LIFE DATA EPIDEMIOLOGY

lect. 4: application of metapopulation models

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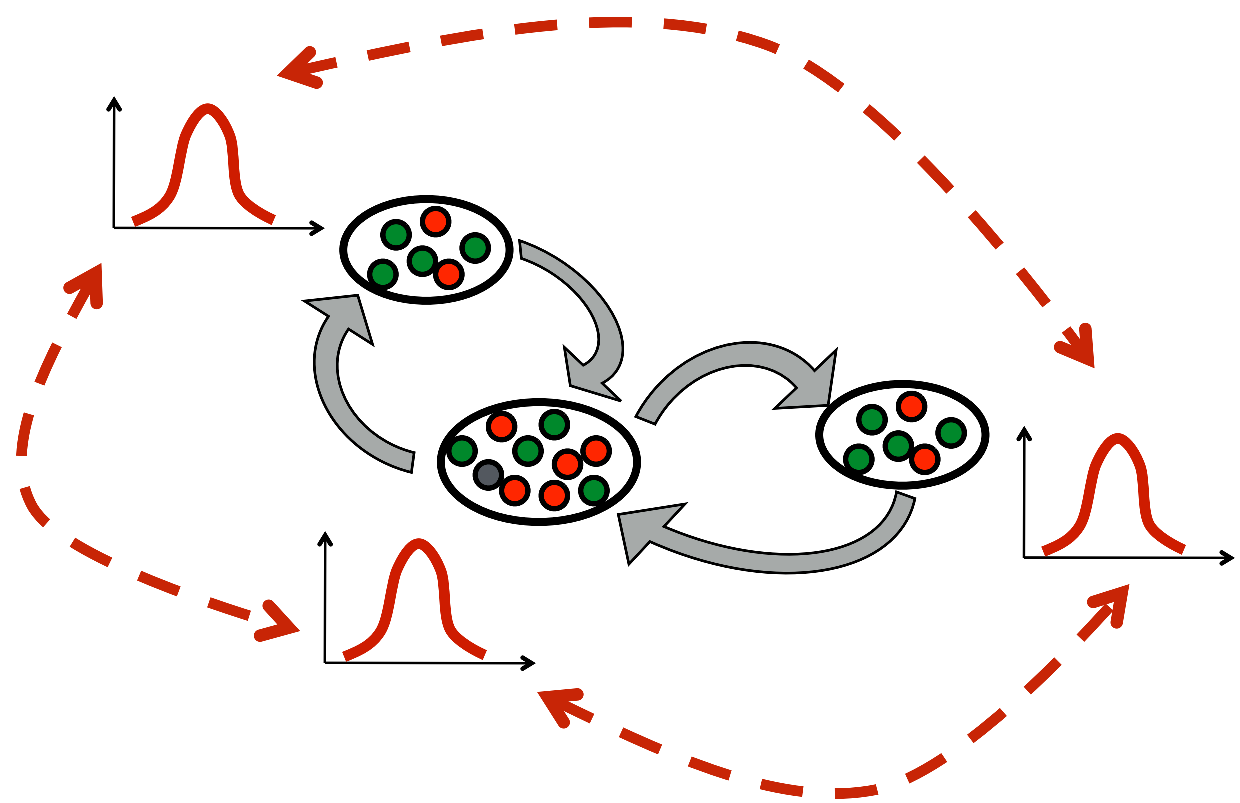
SIR metapopulation model: markovian mobility

$$\frac{dS_i}{dt} = -\beta \frac{I_i(t)S_i(t)}{N_i} + \Omega_i^S$$

$$\frac{dI_i}{dt} = \beta \frac{I_i(t)S_i(t)}{N_i} - \mu I_i(t) + \Omega_i^I$$

$$\frac{dR_i}{dt} = \mu I_i(t) + \Omega_i^R$$

$$\Omega_i^X = \sum_j \frac{w_{ji}}{N_j} X_j - \frac{w_{ij}}{N_i} X_i$$

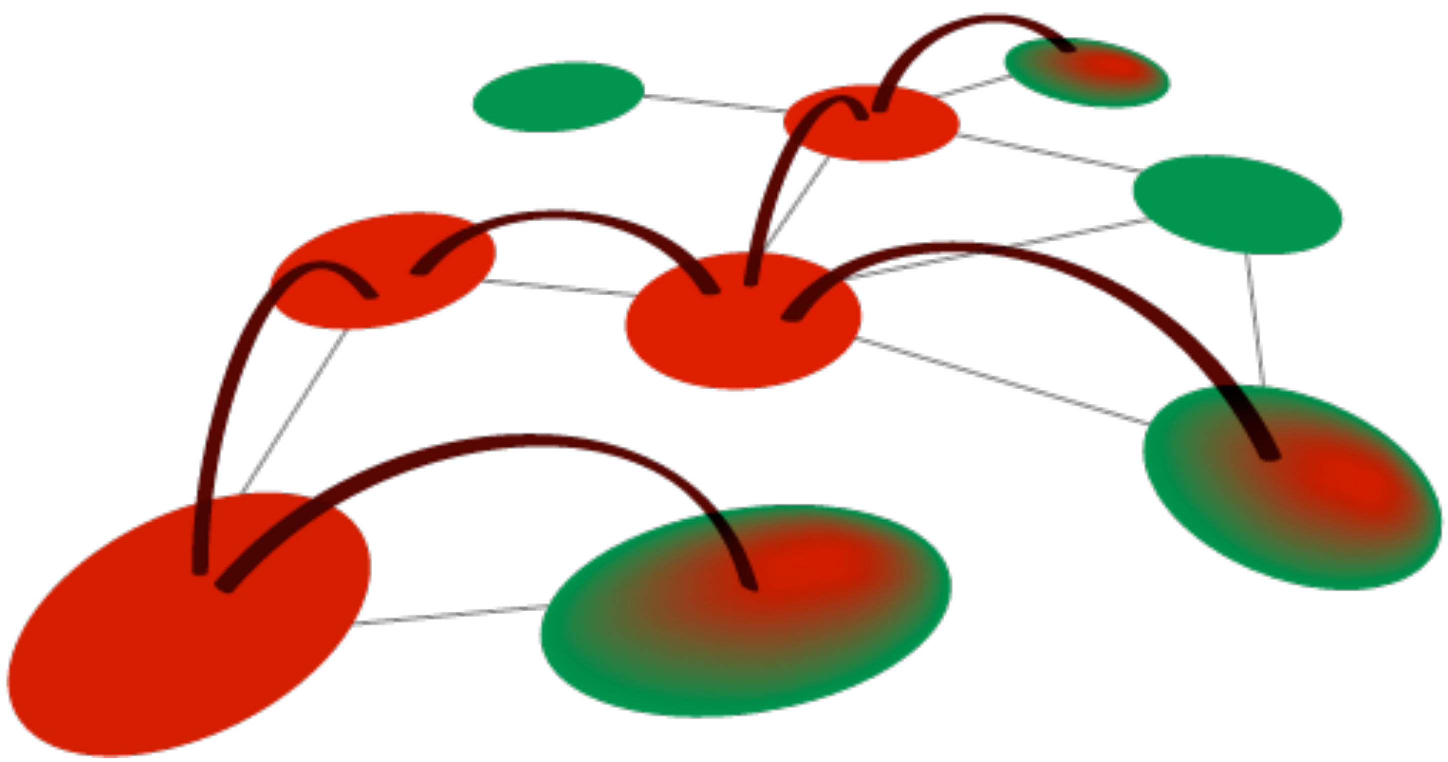


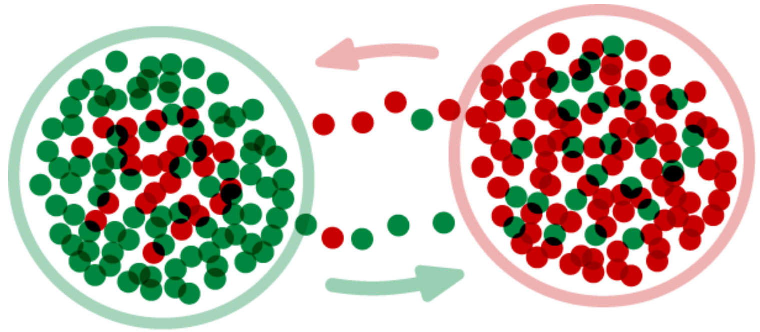
SIR metapopulation model: markovian mobility

What can I do with that?

- analytical understanding
  - spatial propagation & predictability
  - **global invasion threshold**
- computer simulations

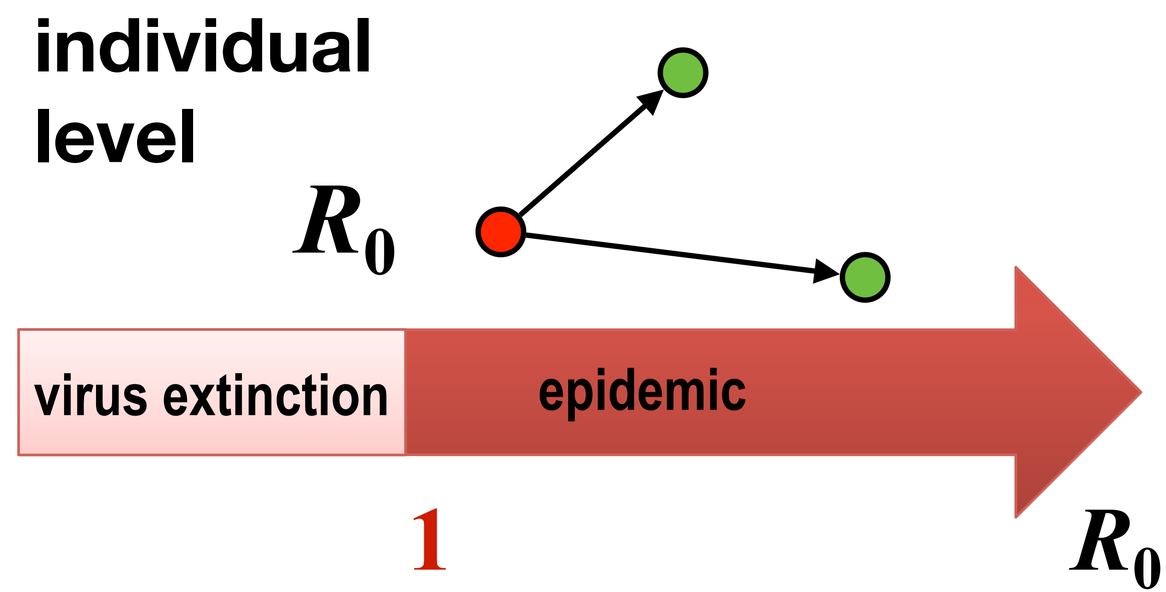
global invasion threshold





which are the conditions for a local outbreak to spread at global proportion?

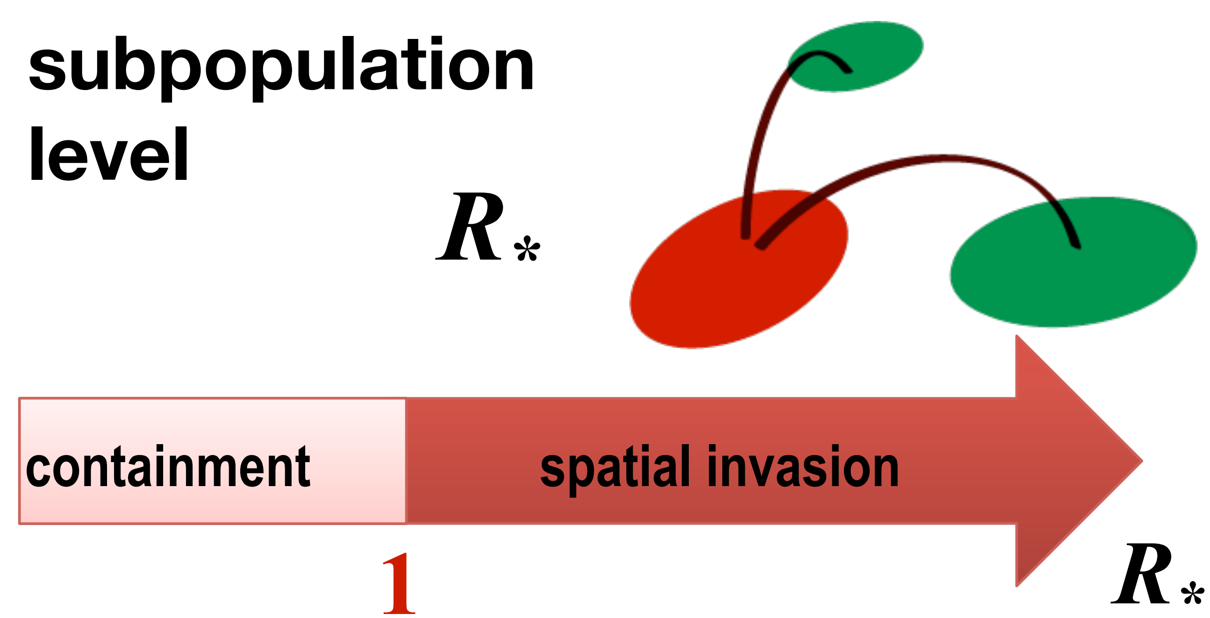
global invasion threshold



coarse graining

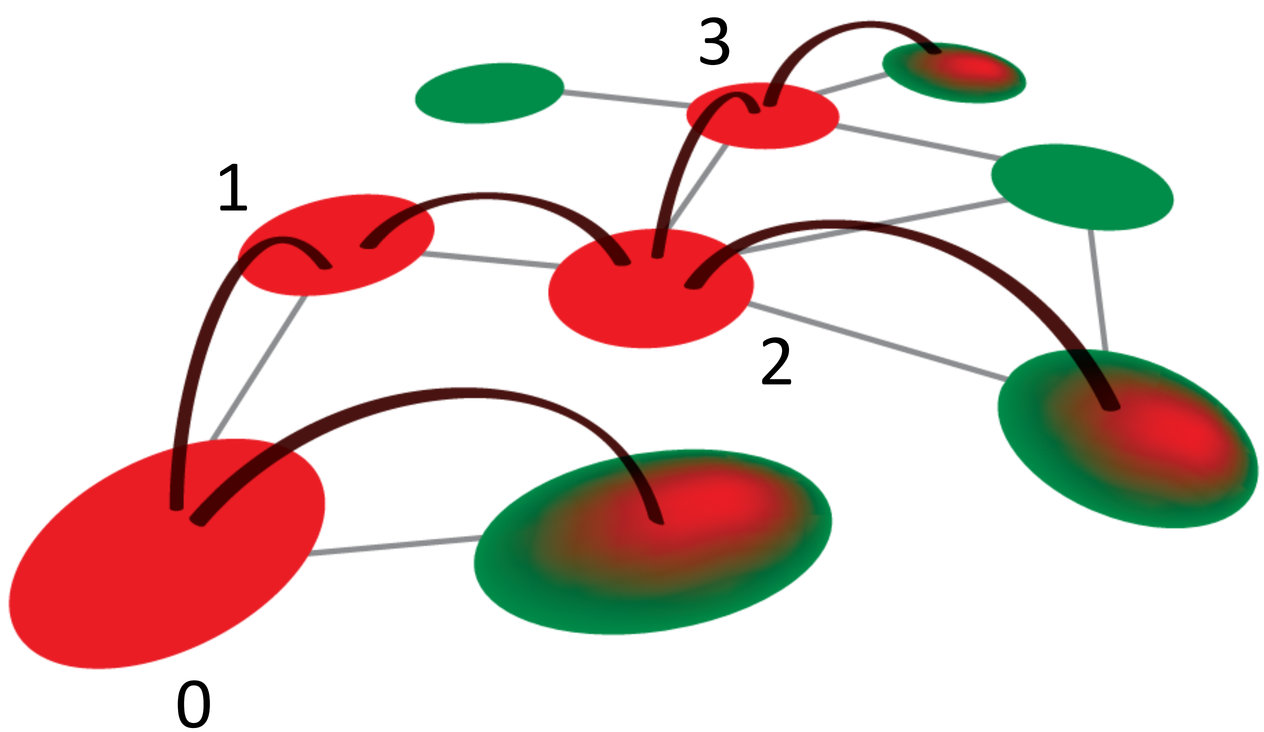
following the spread from one subpopulation to another

mapping the spreading dynamics among subpopulation into the spreading on a network



[Colizza & Vespignani, PRL 2007, JTB 2008;
Cross, et al. JRSoc Interface 2007]

global invasion threshold

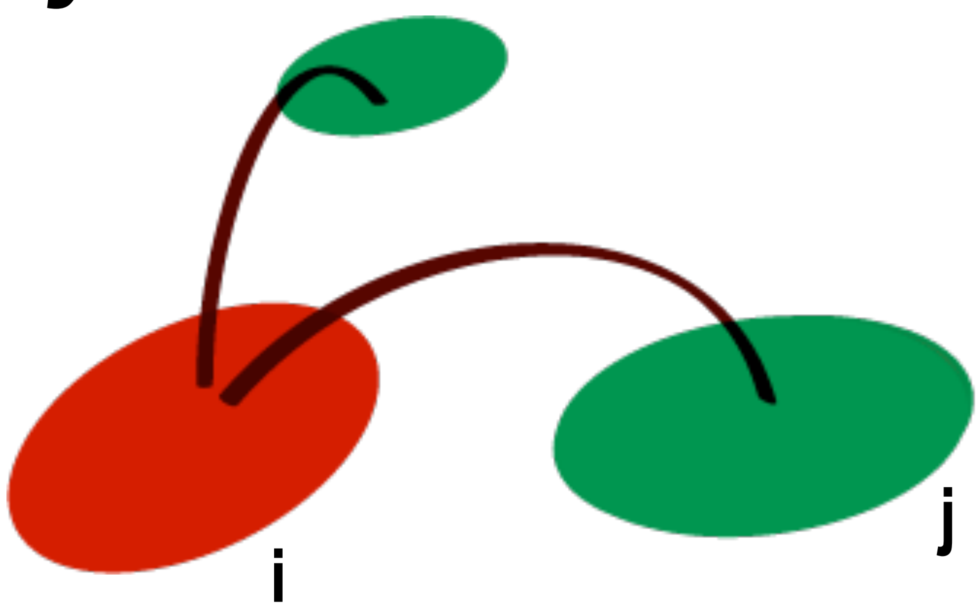


- Invasion dynamics at the subpopulation level
- branching process approximation

D^n: diseased subpopulations at generation n

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: homogeneous systems



w_0 travellers along each link

$\langle k \rangle$ # connection of each subpopulation

N population of each subpopulation

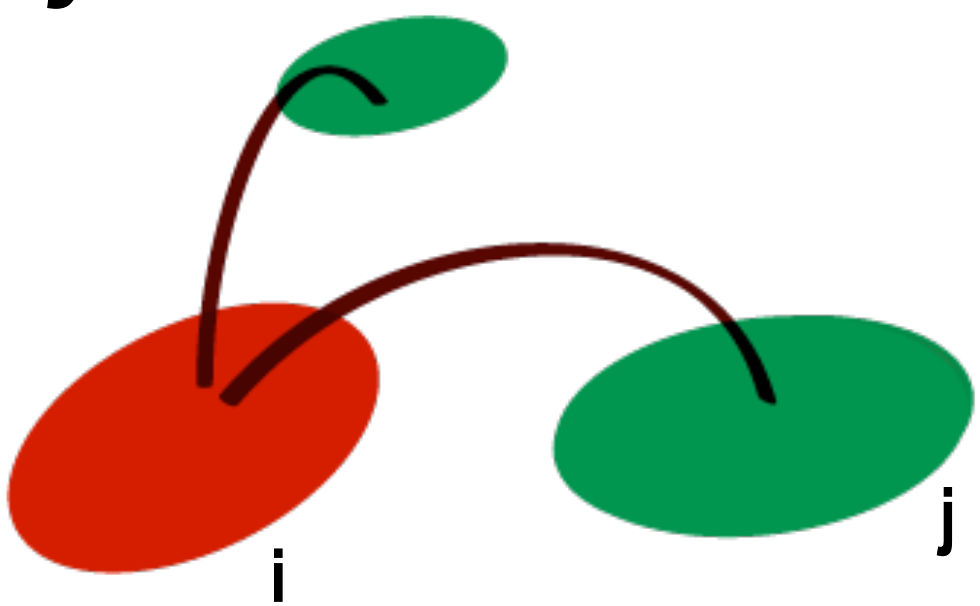
α epidemic attack rate

total # infectious individuals sent from i to j during the local outbreak $\lambda_{ij} = \frac{w_0}{\cancel{N}} \frac{\alpha \cancel{N}}{\mu}$

probability of early extinction $P_{\text{ext}} = \left(\frac{1}{R_0} \right)^{\lambda_{ij}}$

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: homogeneous systems



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$$D^n = (\langle k \rangle - 1)\,(1 - P_{\text{ext}}) \left(1 - \sum_{m=0}^{n-1} \frac{D^n}{V} \right) D^{n-1}$$

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: homogeneous systems

$$D^n = (\langle k \rangle - 1)(1 - P_{\text{ext}}) \left(1 - \cancel{\sum_{m=0}^{n-1} \frac{D^n}{V}} \right) D^{n-1}$$

$$\boxed{R_* = (\langle k \rangle - 1)(1 - P_{\text{ext}})}$$

$$1 - P_{ext} = 1 - \left(\frac{1}{R_0} \right)^{\lambda_{ij}} \simeq \lambda_{ij}(R_0 - 1) = \frac{\alpha w_0}{\mu}(R_0 - 1)$$

$$\boxed{R_* = (\langle k \rangle - 1) \frac{\alpha w_0}{\mu}(R_0 - 1)}$$

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: homogeneous systems

$$R_* = (\langle k \rangle - 1)\frac{\alpha w_0}{\mu}(R_0 - 1)$$

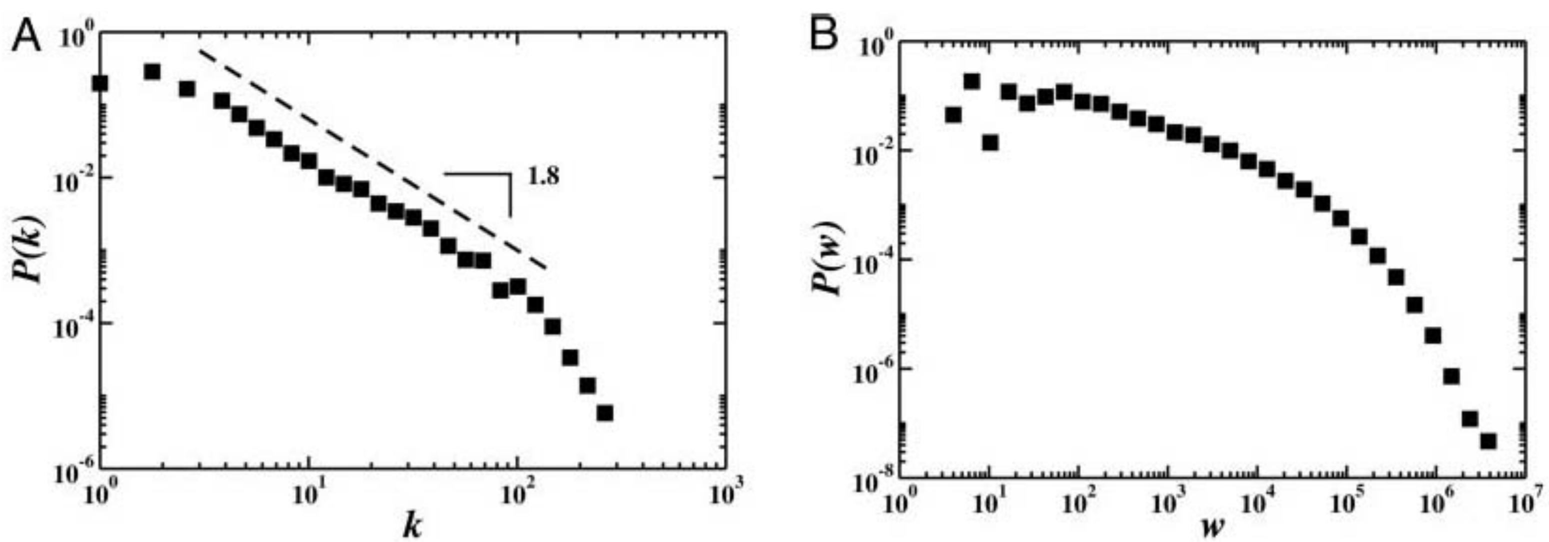
invasion potential growing function of :

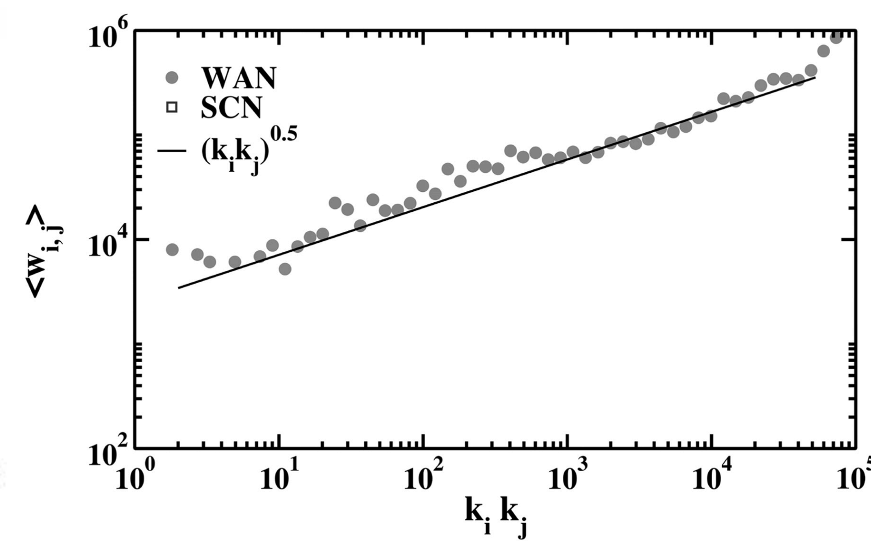
- R_0
- over all traffic rescaling
- average number of connections
- infectious duration

invasion threshold: heterogeneous systems

Real systems are highly heterogeneous. E.g.: air transportation network

Bad news :(	**Good news :)**





- number of connections and travellers along the connections is heterogeneous
- average quantities are not good representative of the properties of patches
- homogenous approximation is bad

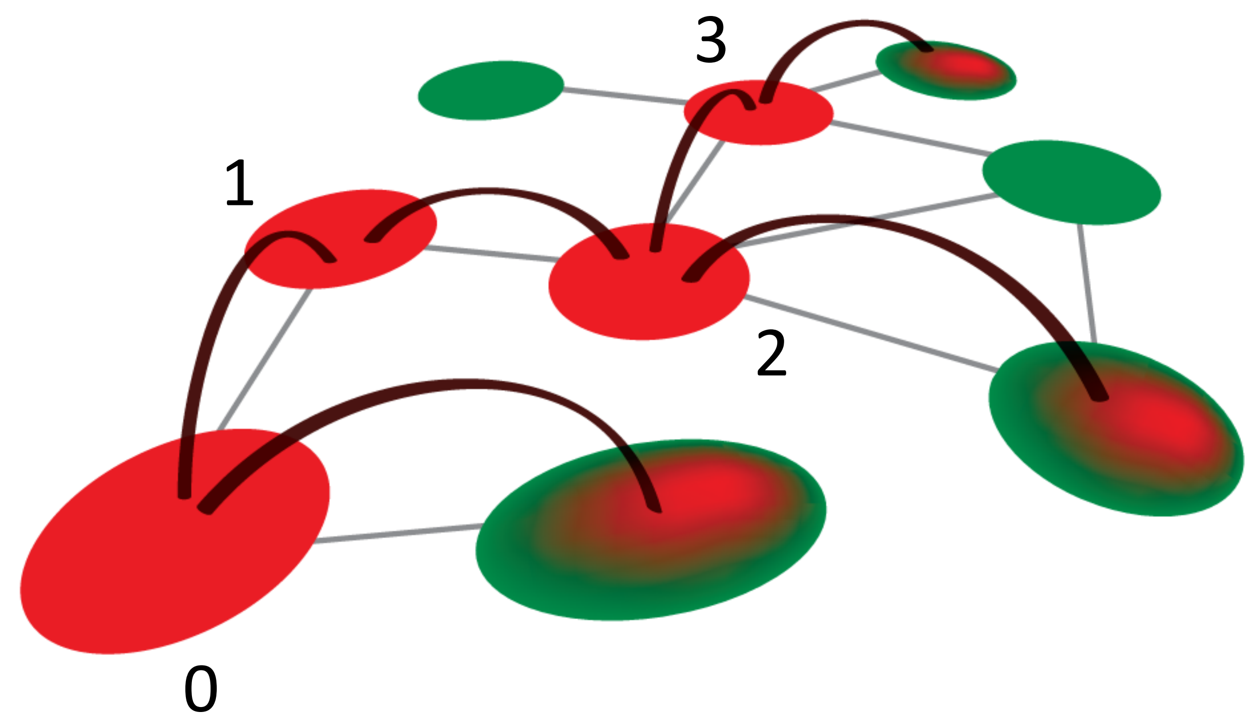
- scaling relations: approximate laws that make possible calculations **and justify the degree-block description**

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: heterogeneous systems

Degree-block description:

we group patches according to their degree and consider patches within the same degree-class homogeneous

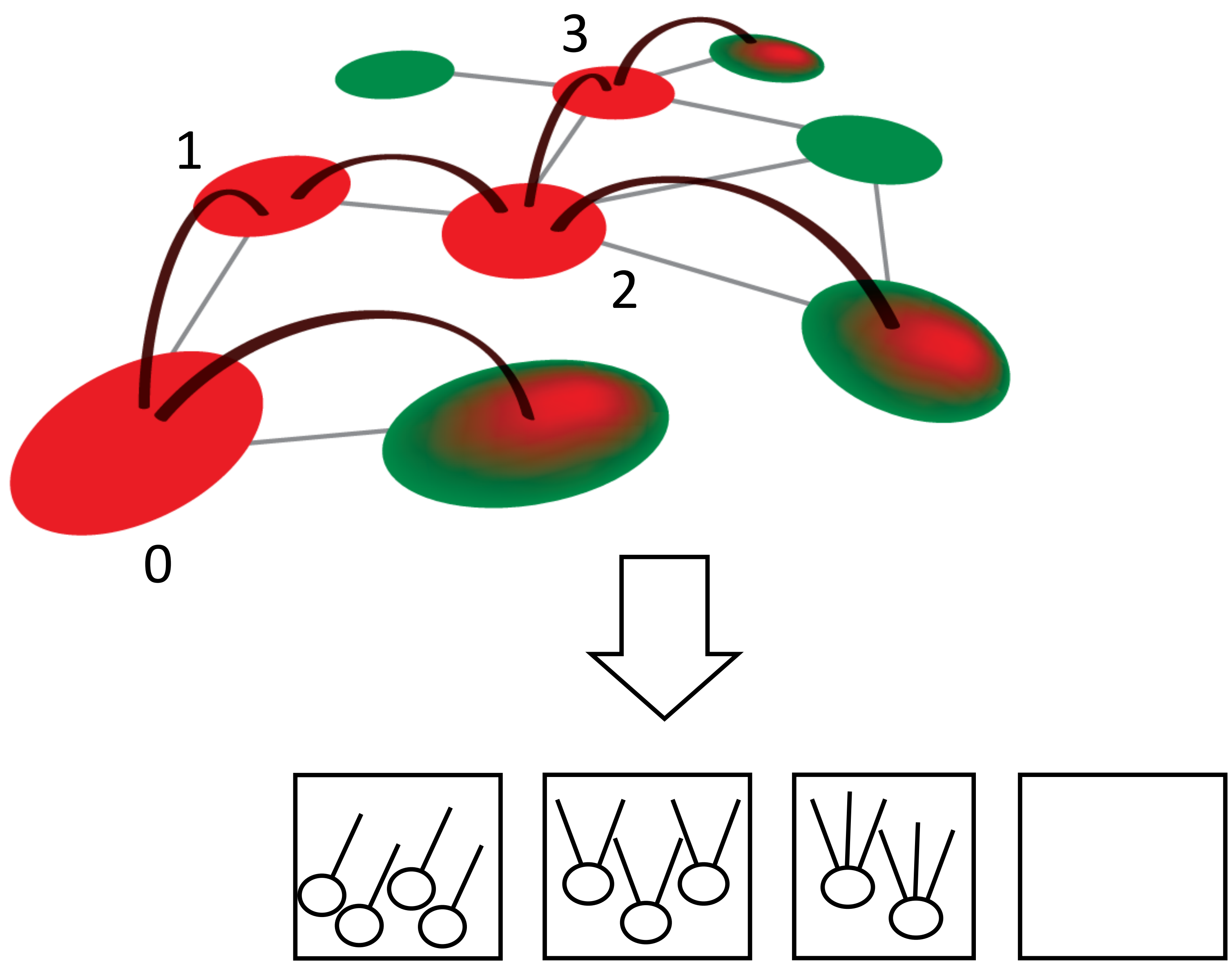


$$N_k = N_0 k^{\phi}$$
$$w_{k\,k'} = w_0 (k\,k')^{\theta}$$
$$p_{k\,k'} = \frac{w_0}{N_0}\frac{(k\,k')^{\theta}}{k^{\phi}}$$

D_k^n: diseased subpopulations at generation *n*, with *k* mobility connections

[Colizza & Vespignani, PRL 2007, JTB 2008]

heterogeneous mean field approach



$k = 1$ $\quad$ $k = 2$ $\quad$ $k = 3$ $\quad$ $k = \ldots.$

$$P(k)$$

Degree distribution

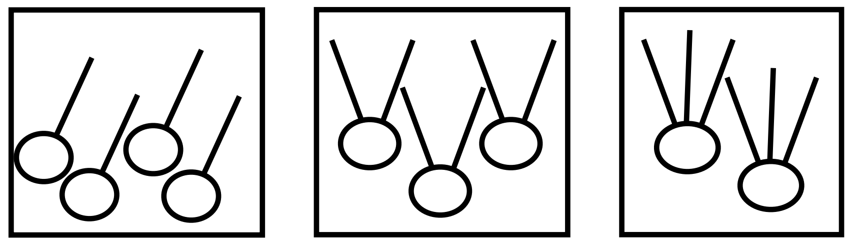
$$N_k = N_0 k^{\phi}$$

$$w_{k\,k'} = w_0 (k\,k')^{\theta}$$

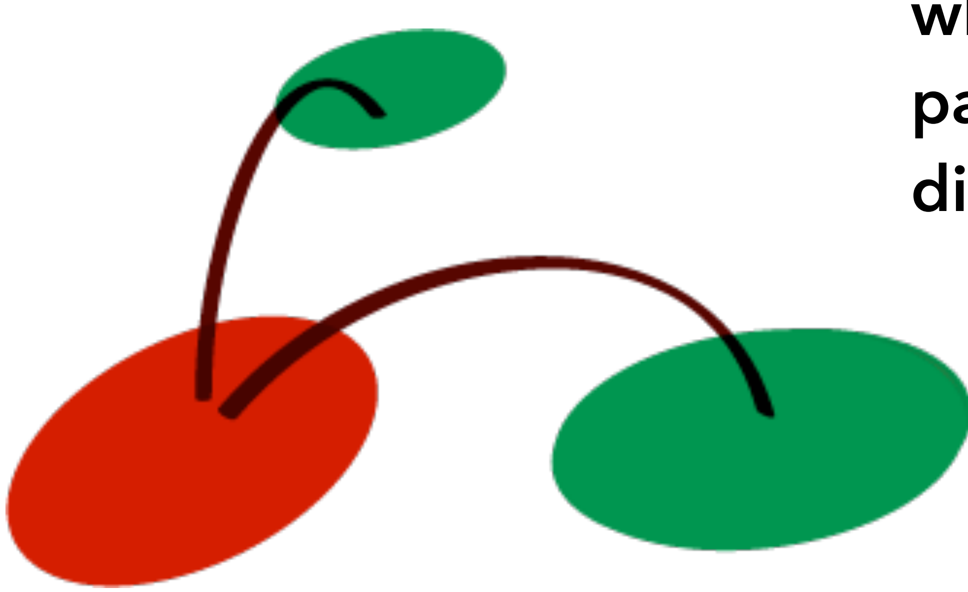
$$p_{k\,k'} = \frac{w_0}{N_0} \frac{(k\,k')^{\theta}}{k^{\phi}}$$

[Pastor-Satorras & Vespignani PRL 2001, review :Pastor Satorras et al. Rev Mod Phys 2015]

heterogeneous mean field approach



$k = 1$ $\quad k = 2$ $\quad k = 3$ $\quad k = ...$

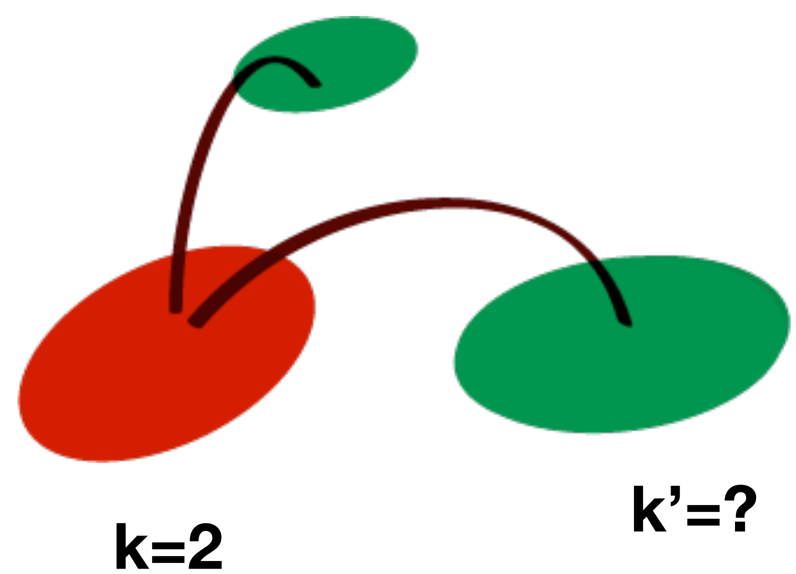


k=2

k'=?

what is the probability that an infected patch with degree *k* is connected to a disease-free patch of degree *k'* ?

heterogeneous mean field approach



number of mobility connections through which the seeding may potentially occur:

$$k-1$$

X

probability that contact has degree k':

$$P(k'|k)$$

X

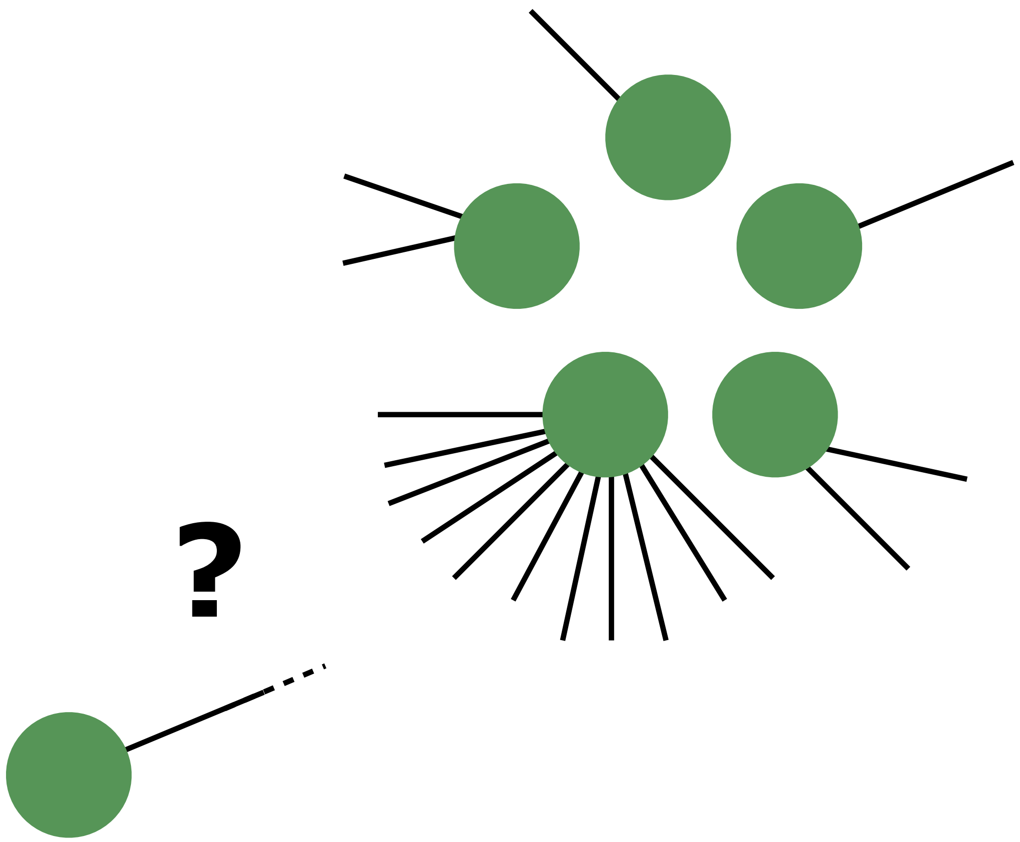
number of disease-free patches within the k'-class:

$$\left(1-\sum_{m=0}^{n-1}\frac{D_k^m}{V_k}\right)$$

heterogeneous mean field approach

$$P(k'|k) = k' \frac{P(k')}{\langle k \rangle}$$

If I make a connection at random I will do it more likely with a node that is well connected (more stubs)



Friendship paradox: my friends have more friends than me!

invasion threshold: heterogeneous systems

$$D^n = \boxed{(\langle k \rangle - 1)} \boxed{(1 - P_{\text{ext}})} \boxed{\left(1 - \sum_{m=0}^{n-1} \frac{D^n}{V}\right)} D^{n-1}$$

$$D^n \rightarrow D^n_k$$

$$D^n_k = \sum_{k'} D^{n-1}_{k'} \boxed{(k'-1)P(k \mid k')} \boxed{\left(1 - \sum_{m=0}^{n-1} \frac{D^m_k}{V_k}\right)} \boxed{(1 - \mathrm{P}_{\text{ext}}(\lambda_{k'k}))}$$

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: heterogeneous systems

$$D_k^n = \sum_{k'} D_{k'}^{n-1} \boxed{(k'-1)P(k|k')} \boxed{\left(1-\cancel{\sum_{m=0}^{n-1}\frac{D_k^m}{V_k}}\right)} \boxed{(1-\mathrm{P}_{\mathrm{ext}}(\lambda_{k'k}))}$$

$$P(k'|k) = k\frac{P(k)}{\langle k\rangle}$$

$$1 - P_{ext}(\lambda_{k'k}) = 1 - \left(\frac{1}{R_0}\right)^{\lambda_{k'k}} \simeq \lambda_{k'k}(R_0 - 1)$$

$$\boxed{\lambda_{ij} = \frac{w_0\alpha}{\mu}}$$

homogenous case

$$\lambda_{k'k} = \frac{w_0(kk')^\theta}{\cancel{N_0k^\phi}}\frac{\alpha}{\mu}\cancel{(N_0k^\phi)}$$

[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: heterogeneous systems

$$D_k^n = \sum_{k'} D_{k'}^{n-1}(k'-1)P(k\,|\,k')\left(1 - \sum_{m=0}^{n-1}\frac{D_k^m}{V_k}\right)\left(1 - P_{ext}(\lambda_{k'k})\right)$$

$$D_k^n = (R_0 - 1)\frac{\alpha w_0}{\mu}\frac{k^{1+\theta}P(k)}{\langle k\rangle}\underbrace{\boxed{\sum_{k'} D_{k'}^{n-1}(k'-1)k'^{\theta}}}_{\Theta^n}$$

$$\Theta^n = (R_0 - 1)\frac{\alpha w_0}{\mu}\frac{\langle k^{2+2\theta}\rangle - \langle k^{1+2\theta}\rangle}{\langle k\rangle}\Theta^{n-1}$$

$$R_* = (R_0 - 1)\frac{\alpha w_0}{\mu}\frac{\langle k^{2+2\theta}\rangle - \langle k^{1+2\theta}\rangle}{\langle k\rangle} > 1$$

invasion threshold: heterogeneous system

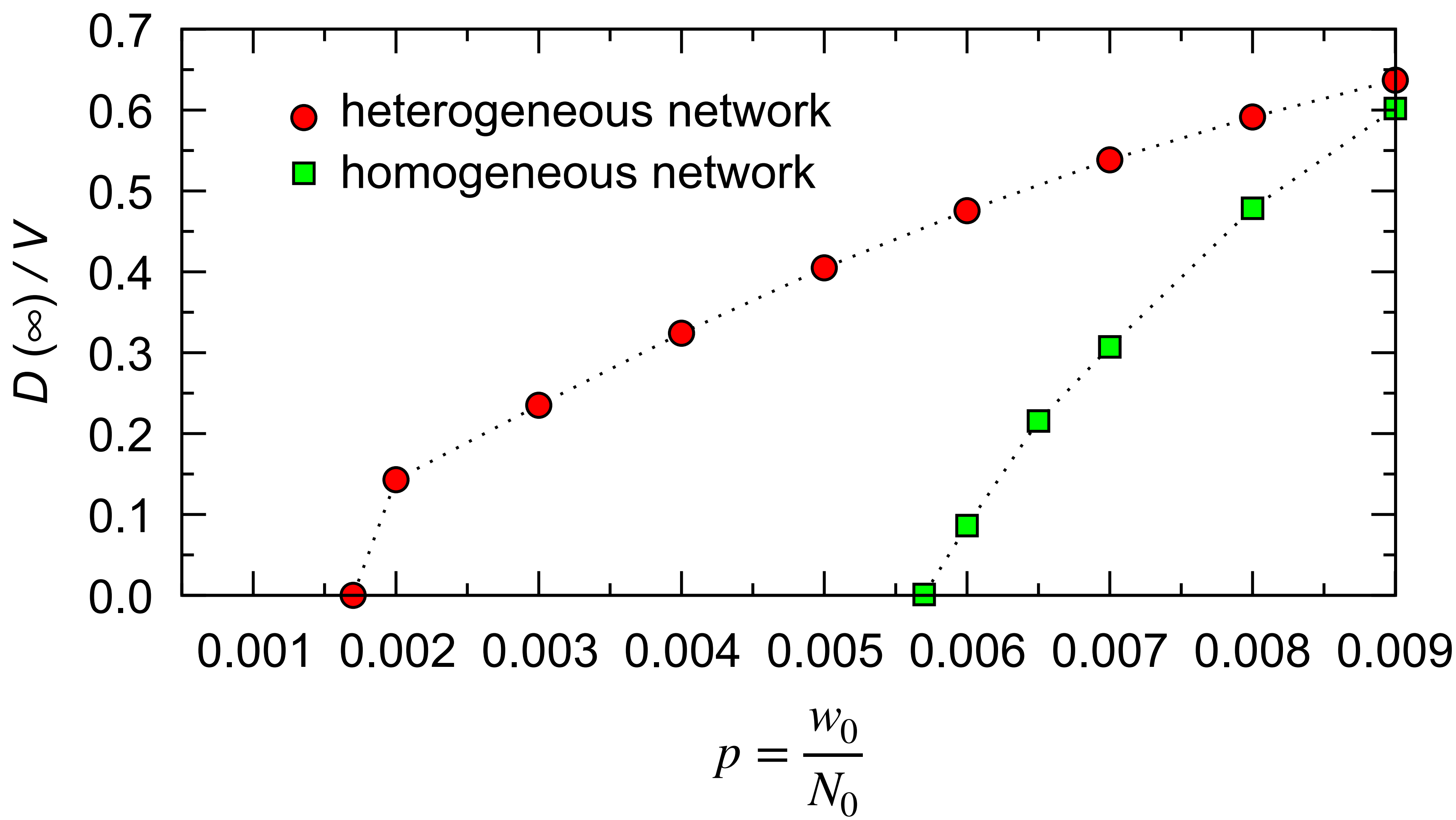
$$R_* = (R_0 - 1)\frac{\alpha w_0}{\mu}\frac{\langle k^{2+2\theta}\rangle - \langle k^{1+2\theta}\rangle}{\langle k\rangle} > 1$$

invasion potential growing function of :

- R_0
- over all traffic rescaling
- average number of connections
- infectious duration
- <u>moments of the degree distribution and its fluctuations</u>

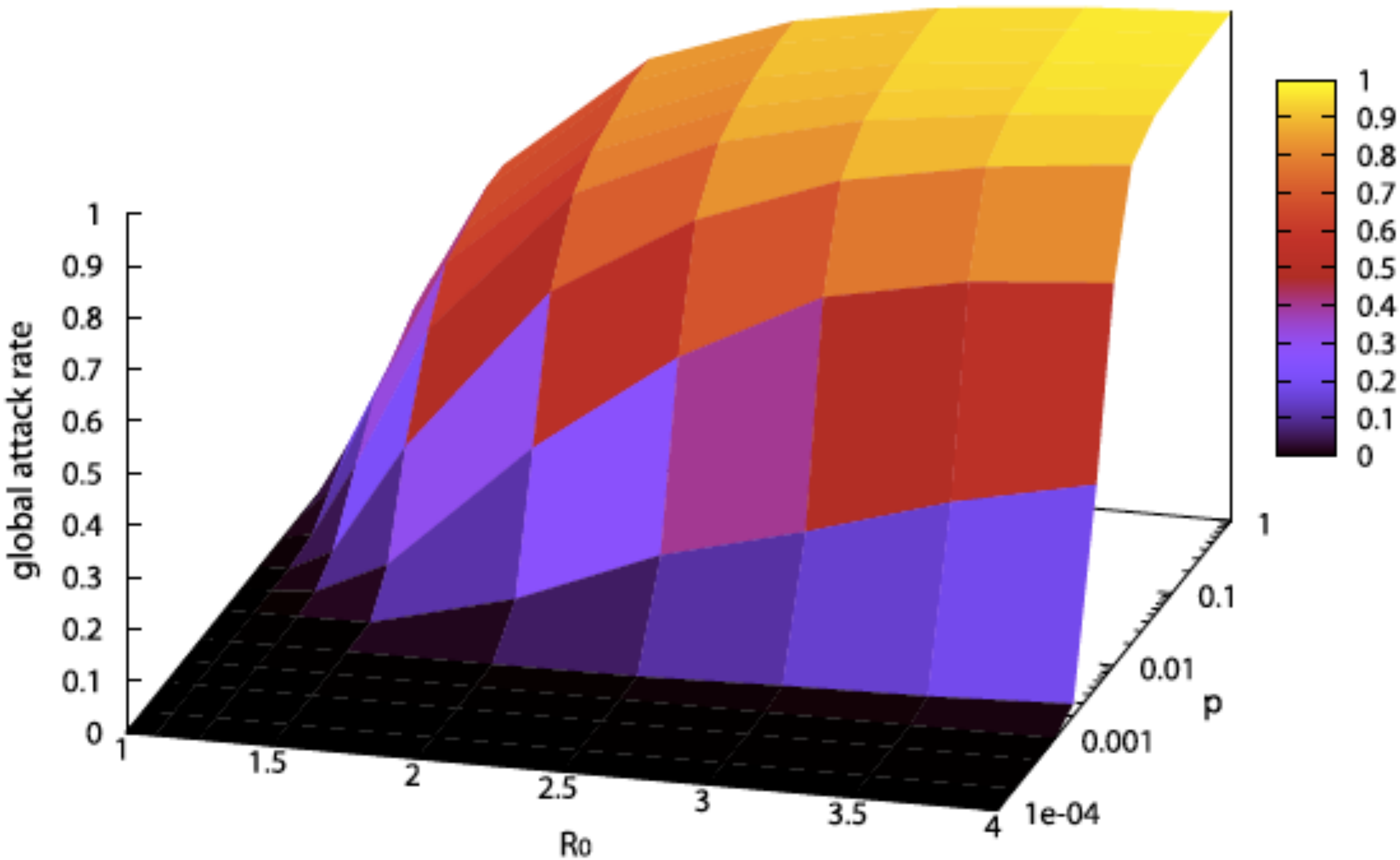
$$\langle k^{2+2\theta}\rangle - \langle k^{1+2\theta}\rangle \simeq 7\ 10^4$$
$$\langle k\rangle \simeq 10$$

invasion threshold: heterogeneous system



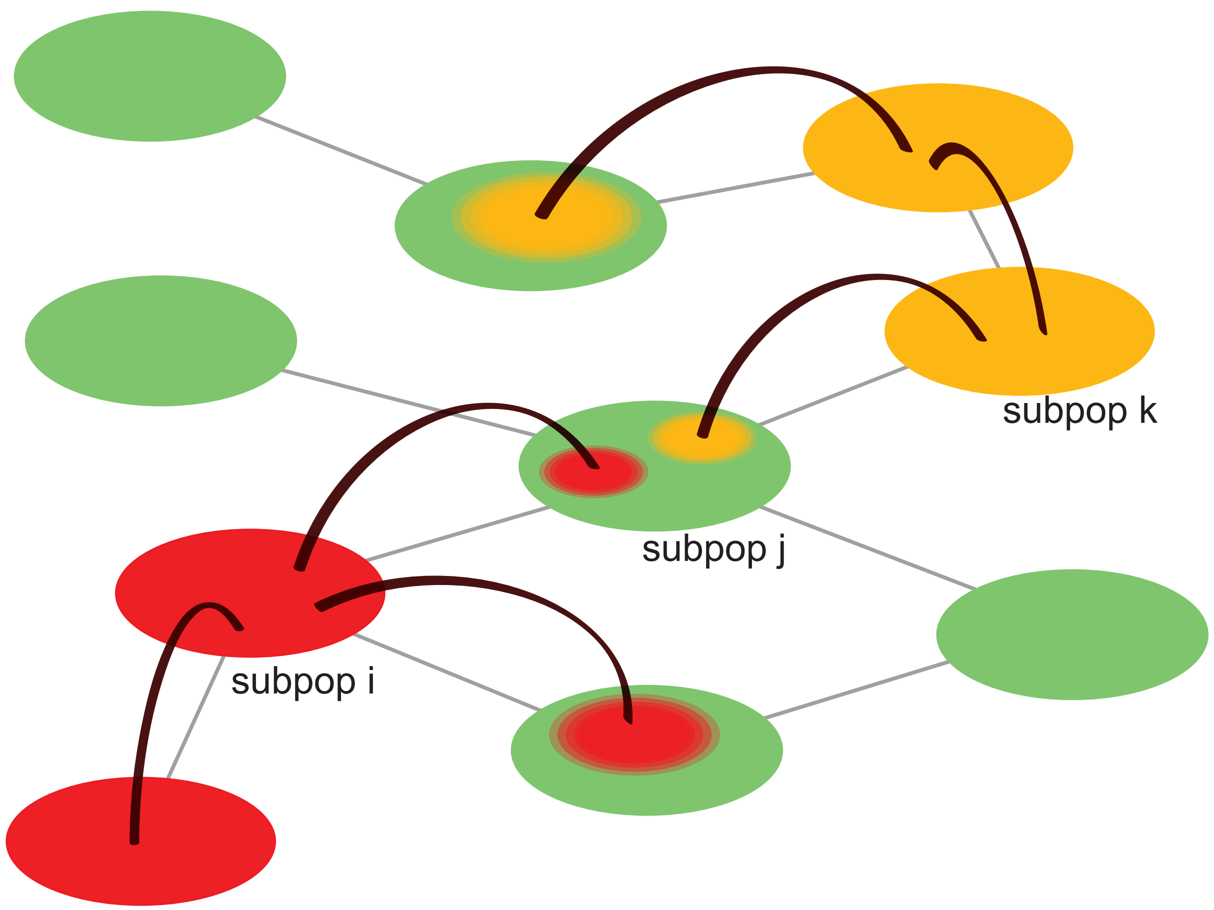
[Colizza & Vespignani, PRL 2007, JTB 2008]

invasion threshold: heterogeneous system



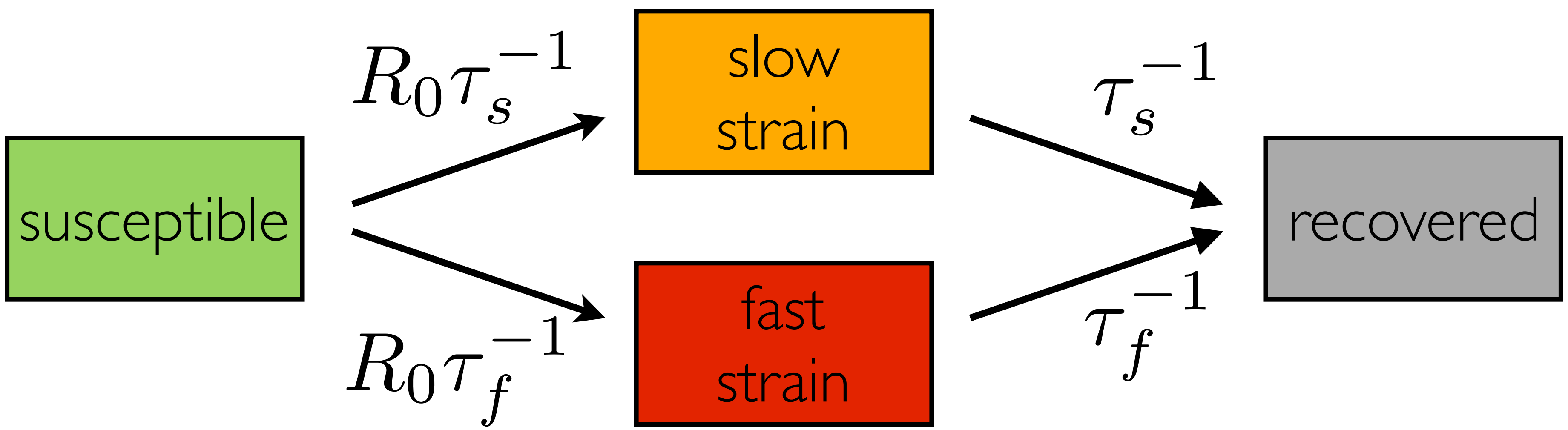
[Colizza & Vespignani, PRL 2007, JTB 2008]

competition between two strains at the spatial level



the model: 2strain transmission

- same R_0
- different infectious period → $\tau_s > \tau_f$
- full cross-immunity



[Poletto, PLOS Comp Biol PRL 2007, JTB 2008]

the model: 2strain transmission

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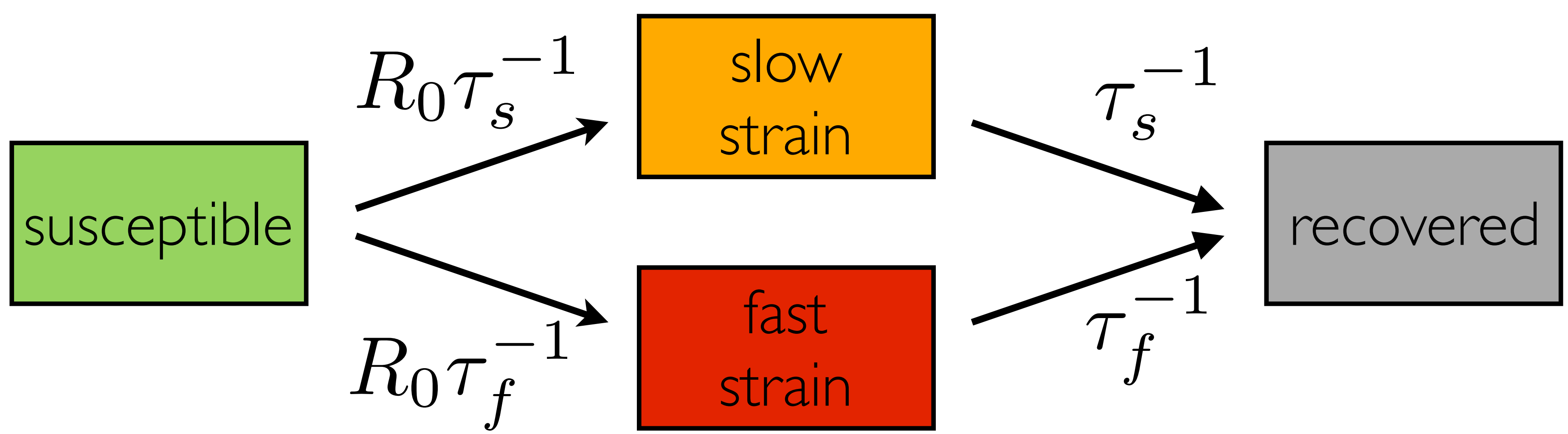
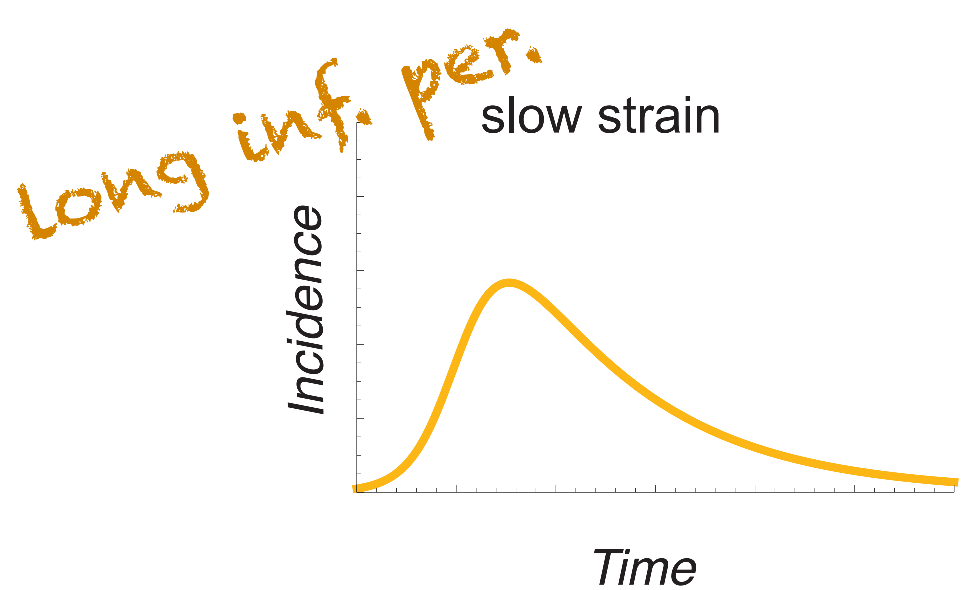
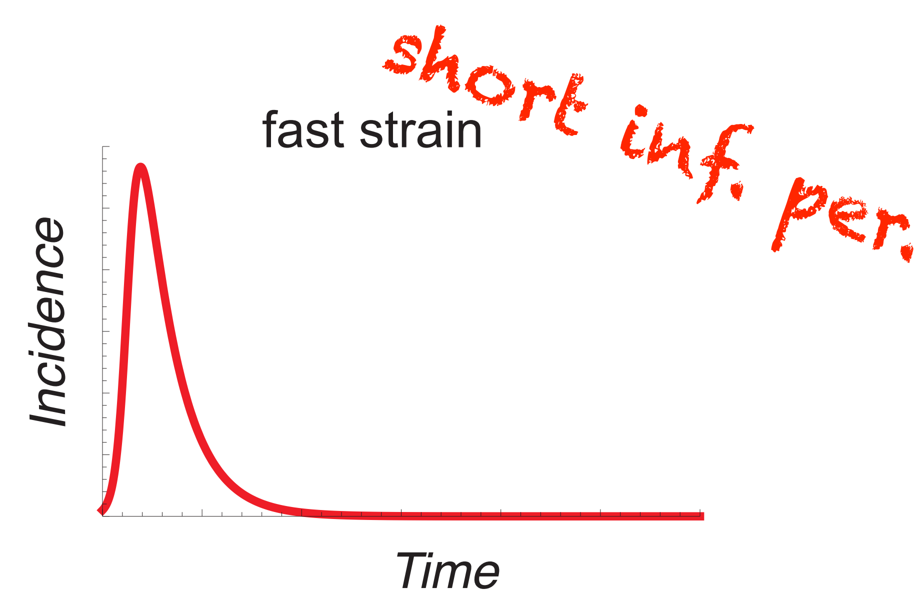
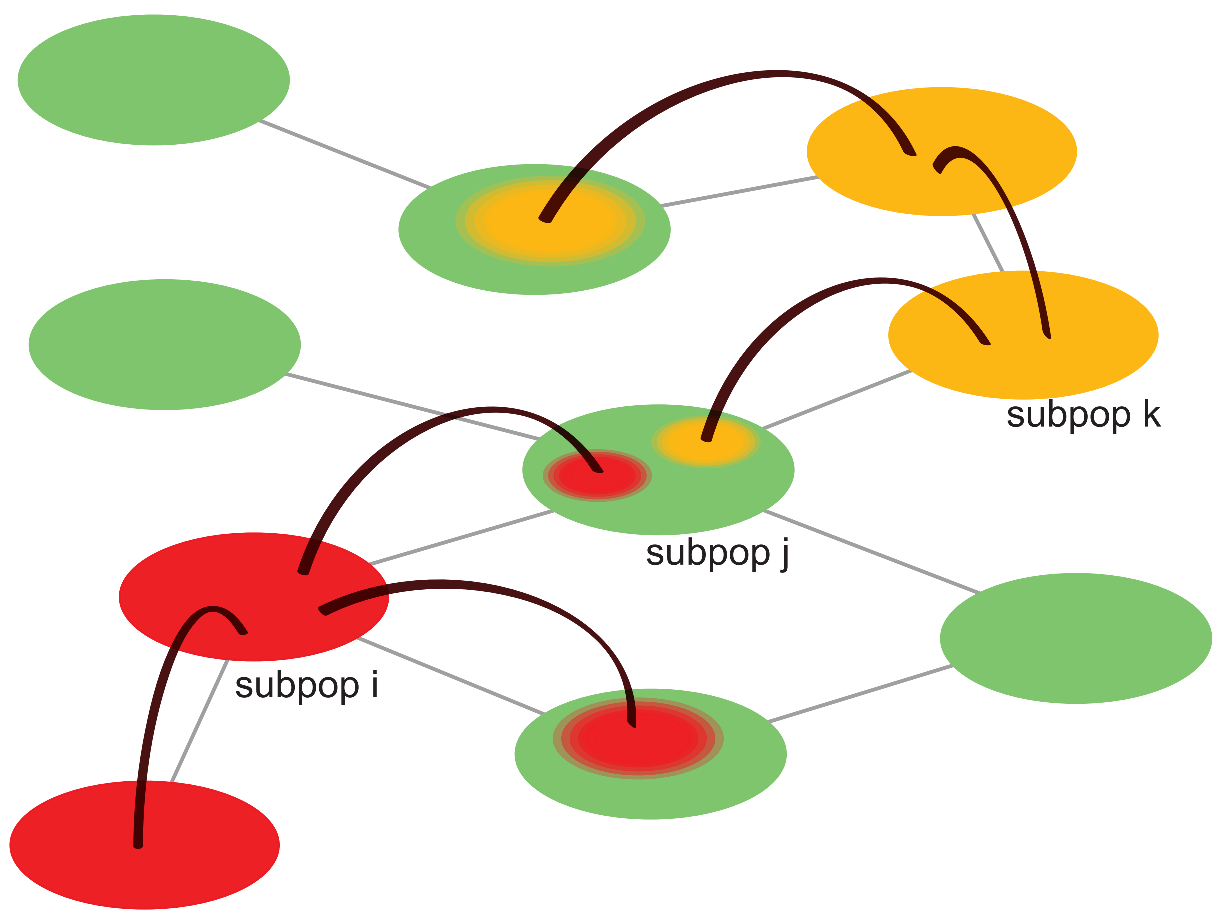




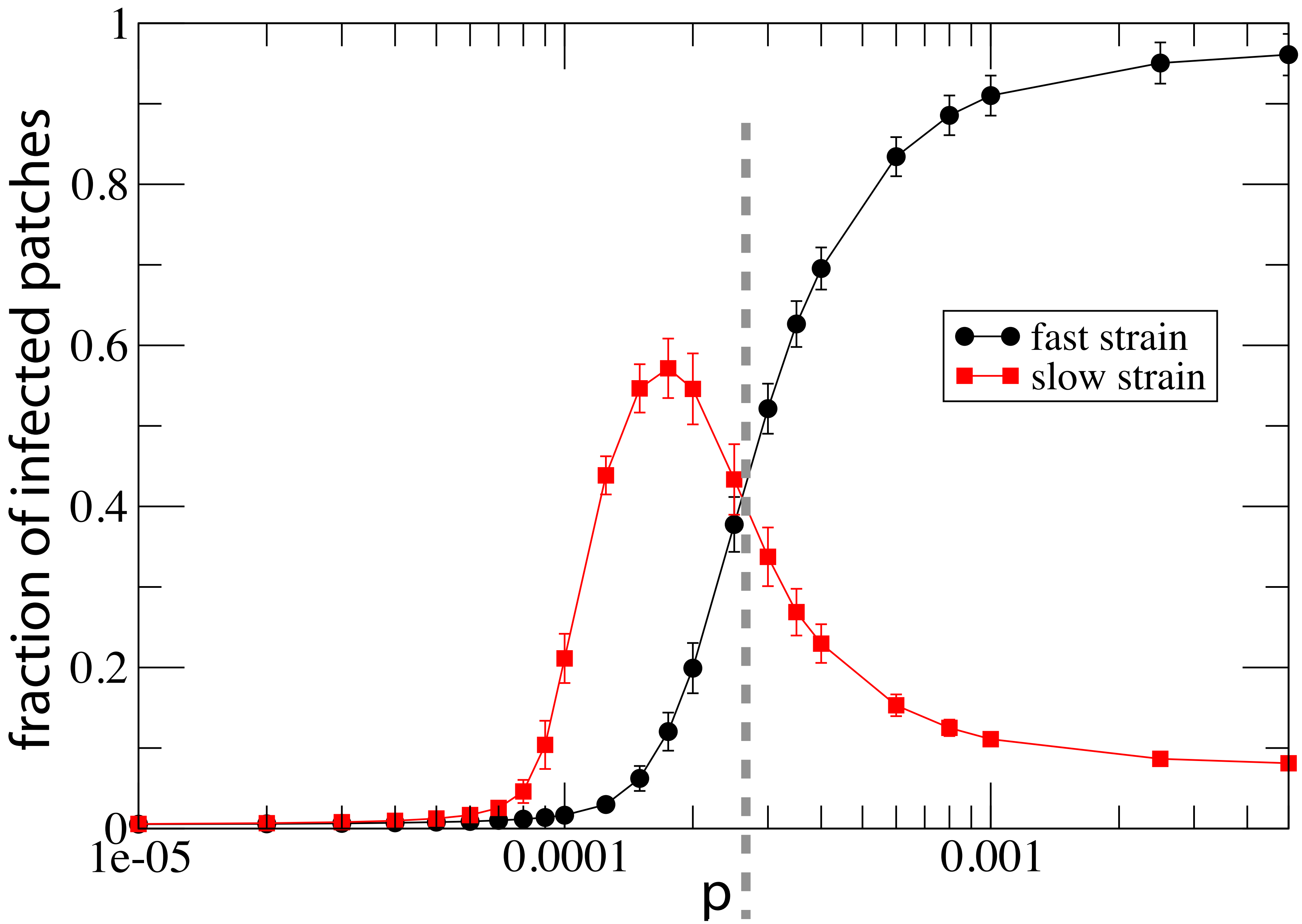


stochastic numerical simulations



- the 2 strains originate from different locations
- markovian dynamics
- probability of traveling: p

results: simulations

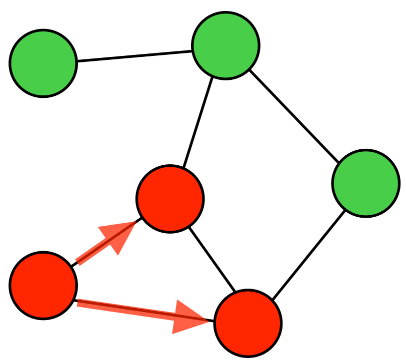


p_c the 2 strains coexist

results: analytical understanding

scale of individuals

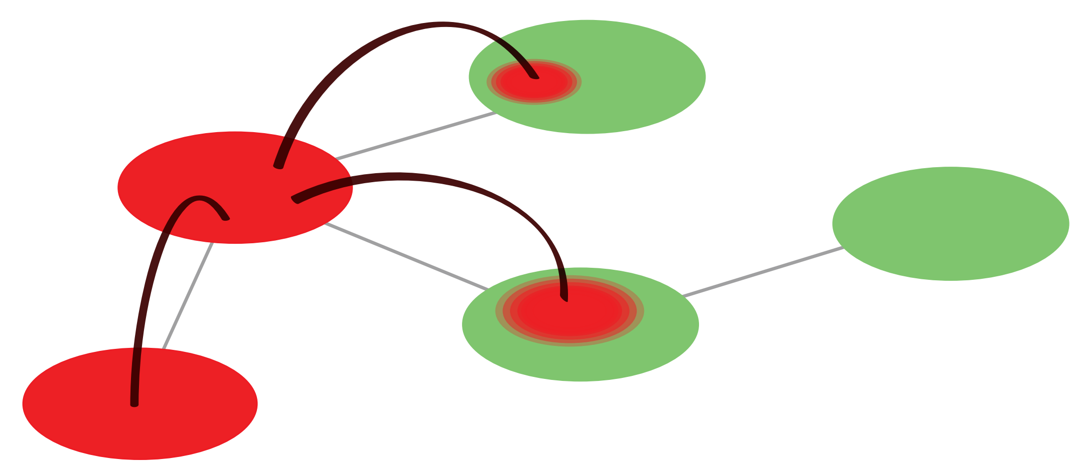
- μ^{-1} = infection duration
- R_0



$$I(t) \sim e^{\mu(R_0-1)t}$$

scale of patches

- T = outbreak duration
- R_*



D = # diseased patches

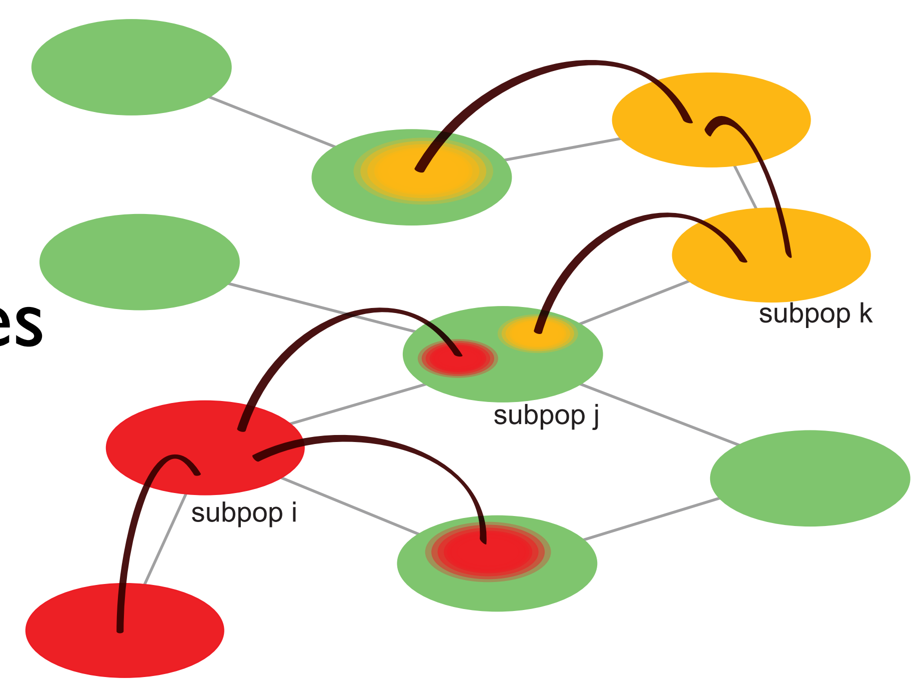
$$D(t) \sim e^{\frac{1}{T}(R_*-1)t}$$

$$R_* = (\bar{k}-1)\left[1-\left(\frac{1}{R_0}\right)^{\frac{pS_\infty}{\mu\bar{k}}}\right]$$

results: analytical understanding

$$R_* = (\bar{k}-1)\left[1-\left(\frac{1}{R_0}\right)^{\frac{pS_\infty}{\mu\bar{k}}}\right]$$

R_* increasing function of μ^{-1} $\Longrightarrow$ $R_*^s > R_*^f$



large *p*:

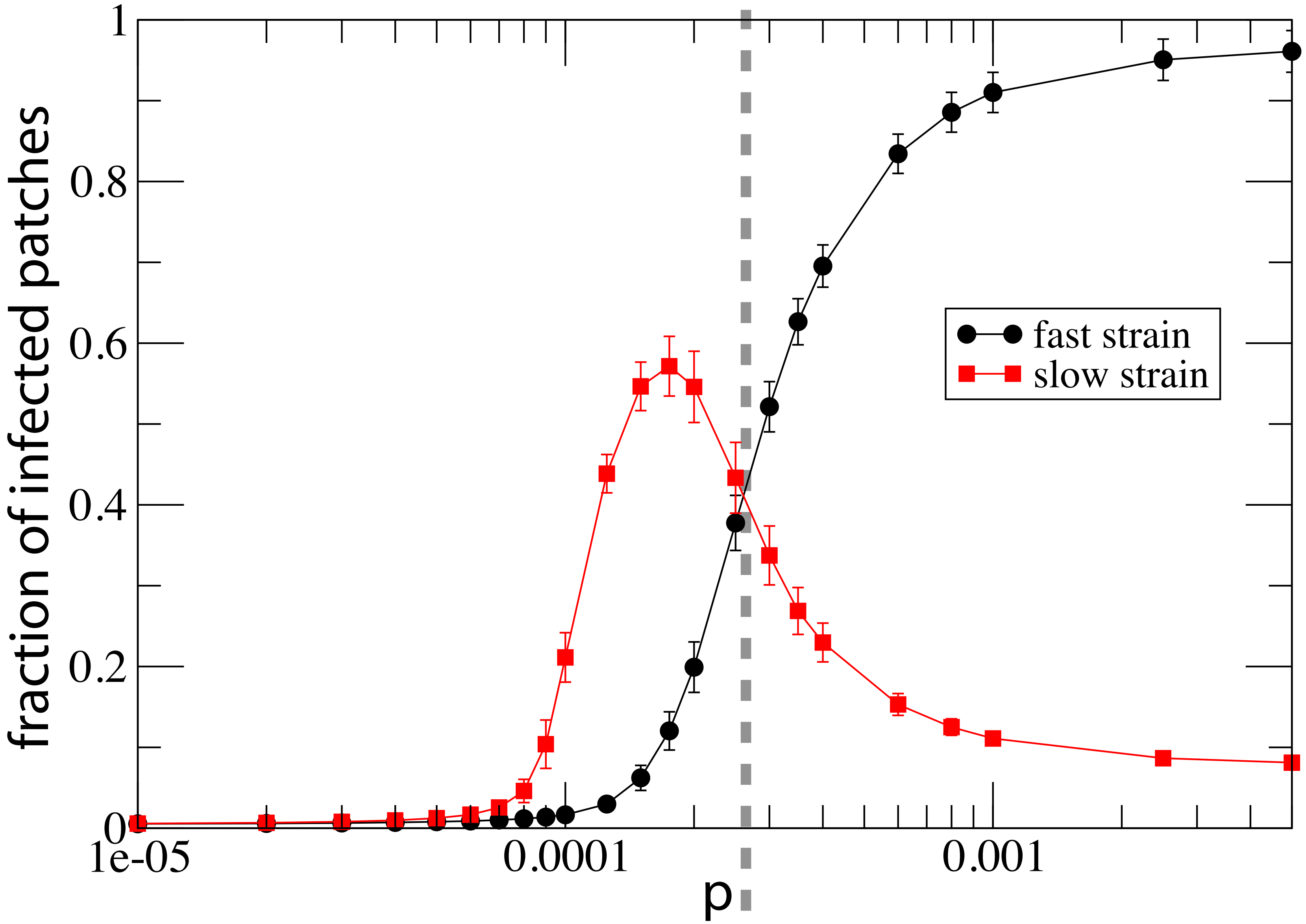
R_*^s and $R_*^f >> 1$

fast strain reaches more rapidly new patches

small *p*:

$R_*^s > R_*^f$ $\Longrightarrow$ more able to percolate

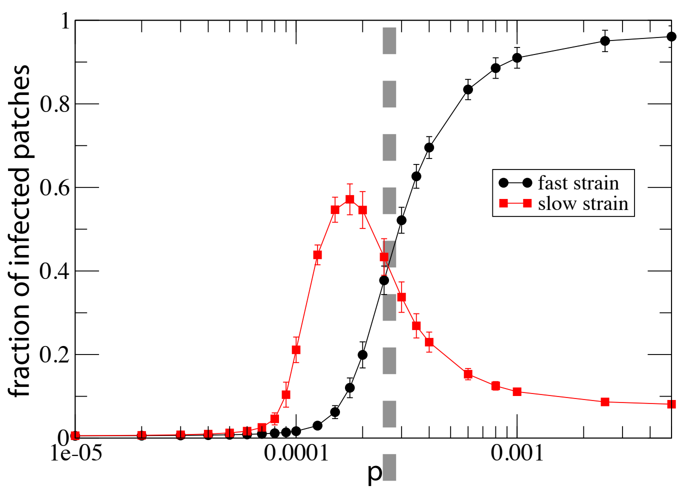
results: analytical understanding



p_c ?

$$\frac{D_s(t)}{D_f(t)} \sim e^{\left(\frac{(R_*^s-1)}{T_s} - \frac{\left(R_*^f-1\right)}{T_f}\right)t} = 1$$

results: analytical understanding



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