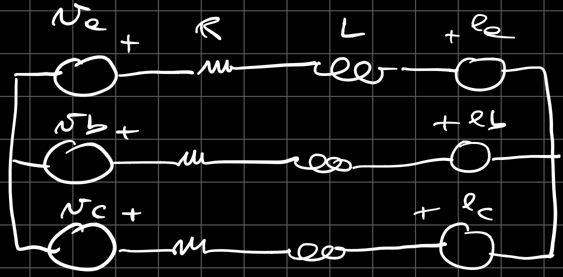
$$\begin{cases} v_a = R i_a + L \dfrac{di_a}{dt} + \dfrac{d\lambda_{am}}{dt} \\ v_b = R i_b + L \dfrac{di_b}{dt} + \dfrac{d\lambda_{bm}}{dt} \\ v_c = R i_c + L \dfrac{di_c}{dt} + \dfrac{d\lambda_{cm}}{dt} \end{cases}$$

$$\begin{cases} \dfrac{d\lambda_{am}}{dt} = -\omega_m^e \Lambda_m \operatorname{sen}\vartheta_m^e = \omega_m^e \Lambda_m \cos(\vartheta_m^e + \frac{\pi}{2}) \\ \dfrac{d\lambda_{bm}}{dt} = \quad \dots \quad = \omega_m^e \Lambda_m \cos(\vartheta_m^e + \frac{\pi}{2} - \frac{2}{3}\pi) \\ \dfrac{d\lambda_{cm}}{dt} = \quad \dots \quad = \omega_m^e \Lambda_m \cos(\vartheta_m^e + \frac{\pi}{2} - \frac{4}{3}\pi) \end{cases}$$

$$\begin{cases} e_a = \omega_m^e \Lambda_m \cos(\vartheta_m^e + \frac{\pi}{2}) \\ e_b = \omega_m^e \Lambda_m \cos(\vartheta_m^e + \frac{\pi}{2} - \frac{2}{3}\pi) \\ e_c = \quad \dots \end{cases}$$

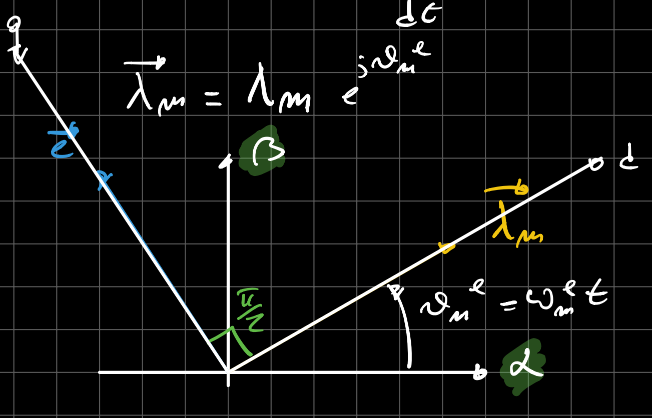
$$\begin{cases} v_a = R i_a + L \dfrac{di_a}{dt} + e_a \\ v_b = R i_b + L \dfrac{di_b}{dt} + e_b \\ v_c = R i_c + L \dfrac{di_c}{dt} + e_c \end{cases}$$




SPACE VECTOR

$$\vec{v} = R\vec{i} + L\frac{d\vec{i}}{dt} + \vec{e} \qquad \alpha\beta$$

$$\vec{\lambda}_m = \Lambda_m e^{j\vartheta_m^e} \qquad \vec{e} = \frac{d\vec{\lambda}_m}{dt} = j\omega_m^e \Lambda_m e^{j\omega_m^e t} = j\omega_m^e \vec{\lambda}_m$$




$$\vec{v} = R\vec{i} + L\frac{d\vec{i}}{dt} + j\omega_m^e \vec{\lambda}_m$$

$$\begin{cases} v_\alpha = R i_\alpha + L \dfrac{di_\alpha}{dt} - \omega_m^e \lambda_{m\beta} \\ v_\beta = R i_\beta + L \dfrac{di_\beta}{dt} + \omega_m^e \lambda_{m\alpha} \end{cases}$$

Pongo nel sistema dq sincrono col rotore $\vec{\lambda}_m$

$$\vec{g}^R = \vec{g}^S e^{-j\vartheta_m^e}$$

R: dq
S: $\alpha\beta$

$$\vec{v}^S = R\vec{i}^S + L\frac{d\vec{i}^S}{dt} + j\omega_m^e \vec{\lambda}_m^S \qquad \alpha\beta$$

$$\vec{v}^R \cancel{e^{j\vartheta_m^e}} = R\vec{i}^R e^{j\vartheta_m^e} + L\frac{d}{dt}\left(\vec{i}^R e^{j\vartheta_m^e}\right) + j\omega_m^e \vec{\lambda}_m^R e^{j\vartheta_m^e}$$

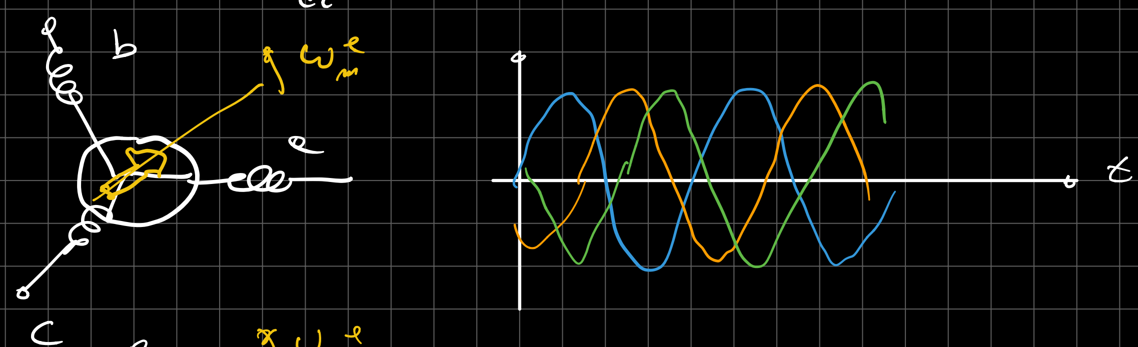
$$= R\vec{i}^R \cancel{e^{j\vartheta_m^e}} + L\frac{d\vec{i}^R}{dt}\cancel{e^{j\vartheta_m^e}} + L\vec{i}^R j\omega_m^e \cancel{e^{j\vartheta_m^e}} + j\omega_m^e \vec{\lambda}_m^R \cancel{e^{j\vartheta_m^e}}$$

$$\vec{v}^R = R\vec{i}^R + L\frac{d\vec{i}^R}{dt} + j\omega_m^e \underbrace{\left(L\vec{i}^R + \vec{\lambda}_m^R\right)}_{\vec{\lambda}^R}$$

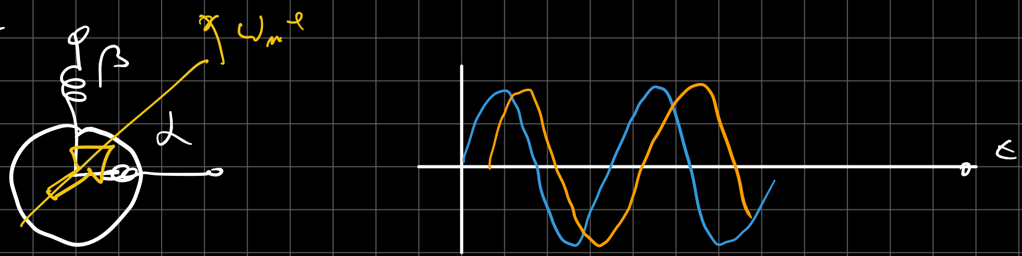
$$\vec{\lambda}^R = L\vec{i}^R + \vec{\lambda}_m^R$$
$$\vec{\lambda}_m^R = \Lambda_m + j0$$

$$\begin{cases} v_d = Ri_d + L\dfrac{di_d}{dt} - \omega_m^e L i_q \\ v_q = Ri_q + L\dfrac{di_q}{dt} + \omega_m^e (L i_d + \Lambda_m) \end{cases}$$

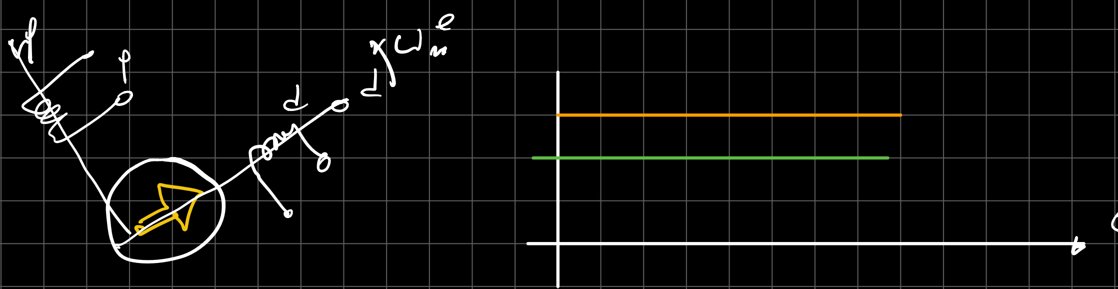
abc




$\alpha\beta$




dq



BILANCIO POTENZA / ENERGIA

$$\begin{cases} v_\alpha = R i_\alpha + L \dfrac{d i_\alpha}{dt} - \omega_m^e \lambda_{m\beta} \\ v_\beta = R i_\beta + L \dfrac{d i_\beta}{dt} + \omega_m^e \lambda_{m\alpha} \end{cases}$$

$$\underbrace{\frac{3}{2}\left(v_\alpha i_\alpha + v_\beta i_\beta\right)}_{①\ P\ \text{ingresso}} =$$

$$\underbrace{\frac{3}{2}\left(R i_\alpha^2 + R i_\beta^2\right)}_{②\ P\ \text{joule}} +$$

$$\underbrace{\frac{3}{2}\left\{\frac{d}{dt}\left[\frac{1}{2} L i_\alpha^2 + \frac{1}{2} L i_\beta^2\right]\right\}}_{③\ W_m} +$$

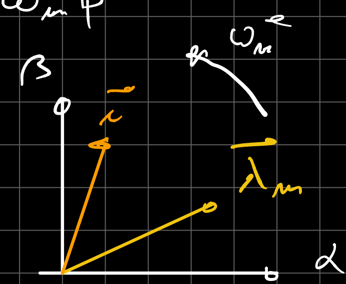
$$\underbrace{\frac{3}{2}\left(\omega_m^e \lambda_{m\alpha} i_\beta - \omega_m^e \lambda_{m\beta} i_\alpha\right)}_{④}$$

④ Potenza Meccanica $= m \cdot \omega_m$

$$m \cdot \omega_m = \frac{3}{2}\omega_m^e \left(\lambda_{m\alpha} i_\beta - \lambda_{m\beta} i_\alpha\right) \qquad \omega_m^e = \omega_m p$$

$$m = \frac{3}{2} p \left(\lambda_{m\alpha} i_\beta - \lambda_{m\beta} i_\alpha\right)$$

sono funzioni del tempo




$$\vec{\lambda}_m^s = \lambda_{m\alpha} + j\lambda_{m\beta} \qquad \vec{i}^s = i_\alpha + j i_\beta$$

$$\vec{\lambda}_m^{s*} = \lambda_{m\alpha} - j\lambda_{m\beta}$$

$$m = \frac{3}{2} p\, \mathrm{Im}\left[\vec{i}^s\ \vec{\lambda}_m^{s*}\right] = \frac{3}{2} p\, \mathrm{Im}\left[(i_\alpha + j i_\beta)(\lambda_{m\alpha} - j\lambda_{m\beta})\right]$$

$$= \frac{3}{2} p \left(\lambda_{m\alpha} i_\beta - \lambda_{m\beta} i_\alpha\right)$$

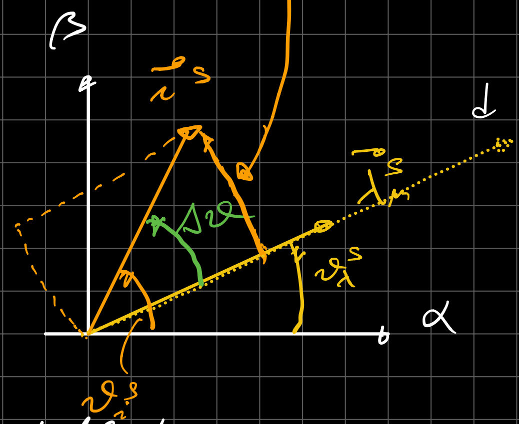
$$m = \frac{3}{2} p \, I_m \left[ \vec{i}^{\,s} \; \vec{\lambda}_m^{s\,*} \right]$$

$$= \frac{3}{2} p \, I_m \left[ |\vec{i}^{\,s}| \, e^{j\vartheta_i^s} \; |\vec{\lambda}_m^s| \, e^{-j\vartheta_\lambda^s} \right]$$

$$= \frac{3}{2} p \, |\vec{i}^{\,s}| \, |\vec{\lambda}_m^s| \; I_m \left[ e^{j(\vartheta_i^s - \vartheta_\lambda^s)} \right]$$

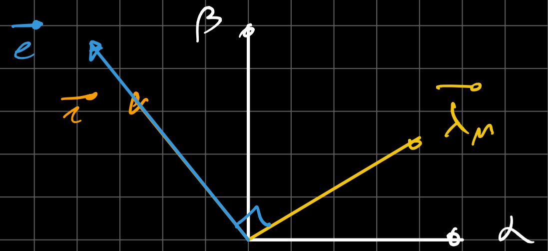
$$= \frac{3}{2} p \, |\vec{i}^{\,s}| \, |\vec{\lambda}_m^s| \; \text{sen}\, \Delta\vartheta$$

$$m = \frac{3}{2} p \, |\vec{\lambda}_m^s| \, \underbrace{|\vec{i}^{\,s}| \, \text{sen}\, \Delta\vartheta}$$




① $|\vec{\lambda}_m|$ è fisso $\quad m$ è controllato controllando $\vec{i}$

② $\forall \, |\vec{i}|$ Avrò $m$ massima per $\Delta\vartheta = \pi/2$




Se facciamo lo stesso bilancio nel sistema dq

$$\underbrace{\frac{3}{2}(v_d i_d + v_q i_q)} = \underbrace{\frac{3}{2}(R i_d^2 + R i_q^2)} + \underbrace{\frac{3}{2}\left(L \frac{di_d}{dt} i_d + L \frac{di_q}{dt} i_q\right)} +$$

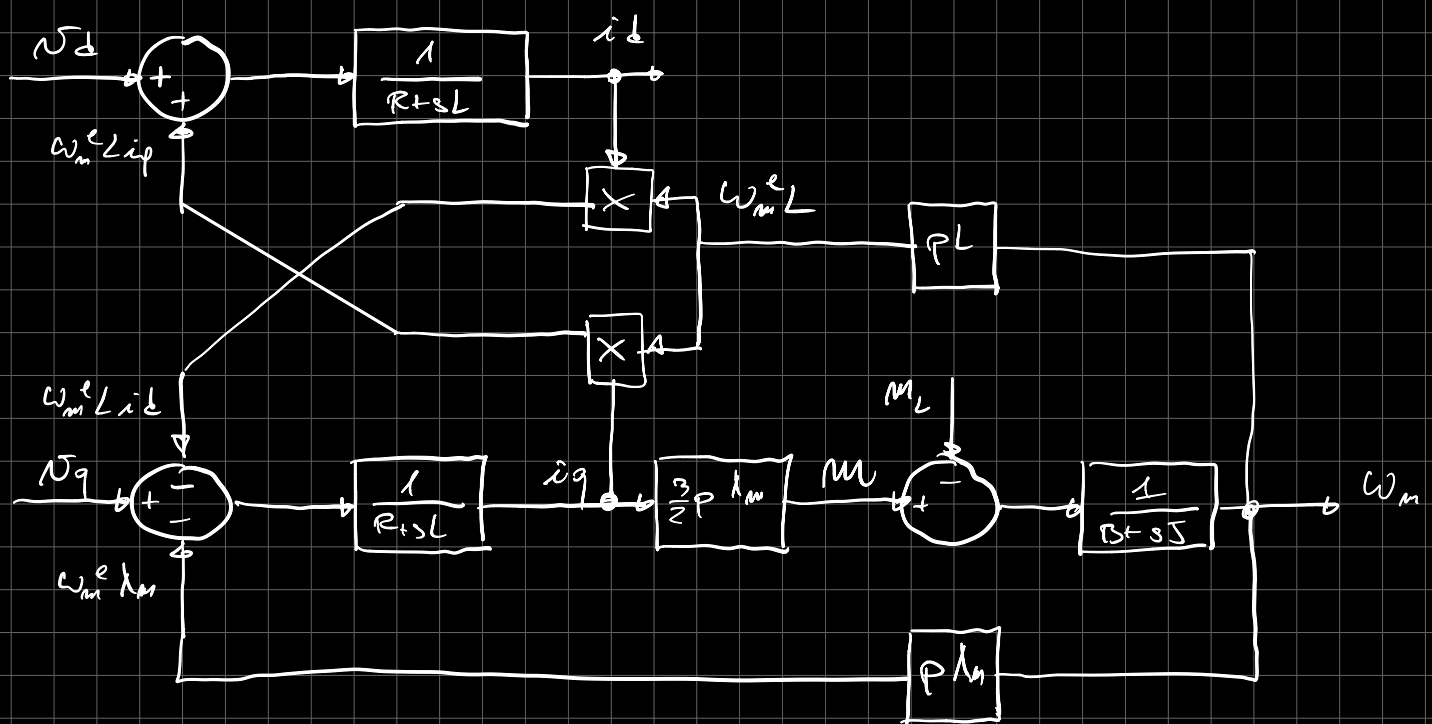
$$\underbrace{- \frac{3}{2} \omega_m^e L i_q i_d + \frac{3}{2} \omega_m^e (L i_d + \lambda_m) i_q}_{m \cdot \omega_m}$$

$$m = \frac{3}{2} p \left( \cancel{L i_q i_d} - \cancel{L i_q i_d} + \lambda_m i_q \right) = \frac{3}{2} p \lambda_m i_q$$

Sintesi:

$$\vec{v}^R = R\vec{i}^R + L\frac{d\vec{i}^R}{dt} + j\omega_m^e \vec{\lambda}^R \qquad \vec{\lambda}^R = \lambda_m + L\vec{i}^R \qquad \begin{cases} \lambda_d = \lambda_m + L i_d \\ \lambda_q = L i_q \end{cases}$$

$$\begin{cases} v_d = R i_d + L\frac{di_d}{dt} - \omega_m^e L i_q \\ v_q = R i_q + L\frac{di_q}{dt} + \omega_m^e (L i_d + \lambda_m) \\ m = \frac{3}{2} p \lambda_m i_q = m_L + B\omega_m + J\frac{d\omega_m}{dt} \end{cases}$$

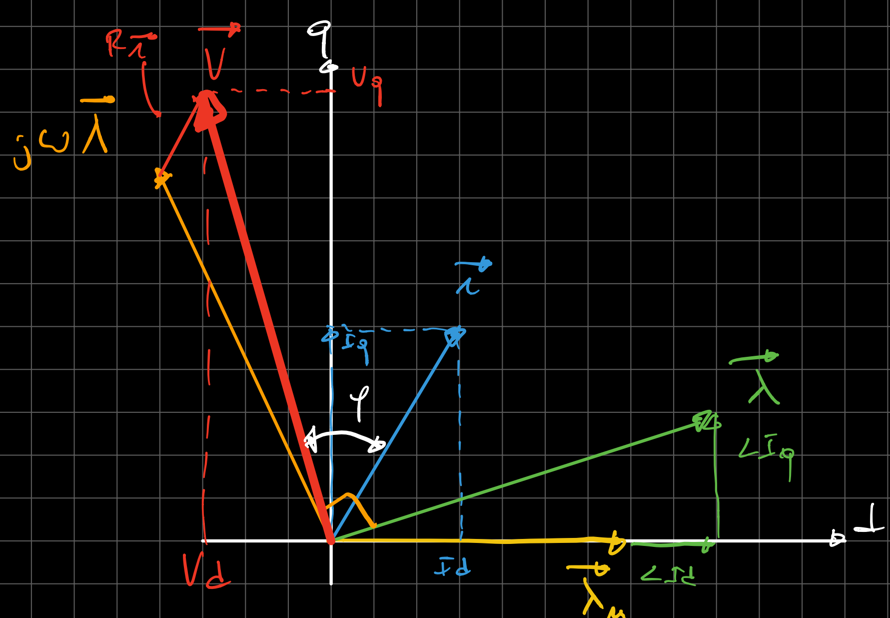



FUNZIONAMENTO A REGIME

$$\begin{cases} V_d = R I_d - \omega_m^e \lambda_q \\ V_q = R I_q + \omega_m^e \lambda_d \end{cases}$$

$$\begin{cases} \lambda_d = \lambda_m + L I_d \\ \lambda_q = L I_q \end{cases}$$

$$m = \frac{3}{2} p \lambda_m I_q$$



Esempio:

$\lambda_m = 0,3\ Vs$ $\qquad I = 15\ A \quad \alpha_i\ 60°$

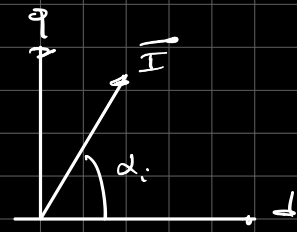
$R = 0,45\ \Omega$

$L = 18\ mH$

$p = 3$

$\omega_m = 1000\ rpm \qquad \omega_m^e = 314\ \frac{rad}{s}$

$f = 50\ Hz$




$I_d = I \cos\alpha_i = 7,5\ A$

$I_q = I \operatorname{sen}\alpha_i = 13\ A$

① $M = \frac{3}{2} p \lambda_m I_q = \frac{3}{2} \cdot 3 \cdot 0,3 \cdot 13 = 17,55\ Nm$

$P_m = M \cdot \omega_m = 17,55 \cdot \underbrace{1000 \cdot \frac{2\pi}{60}}_{104,7 = \omega_m} = 1837,8\ W$

② $V_d = R I_d - \omega_m^e L I_q$

$= 0,45 \cdot 7,5 - 314 \cdot 0,018 \cdot 13 = \overbrace{3,375}^{RI_d} - 73,51 = -70,13\ V$

$V_q = R I_q + \omega_m^e (\lambda_m + L I_d)$

$= 0,45 \cdot 13 + 314 (0,3 + 0,018 \cdot 7,5)$

$= 5,85 + 136,7 = 142,5\ V$

$V = \sqrt{V_d^2 + V_q^2} = 158,8\ V \qquad$ di cui $RI = 6,75\ V$

$V_{RMS} = \frac{V}{\sqrt{2}} = 112,3\ V$

$I_{RMS} = 15/\sqrt{2} = 10,6\ A$

$S = 3 V_{RMS} I_{RMS} = 3 \cdot 112,3 \cdot 10,6 = 3571\ VA$

$\vec{V} = V_d + jV_q = 158,8\ e^{j116,2°}$

$P = 3 \frac{VI}{2} \cos(116,2 - 60) = 1987\ W$

$P_J = 3 R \frac{I^2}{2} = 3 \cdot 0,45 \cdot \frac{15^2}{2} = 151,8\ W$

$P_m + P_J = 1837,8 + 151,8 = 1988 \cong 1987$

(3) $m_{max} = \frac{3}{2} p \lambda_m I = \frac{3}{2} \cdot 3 \cdot 0{,}3 \cdot 15 = 3{,}25 \ Nm$

$P_{m,max} = m_{max} \omega_m = 2120{,}6 \ W$

(5) $\omega_m = 1000 \ rpm$

$\omega_m' = 2 \omega_m = 2000 \ rpm \qquad \omega_m^{e\,\prime} = 628 \ \frac{rad}{s}$

$V_d' \cong 2 V_d = -140 \ V$

$V_q' \cong 2 V_q = 285 \ V$

$V' \cong 2V = 317 \ V$