**Life Data Epidemiology exercises – 2019-20**
**University of Padova – L. Badia**

**Exercise 5** An online social network contains $N \approx 10^5$ users and has a degree distribution $p_k(k)$ as follows:

| $k$ | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
|---|---|---|---|---|---|---|---|
| $p_k$ | 0.6 | 0.2 | 0.1 | 0.05 | 0.02 | 0.007 | 0.003 |

(all the values of $k$ not reported in the table have $p_k = 0$)

A trend of replacing your profile picture with a flag is spreading over this network, following these rules:

- If a node decides to adopt the trend, it does so for an exponentially distributed time with average of one week; after that, the node restores the original profile picture and does not revert back to the flag
- Every day, for any pair of neighbors $(i, j)$ where $i$ is adopting the trend and $j$ is not, there is a probability $b = 0.02$ that $j$ adopts the trend from $i$

Answer the following questions:

1. What kind of epidemic model can describe this trend? Write down its differential equations of the compartmental model with homogeneous mixing.
2. For an average degree approximation: does the trend catch on the network?
3. For a block degree approximation: does the trend catch on the network?

(the trend "catches" on the network if we expect that a giant component of the nodes is adopting it).

**Exercise 6** In a plantation, trees are distributed according to a Poisson point process, so the number of trees in a radius of 100 meters is Poisson distributed with average 5 trees. A parasitic pest is spreading over the plantation and they can move from a tree to another. The spreading radius of the disease from a tree to another is 100 meters (note: this means that the average tree has 4 neighbors able to infect it). For each pair of trees in close proximity, the event that one tree infected by parasites infect the other one is drawn each day with probability $\beta = 1\%$. A fraction $q$ of the trees have flowers. Trees with flowers attract more parasites: they can reach the tree from a distance that is 3 times higher. Parasites infect a tree for ten days of duration, on average. A tree that is free from parasites, is susceptible again.

1. Write down the model for homogeneous mixing without considering the network structure.
2. Write down the model with either a degree-block approximation or a risk structure.
3. What is the minimum value of $q$ that causes the disease to spread?

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**Exercise 7** A disease is spreading over a population of 5000 individuals. The disease is spread through exchange of bodily fluids that can be represented with a proper contact network. We distinguish between *dangerous* (i.e., potentially infectious) contacts, and actually infectious contacts, where the disease actually spreads. The pattern of dangerous contacts can be adequately represented with the Erdős-Rényi model, where the average number of dangerous contacts among two network individuals is 8. Infected individuals can pass the disease to any other with whom they are engaging in a dangerous contact; this happens with probability $p = 0.4$. That is, 4 out of 10 dangerous contacts, determined with i.i.d distribution, are actually infectious if either of the contacted individuals is infectious.

1. Find the maximum degree of the network of dangerous contacts.
2. Find the average and maximum degree of the network of infectious contacts.
3. Find what is the fraction of random vaccinations that need to be performed to break the giant component of the network of dangerous contacts.