx_k}{dt} = \beta k (1-x_k)\Theta_k$$

with Θ_k almost indep. of k

$\Rightarrow$ so, all that matters is the in-degree

(this is why trees with are high-risk individuals without being super-spreaders)

Exercise 7 A disease is spreading over a population of 5000 individuals. The disease is spread through exchange of bodily fluids that can be represented with a proper contact network. We distinguish between *dangerous* (i.e., potentially infectious) contacts, and actually infectious contacts, where the disease actually spreads. The pattern of dangerous contacts can be adequately represented with the Erdős-Rényi model, where the average number of dangerous contacts among two network individuals is 8. Infected individuals can pass the disease to any other with whom they are engaging in a dangerous contact; this happens with probability $p = 0.4$. That is, 4 out of 10 dangerous contacts, determined with i.i.d distribution, are actually infectious if either of the contacted individuals is infectious.

$p_k(k)$ is Poisson

$$p_k = e^{-\lambda_0} \lambda_0^{k} / k! \qquad \lambda_0 = 8$$

Max degree you expect in this network is k_{max} where

$$p_{k_{max}} \simeq 1/5000$$

$$p_{20} < 1/5000 \qquad p_{19} > 1/5000$$

In the infectious network denote the degree with k'

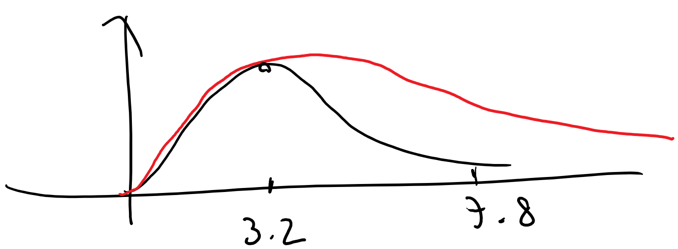
$\langle k' \rangle = 0.4 \langle k \rangle = 3.2$

$$p_{k'}(k) = \sum_{j=k}^{\infty} p_j \binom{j}{k} g^{j-k} (1-g)^k$$

You must find

$p_{k'} \simeq 1/5000$

After some trials $k'_{max} = 15$



(3) Random vaccinations to break the network of dangerous contacts

In the specific case of Erdős-Rényi

$$f = 1 - \frac{1}{\langle k \rangle} = 87.5\%$$

In general we need f s.t. $\kappa < 2$

(Molloy-Reed criterion)

$$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle}$$