BEYOND THE DP UNIVERSALITY CLASS

(out-of-eq. phase transitions with absorbing phases)

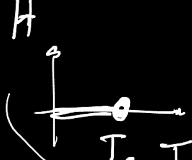
Eq. phase transitions | Non eq. phase transitions

- space dimension (d) ⟷ - space dimension (d)
- symmetry and dimensions of order parameter (→ spontaneous symmetry breaking) || - # of absorbing states
- conservation laws

in eq. conservation laws NOT relevant:
/ density is conserved (lattice gas)
\ magnetization not conserved (Ising)

out-of-eq. (with absorbing states) → NO spontaneous symmetry breaking (SSB)

[but SSB possible in *driven* phase transitions]



external symmetry breaking
/ eq. (Ising model) → go away from the critical point
\ NON EQ. → changing universality class

4 examples of universality classes beyond DP

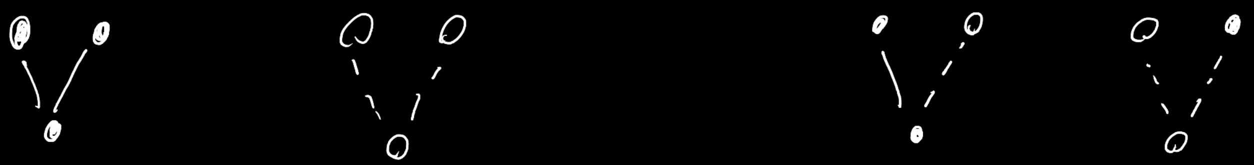
- CDP (Compact Directed Percolation)
- DP2 (DP with 2 absorbing inactive phases)
- DyP (Dynamical percolation)

↳ contact process with immunization (∞ # of absorbing states)

- PC (Parity Conserving model)

CDP ($q = 1$ in Domany-Kinzel CA)

symmetry between active ↔ inactive phase



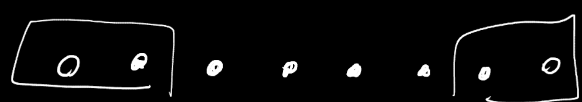
$q = 1$ prob. 1 p $1 - p$

2 absorbing states (1 active, 1 inactive)

invariance under ● → ○ AND $p \to 1 - p$

p_c determined by $p_c = 1 - p_c \to \boxed{p_c = 1/2}$

"active spots"
stretches of consecutive active sites



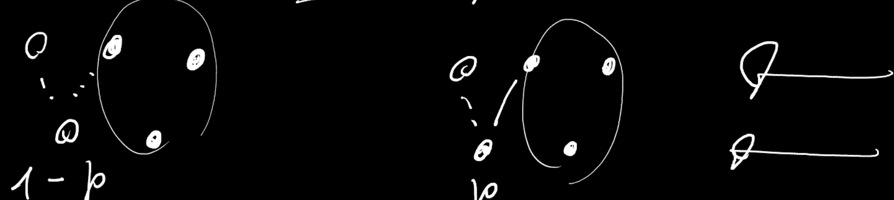
Domain walls Domain wall

$\underline{\underline{L = 6}}$

CDP → Dynamics of Domain Walls

→ <u>RW</u> dy<u>namics</u>

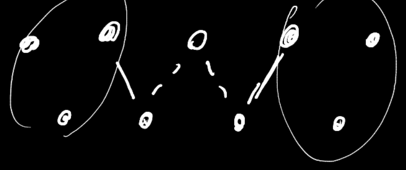
- Domain walls diffuse



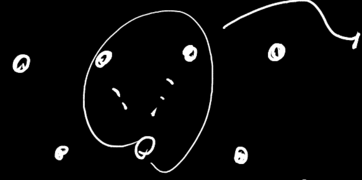
$1 - p$ (size of the active spot → $L - 1$) p ($L \to L + 1$)

- Domain walls can annihilate →

$(1-p)^2$ → <u>disappearance</u> of the active spots ($L \to 0$)

$\to$  $\to$ fusion of two active spots ($L = L_1 + L_2$)

p^2

- Domain walls cannot be created  $\to$ prob. $1-q$

$q = 1 \to$ this cannot happen

RW dynamics for $L(t)$: (just 1 active spot)

$$L(t+1) = \begin{cases} L(t)+1 & \text{with prob. } p^2 \\ L(t) & \text{with prob. } 2p(1-p) \\ L(t)-1 & \text{prob. } (1-p)^2 \end{cases}$$

with an absorbing boundary at $L = 0$ (absorbing trap)

RW in $[0, +\infty)$ with a trap at $L = 0$

- $p < p_c = 1/2 \to$ RW is positive-recurrent

(it goes back to $L = 0$ with prob. 1 in a time $\tau \sim 1/|\delta|$ (trapping time))

δ = asymmetry

$r = \dfrac{p^2}{p^2 + (1-p)^2}$

effective prob. of moving to the right ($L \to L+1$)

$\to \delta = r - (1-r) \overset{!}{=} \dfrac{1-2p}{2p(1-p)-1}$

$p < 1/2 \to \delta < 0$

- $p = p_c = 1/2$ ($\delta = 0$) RW is null-recurrent (recurrent but not positive recurrent)

$r = +\infty$; prob. 1 of getting trapped at $L = 0$

• $p > p_c = 1/2$ RW is transient

$(\delta > 0)$ prob. of getting trapped at $L = 0$

$$p_t = \frac{1 - r}{r}$$

NOT RECURRENT

$$\beta' \longrightarrow P(\underset{\text{infinite time}}{\infty}) \simeq (p - p_c)^{\beta'} \quad (p > p_c)$$

ρ

survival of an active spot with $L(0) = 1$ NOT TRAPPED (at $L = 0$)

$$P(\infty) = 1 - p_t \overset{!}{=} \frac{2}{p^2}(p - 1/2)$$

$$\Rightarrow \boxed{\beta' = 1}$$

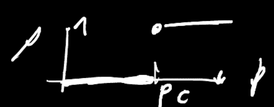
$\beta \to$ fraction of active sites $\quad \rho(\infty) \simeq (p - p_c)^{\beta} \quad p > p_c$

different active spots will merge $\to \rho(\infty) = 1$

$(p > p_c)$

$$\Rightarrow \boxed{\beta = 0}$$

(ρ is discontinuous at the transition)



$$\boxed{\beta \neq \beta'}$$

CDP

(in DP: $\beta = \beta'$)

$$\boxed{\vartheta} \longrightarrow p = p_c \quad L(t) \sim t^{\vartheta}$$

$$L(t) \sim \text{const} \to \boxed{\vartheta = 0}$$

$\nu_\perp, \nu_\parallel \rightarrow \nu_\parallel \quad p < p_c \quad \xi_\parallel \sim (p_c - p)^{-\nu_\parallel}$

correlation time = trapping time

$\tau \sim 1/|\delta| \sim \dfrac{1}{p_c - p} \Rightarrow \boxed{\nu_\parallel = 1}$

$\nu_\parallel - \xi_\perp \sim (p_c - p)^{-\nu_\perp}$

RW scaling (diffusion) $\quad \xi_\perp \sim \xi_\parallel^{1/2} \sim (p_c - p)^{-1/2}$

$\Rightarrow \boxed{\nu_\perp = 1/2} \rightarrow \underbrace{z = \dfrac{\nu_\parallel}{\nu_\perp} = 2}$

exponents for CDP are the same as DP mean field values EXCEPT for $\beta = 0$ $(\beta \neq \beta')$

<u>DP2</u> (2 absorbing inactive phases)
symmetric

symmetry between inactive phases broken

$\rightarrow$ back to DP universality class

2 inactive states I_1, I_2 ; active states A

$P(I_k \mid I_k, I_k) = 1$ (INACTIVE STATES!)

$P(A \mid I_k, A) = p \quad (\Rightarrow P(I_k \mid I_k, A) = 1 - p)$

$P(A \mid A, A) = q \; ; \; P(I_1 \mid A, A) = P(I_2 \mid A, A) = \dfrac{1-q}{2}$

symmetry between I_1, I_2

$P(A|I_1, I_2) = P(A|I_2, I_1) = 1 \rightarrow$ "interfacial noise"

asymmetry $\left(P(I_1|A,A) > P(I_2|A,A) \right)$

$\Rightarrow$ go back to DP

- **PC** parity conserving

DP $\rightarrow$ no conservation law

↳ $(A \rightarrow A + A)$; $(A + A \rightarrow A)$

offspring generation ; coalescence

$(A\emptyset \leftrightarrows \emptyset A)$ diffusion

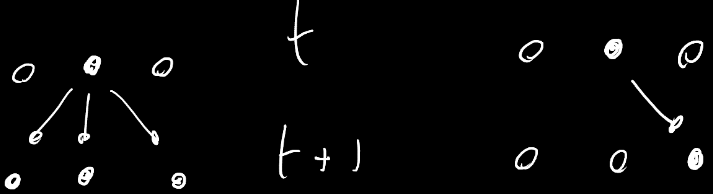
$\boxed{PC}$: modify dynamics to conserve the parity of the number of active sites

$(A \rightarrow A + A + A)$; $(A + A \rightarrow 0)$

generation ; annihilation

+ diffusion (same as DP)

New tilted lattice

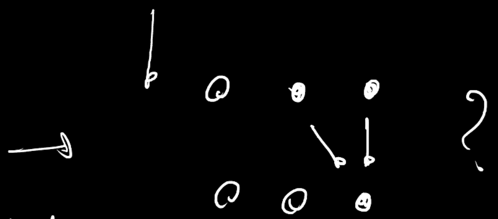


diffusion with nearby site already occupied

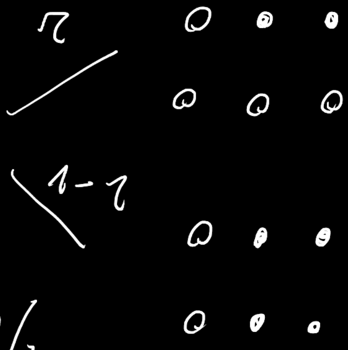
↓

generation with prob. $1-p$

diffusion with prob. $p/2$ (to the right) $p/2$ (to the left)



diffusion event already selected with prob. $p/2$



annihilation

don't do anything