2021-12-17

giovedì 16 dicembre 2021 18:51

ALCUNI ESERCIZI

1) RISOLVI I SEGUENTI SISTEMI

- $\begin{cases} x_2 + x_3 = 1 \\ x_1 - x_2 = 0 \\ x_1 + x_3 = 2 \end{cases}$ (IMPOSSIBILE)

- $\begin{cases} x_1 + x_3 = 2 \\ 2x_2 + x_3 = 4 \\ x_1 + 2x_2 = 6 \end{cases}$ $S = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

- $\begin{cases} x_1 + x_3 = 2 \\ 2x_2 - 2x_3 = -2 \\ x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 2 \end{cases}$ $S = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

2) RICAVA UNA BASE E CALCOLA LA DIMENSIONE DEL SEGUENTE SOTTOSPAZIO di $\mathbb{R}[x]_{\leq 4}$

$W = \langle 2x^3 + x,\ x^4 - 2x^2,\ 2x,\ x^4 - 2x^2 + 2x \rangle$

$\left(\dim W = 3, \quad B = \{ 2x^3 + x,\ x^4 - 2x^2,\ 2x \} \right)$

3) DATI I SOTTOSPAZI W e U di $\mathbb{R}^5$ DETERMINARE UNA LORO BASE, E I SOTTOSPAZI $U \cap W$ e $U + W$

$$W = \left\langle \begin{pmatrix}1\\2\\0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\-1\\0\\0\end{pmatrix} \right\rangle \qquad U : \begin{cases} x_4 - x_5 = 0 \\ x_1 - x_2 = 0 \end{cases}$$

RISPOSTA: Di W abbiamo già una base

$$U = \left\langle \begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\1\\1\end{pmatrix} \right\rangle$$

$$U \cap W = \left\langle \begin{pmatrix}1\\1\\-1\\1\\1\end{pmatrix} \right\rangle$$

$$U + W = \mathbb{R}^5$$

(4) CALCOLA IL DETERMINANTE DI

$$A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix} \qquad (R = 0)$$

(5) DETERMINARE $\operatorname{Ker}(f)$, $\operatorname{Im}(f)$ e la MATRICE ASSOCIATA NELLE BASI CANONICHE

$$f : \mathbb{R}^4 \to \mathbb{R}^3$$

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} \mapsto \begin{pmatrix}x_1 + x_2 + x_4 \\ x_2 + x_3 \\ x_1 - x_3 + x_4\end{pmatrix}$$

R: $\operatorname{Ker}(f) = \left\langle \begin{pmatrix}1\\-1\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\0\\1\end{pmatrix} \right\rangle$

$$Im(f) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\rangle$$

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{pmatrix}$$

⑥ DATA $f: \mathbb{R}^3 \to \mathbb{R}^3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} 3x_1 - x_2 \\ 2x_2 + x_3 \\ 3x_1 + x_2 + x_3 \end{pmatrix}$$

Scrivere la matrice $A = M_{\mathcal{E}_3}^{\mathcal{E}_3}(f)$ e $B = M_{\mathcal{E}_3}^{\mathcal{C}}(f)$

con $\mathcal{E}_3$ BASE CANONICA e

$$\mathcal{C} = \{ e_1 + e_3,\ e_2 + e_3,\ 2e_1 + e_2 \}$$

R: $A = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

⑦ $\mathcal{B} = \{ e_1 + e_2,\ e_1 - e_2 \}$ base di $\mathbb{R}^2$

$\mathcal{C} = \{ e_1,\ e_1 + e_2,\ e_1 + e_2 + e_3 \}$ base di $\mathbb{R}^3$

$f: \mathbb{R}^2 \to \mathbb{R}^3$

$f(e_1) = 2e_1 + 3e_2 + e_3$

$f(e_2) = e_1 + e_2 + e_3$

DARE $A = M_{\mathcal{B}}^{\mathcal{C}}(f)$

R: $A = \begin{pmatrix} -1 & -1 \\ 2 & 2 \\ 2 & 0 \end{pmatrix}$

$\mathbb{R}^3$ $v_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$ $v_3 = \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix}$

$$v_1 \cdot v_2 = 0 - 2 - 3 = -5$$

$$\|v_1\| = \sqrt{v_1 \cdot v_1} = \sqrt{\sum x_i^2} = \sqrt{1+4+1} = \sqrt{6}$$

$$\cos\vartheta = \frac{v_1 \cdot v_2}{\|v_1\|\,\|v_2\|} \overset{*}{=} \frac{-\cancel{5}\sqrt{5}}{\sqrt{6}\cdot\sqrt{\cancel{10}\,2}} = -\frac{1}{2}\sqrt{\frac{5}{3}}$$

ϑ angolo compreso tra v_1 e v_2 $\quad \vartheta \in [0, \pi]$

$$* \; -\frac{5}{\sqrt{6}\sqrt{10}} = -\sqrt{\frac{\cancel{25}\,5}{6 \cdot \cancel{10}\,2}} = -\sqrt{\frac{5}{2^2 \cdot 3}} = -\frac{\sqrt{5}}{2\sqrt{3}}$$

$$\cos\alpha = \frac{v_2 \cdot v_3}{\|v_2\|\,\|v_3\|} = \frac{0-1-6}{\sqrt{10}\cdot\sqrt{14}} = -\frac{7}{\sqrt{10}\sqrt{14}} =$$

α angolo tra v_2 e v_3

$$= -\sqrt{\frac{\cancel{49}\,7}{5 \cdot 2 \cdot \cancel{14}\,2}} = -\frac{1}{2}\cdot\sqrt{\frac{7}{5}}$$

$$\cos\beta = \frac{v_1 \cdot v_3}{\|v_1\|\,\|v_3\|} = \frac{-3+2+2}{\sqrt{6}\cdot\sqrt{14}} = \frac{1}{2\sqrt{21}}$$

β angolo tra v_1 e v_3

$$\left(\cos\vartheta = \frac{v \cdot w}{\|v\|\,\|w\|} \right)$$

$$\mathbb{R}^5 \qquad U = \left\langle \begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}2\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\1\\0\end{pmatrix} \right\rangle$$

1) ORTONORMALIZZARE UNA BASE DI U

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \xrightarrow[\text{III}-2\text{I}]{\text{II}-\text{I}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{III}-\text{II}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \xrightarrow{\text{IV}+\text{III}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$rk = 4$

$$\mathcal{B} = \left\{ \underset{u_1}{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}}, \underset{u_2}{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}, \underset{u_3}{\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}, \underset{u_4}{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}} \right\}$$

$$v_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$$

$$v_2' = u_2 - (u_2 \cdot v_1)\, v_1$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 \\ 1 \\ -1/2 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} \qquad \sqrt{\frac{1}{4} + 1 + \frac{1}{4}} = \frac{\sqrt{6}}{2}$$

$$\begin{pmatrix} -1/2 \\ 0 \\ 0 \end{pmatrix} \qquad 2 \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \qquad \sqrt{\frac{1}{4}+1+\frac{1}{4}} = \frac{\sqrt{6}}{2}$$

$$v_2 = \frac{v_2'}{\|v_2'\|} = \cancel{\frac{1}{2}}\,\frac{\cancel{2}}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

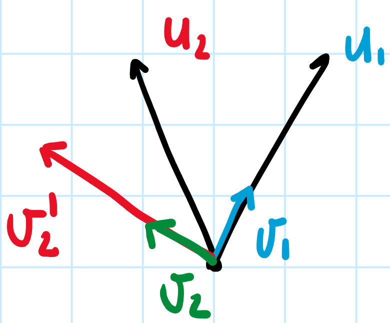
NB

$$w = \begin{pmatrix} \alpha a \\ \alpha b \\ \alpha c \end{pmatrix} = \alpha \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\|w\| = \sqrt{\alpha^2 a^2 + \alpha^2 b^2 + \alpha^2 c^2} = \alpha \sqrt{a^2+b^2+c^2}$$

$$v_3' = u_3 - (u_3 \cdot v_1)\,v_1 - (u_3 \cdot v_2)\,v_2$$

$$= \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{\cancel{2}}{\cancel{\sqrt{2}}} \cdot \frac{1}{\cancel{\sqrt{2}}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \frac{4}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$



$$= \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \frac{2}{3} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 \\ -1/3 \\ -1/3 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$v_3 = \frac{v_3'}{\|v_3'\|} = \frac{\cancel{3}}{\sqrt{3}} \cdot \frac{1}{\cancel{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$v_3 = \frac{v_3'}{\|v_3'\|} = \frac{\cancel{3}}{\sqrt{3}} \cdot \frac{1}{\cancel{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$v_4' = u_4 - (u_4 \cdot v_1) v_1 - (u_4 \cdot v_2) v_2 - (u_4 \cdot v_3) v_3$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \left(-\frac{1}{\sqrt{6}}\right) \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} - \left(-\frac{1}{\sqrt{3}}\right) \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{6} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad v_4 = \frac{v_4'}{\|v_4\|} = v_4'$$

$$B' = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

b) Determinare una base per $U^\perp$

$$w = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \quad \text{tc} \quad \begin{cases} w \cdot v_1 = 0 \\ w \cdot v_2 = 0 \\ w \cdot v_3 = 0 \\ w \cdot v_4 = 0 \end{cases}$$

$$\begin{cases} x_1+x_3=0 & ; \quad x_1=-x_3 \quad ; \quad x_3=0 \\ x_1+2x_2-x_3=0 & ; \quad 2x_1+2x_2=0 \quad ; \quad x_1=-x_2 \quad ; \quad x_2=0 \\ x_1-x_2-x_3=0 & ; \quad x_1+x_1+x_1=0 \quad ; \quad x_1=0 \\ x_4=0 \end{cases}$$

$$W = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ x_5 \end{pmatrix} \qquad U^\perp = \left\langle \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$\mathbb{R}^3$

$$U = \left\langle \underset{u_1}{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}, \underset{u_2}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}} \right\rangle$$

1) TROVARE UNA BASE ORTONORM.

$$v_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$v_2' = u_2 - (u_2 \cdot v_1)\, v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \\ -1/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$v_2 = \frac{v_2'}{\|v_2'\|} = \frac{\cancel{2}}{\sqrt{6}} \cdot \frac{1}{\cancel{2}} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

- DETERMINARE UNA BASE DI $U^\perp$

$$B = \left\{ \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\-1\end{pmatrix} \right\}$$

$$w = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \quad \text{t.c.} \quad \begin{cases} x_1 + x_3 = 0 \\ x_1 + 2x_2 - x_3 = 0 \end{cases} \quad \begin{cases} x_1 = -x_3 \\ 2x_2 - 2x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = -x_3 \\ x_2 = x_3 \end{cases}$$

$$w = \begin{pmatrix}-x_3\\x_3\\x_3\end{pmatrix} \qquad U^\perp = \left\langle \begin{pmatrix}-1\\1\\1\end{pmatrix} \right\rangle = \left\langle \frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\1\end{pmatrix} \right\rangle$$

- MATRICE DELLA PROIEZIONE SU U (Base canonica)

$$\mathbb{R}^3 = U \oplus U^\perp = \left\langle \overset{v_1}{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}}, \overset{v_2}{\frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\-1\end{pmatrix}} \right\rangle \oplus \left\langle \overset{w}{\frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\1\end{pmatrix}} \right\rangle$$

$$e_1 = (e_1 \cdot v_1)\, v_1 + (e_1 \cdot v_2)\, v_2 + (e_1 \cdot w)\, w$$

$$= \frac{1}{\sqrt{2}} \cdot v_1 + \frac{1}{\sqrt{6}} v_2 - \frac{1}{\sqrt{3}} w$$

$$\pi_U^{U^\perp}(e_1) = \frac{1}{\sqrt{2}} v_1 + \frac{1}{\sqrt{6}} v_2 = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix} + \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\-1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1\\0\\1\end{pmatrix} + \frac{1}{6}\begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}2/3\\1/3\\1/3\end{pmatrix} \quad \text{PRIMA COLONNA}$$

$$e_2 = (e_2 \cdot v_1)\, v_1 + (e_2 \cdot v_2)\, v_2 + (e_2 \cdot w)\, w$$

$$\pi_U^{U^\perp}(e_2) = (e_2 \cdot v_1)v_1 + (e_2 \cdot v_2)v_2$$

$$= 0 + \frac{2}{\sqrt{6}} v_2 = \frac{2}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \\ -1/3 \end{pmatrix}$$

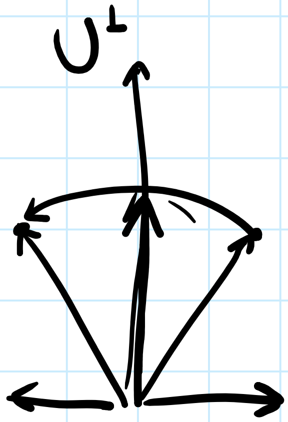
$$\pi_U^{U^\perp}(e_3) = (e_3 \cdot v_1)v_1 + (e_3 \cdot v_2)v_2$$

$$= \frac{1}{\sqrt{2}} v_1 - \frac{1}{\sqrt{6}} v_2 = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{\sqrt{6}} \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{6} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ -1/3 \\ 2/3 \end{pmatrix}$$

$$M_{\mathcal{E}_3}^{\mathcal{E}_3}(\pi_U^{U^\perp}) = \begin{pmatrix} 2/3 & 1/3 & 1/3 \\ 1/3 & 2/3 & -1/3 \\ 1/3 & -1/3 & 2/3 \end{pmatrix}$$

- MATRICE RIFLESSIONE DI ASSE $U^\perp$



$\mathbb{R}^2$