2022-01-14

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$\mathbb{A}^3(\mathbb{R})$

piano M: $x-y-z=1$

$$\begin{pmatrix}1\\0\\0\end{pmatrix} + \left\langle \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix} \right\rangle = \begin{pmatrix}1+\lambda+\mu\\ \lambda\\ \mu\end{pmatrix}$$

retta r: $\begin{pmatrix}1\\-1\\0\end{pmatrix} + \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix} \right\rangle = \begin{pmatrix}1+a\\-1\\a\end{pmatrix}$ $\begin{cases} x-z=1 \\ y=-1 \end{cases}$

retta t: $\begin{cases} x-y=1 \\ z-y=1 \end{cases}$ $\begin{pmatrix}1\\0\\1\end{pmatrix} + \left\langle \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rangle = \begin{pmatrix}1+b\\b\\1+b\end{pmatrix}$

punto $P = \begin{pmatrix}1\\1\\0\end{pmatrix}$

a) Determinare $M \cap r$ e $M \cap t$

$$\begin{cases} x-y-z=1 \\ x-z=1 \\ y=-1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{II-I} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{III-II} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

$rk(A) = 2$

$rk(A|b) = 3$

$r // M$ e $r \notin M$

$$\begin{cases} x-y-z=1 \\ x-y=1 \\ z-y=1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{\text{II}-\text{I}} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{array}\right) \xrightarrow{\text{II} \leftrightarrow \text{III}} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$rk(A) = rk(A|b) = 3$$

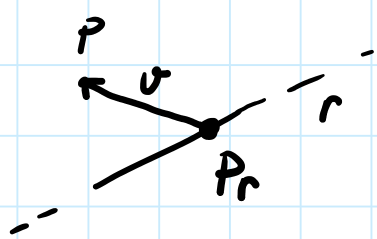
$$\begin{cases} x-y-z=1 \\ -y+z=1 \\ z=0 \end{cases} \qquad \begin{cases} x=0 \\ y=-1 \\ z=0 \end{cases} \qquad M \cap t = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

b) Determinare il piano M' contiene r sia P

$$r = \underbrace{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}_{P_r} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle \qquad P = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{cases} x-z=1 \\ y=-1 \end{cases} \qquad P \notin r \text{ infatti } \begin{cases} 1-0=1 \\ 1=-1 \quad \times \end{cases}$$

$$P - P_r = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = v$$



$$M' = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \right\rangle = \begin{pmatrix} 1+c \\ -1+2d \\ c \end{pmatrix}$$

$$\begin{cases} x = 1+c \\ y = -1+2d \\ z = c \end{cases}$$

$x - z = 1$ eq. cart.

dim Sp − dim Var = n° eq.

d) Calcolare $M \cap M'$

$$\begin{cases} x-y-z=1 \\ x-z=1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{II-I} \left(\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right) \qquad rk(A) = rk(A|b) = 2$$

$$\begin{cases} x-y-z=1 \\ y=0 \end{cases} \qquad \begin{cases} x-z=1 \\ y=0 \end{cases}$$ eq. cart. retta

$$\begin{cases} x = 1+z \\ y = 0 \end{cases} \qquad \begin{pmatrix} 1+z \\ 0 \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

e) Verificare che r e t sono sghembe e determinare due piani A, B paralleli (A//B) tc $r \subset A$ e $t \subset B$

$$\begin{cases} x-z=1 \\ y=-1 \\ x-y=1 \\ z-y=1 \end{cases} \qquad \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{III-I} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right. \xrightarrow[IV+II]{III+II} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right.$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{array}\right) \xrightarrow{IV+II} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{IV-III} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$rk(A)=3$

$rk(A|b)=4$

$\Rightarrow$ SGHEMBE

$$r: \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle \qquad A: \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$t: \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle \qquad B: \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$A // B \Rightarrow$ avere la stessa giacitura

ESERCIZIO (APPELLO 10/02/2020)

$\mathbb{E}^3(\mathbb{R}) \qquad \pi: 4x-3z=1$

1) Eq. cartesiana e parametrica di r passante per $P=\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ e $\perp \pi$

$$r: \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \right\rangle \leftarrow \text{coef. eq. di } \pi$$

$\pi: 4x-3z=1 \quad ; \quad x = \frac{3}{4}z + \frac{1}{4}$

$$\begin{pmatrix} \frac{3}{4}z + \frac{1}{4} \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/4 \\ 0 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3/4 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad = \begin{pmatrix} 1/4 \\ 0 \\ 0 \end{pmatrix} + \left\langle \underset{w_1}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}, \underset{w_2}{\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}} \right\rangle$$

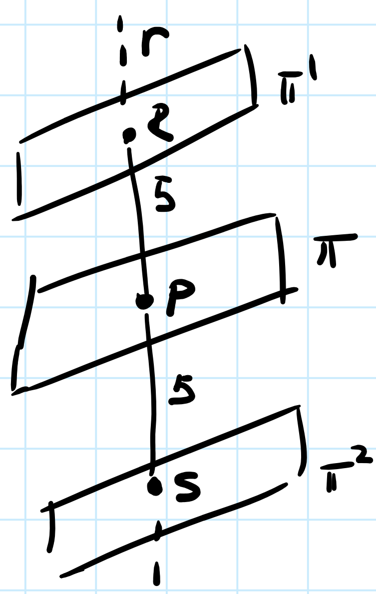
$$\begin{cases} v \cdot w_1 = 0 \\ v \cdot w_2 = 0 \end{cases} \qquad \begin{cases} b = 0 \\ 3a + 4c = 0 \end{cases} \qquad \begin{cases} b = 0 \\ a = -4/3\, c \end{cases}$$

$$v = \begin{pmatrix} -4/3\, c \\ 0 \\ c \end{pmatrix} = \left\langle \begin{pmatrix} -4/3 \\ 0 \\ 1 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \right\rangle$$

2) Determinare la giacitura U di π, dare una base di U e le eq parametriche dei piani π' e π^2 // π tc $d(\pi', \pi) = d(\pi^2, \pi) = 5$

$$\pi : \begin{pmatrix} 1/4 \\ 0 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$

$$B_U = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\}$$



$P = r \cap \pi$

$$r : \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \right\rangle = \begin{pmatrix} 1 + 4a \\ 0 \\ 1 - 3a \end{pmatrix}$$

$$r: \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} \right\rangle = \begin{pmatrix} 1+4a \\ 0 \\ 1-3a \end{pmatrix}$$

$$\begin{cases} x = 1+4a \\ y = 0 \\ z = 1-3a \end{cases} \qquad \begin{cases} x = 1 + 4/3 - 4/3\, z \\ y = 0 \\ a = 1/3 - z/3 \end{cases}$$

$$\begin{cases} x = 7/3 - 4/3\, z \\ y = 0 \\ * \end{cases} \qquad \begin{cases} 3x + 4z = 7 \\ y = 0 \\ * \end{cases}$$

$$P = r \cap \pi : \quad \begin{cases} 4x - 3z = 1 \\ 3x + 4z = 7 \\ y = 0 \end{cases} \qquad \left(\begin{array}{ccc|c} 4 & 0 & -3 & 1 \\ 3 & 0 & 4 & 7 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{II - \frac{3}{4} I} \left(\begin{array}{ccc|c} 4 & 0 & -3 & 1 \\ 0 & 0 & 25/4 & 25/4 \\ 0 & 1 & 0 & 0 \end{array}\right) \xrightarrow{III \leftrightarrow II} \left(\begin{array}{ccc|c} 4 & 0 & -3 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 25/4 & 25/4 \end{array}\right)$$

$$\begin{cases} 4x - 3z = 1 \\ y = 0 \\ \frac{25}{\cancel{4}} z = \frac{25}{\cancel{4}} \end{cases} \qquad \begin{cases} x = 1 \\ y = 0 \\ z = 1 \end{cases} \qquad P = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$R, S \in r \quad \text{tc} \quad d(R,P) = d(S,P) = 5$$

$$R, S = \begin{pmatrix} 1+4a \\ 0 \\ 1-3a \end{pmatrix} \qquad R - P = \begin{pmatrix} \cancel{1}+4a-\cancel{1} \\ 0-0 \\ \cancel{1}-3a-\cancel{1} \end{pmatrix}$$

$$\left\| \begin{pmatrix} 4a \\ 0 \\ -3a \end{pmatrix} \right\| = 5 \quad ; \quad \sqrt{16a^2 + 9a^2} = 5$$

$$25a^2 = 25$$

$$a^2 = 1 \quad ; \quad a = \pm 1$$

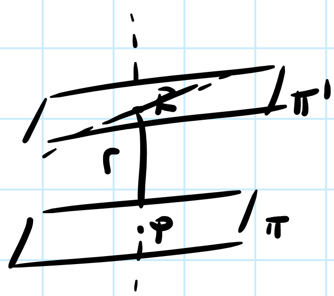
$$R = \begin{pmatrix} 1+4 \\ 0 \\ 1-3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$$

$$S = \begin{pmatrix} 1-4 \\ 0 \\ 1+3 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$$

$$\pi^1 = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$

$$\pi^2 = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$

c) Scrivere le eq param. di una retta s' contenuta in π', incidente a r e ha giacitura $V = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$



Necessariamente $R \in s'$

$$s' = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle = \begin{pmatrix} 5 \\ \lambda \\ -2 \end{pmatrix}$$

d) Eq. parametrica della retta s^2 contenuta in π^2, incidente a r e di giacitura $V^\perp \cap U$

$$V = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad V^\perp = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

Costruire le eq di $V^\perp$

→ scrivo i vettori come righe di una matrice e aggiungo come ultima riga $x\ y\ z$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ x & y & z \end{pmatrix} \xrightarrow{III - xI} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & y & z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & z \end{pmatrix} \xrightarrow{III - zII} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$y = 0$ ← eq $V^\perp$

$$U = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ x & y & z \end{pmatrix} \xrightarrow{II \leftrightarrow I} \begin{pmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ x & y & z \end{pmatrix}$$

$$\xrightarrow{III - \frac{x}{3}I} \begin{pmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & y & z - \frac{4}{3}x \end{pmatrix} \xrightarrow{III - yII} \begin{pmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & \left(z - \frac{4}{3}x\right) \end{pmatrix}$$

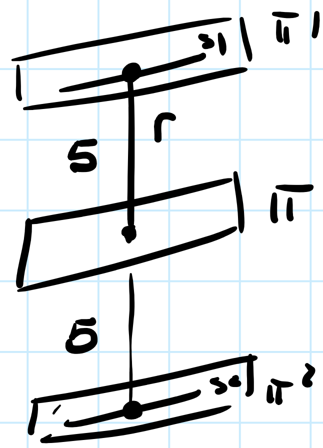
$$\begin{pmatrix} 0 & y & z-\frac{4}{3}x \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & z-\frac{4}{3}x \end{pmatrix}$$

$$z - \frac{4}{3}x = 0 \quad \rightarrow \quad 3z - 4x = 0$$

$$V^{\perp} \cap U \qquad \begin{cases} 3z - 4x = 0 \\ y = 0 \end{cases} \qquad \begin{cases} z = 4/3\, x \\ y = 0 \end{cases}$$

$$\begin{pmatrix} x \\ 0 \\ 4/3x \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 4/3 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$

$$s^2 : \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} + \left\langle \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \right\rangle$$



5) quanto vale $d(s', s'') = ?$

$$d(s', s'') = 10$$

In generale $\quad M = \begin{pmatrix} 5 \\ \lambda \\ -2 \end{pmatrix} \in s'$

$$N = \begin{pmatrix} -3 + 3\mu \\ 0 \\ 4 + 4\mu \end{pmatrix} \in s^2$$

$$M - N = v$$

$$\begin{cases} v \cdot V_{s'} = 0 \\ v \cdot V_{s^2} = 0 \end{cases} \qquad \begin{matrix} \ldots \\ \ldots \end{matrix}$$