Power Flow Solution by a Complex Admittance Matrix Method

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Abstract

The paper describes a newly developed algorithm for power flow solution based on a complex admittance matrix. The conciseness and close-to-reality iterative procedure make the method extremely attractive for self-made software implementation. The complex algorithm approach enables network connections, loads, generators, controlled bus voltages, including slack bus, to be considered in a single complex matrix. The power flow solution is achieved with iterations in complex form without the need of real/imaginary decomposition. The method has shown to have advantages over other traditional and available methods in terms of accuracy and convergence also in highly ill-conditioned cases. The cpu-time is comparable and sometimes competitive with those derived from Newton-Raphson methods.

1 Introduction

The numerical solution of the power flow problem is one of the most widely investigated topics in power system analysis and hundreds of contributions can be found in the literature. As well known, the first numerical approaches for power flow computation were based on Y-matrix and Z-matrix iterative methods [1], the main disadvantages being scarce reliability for the former and high storage requirement and relatively low speed for the latter. Subsequently, the Newton-Raphson method [2] and different techniques derived [3] (made competitive with the application of sparsity programming and optimal-ordering triangular factorization) have shown better properties than the previous techniques and have now practically replaced any other existing method for power flow computation in industrial applications. There are other approaches, including the solution of the power flow problem in complex form applied to the Newton-Raphson method [4]. Overviews of the various methods may be found in review papers and even standard textbooks (see for instance [5], [6]). In this framework it seems hard to propose anything new different from variations or improvements of the existing Newton methods and derivations. Nevertheless, in this work the authors have reconsidered the admittance matrix approach and have developed a novel complex matrix iterative procedure particularly simple for programming and which have shown to have excellent convergence properties. The distinctive performance of this method is the high accuracy of the solutions. It is possible to achieve convergence, within reasonable times, with largest absolute active and reactive power-mismatch equal to 10^{-14} MW/Mvar. The power flow solution is achieved also in those systems with high r/x ratio in some lines and in situations close to voltage collapse. Moreover the procedure can be easily implemented in any commercially available math packages (in this work "Matlab" has been used). Test results and some details of the method's performance on up to 450-bus networks are presented.

2 Underlying Concepts

It is known that networks with shunt admittances which are small with respect to branch admittances are likely to be ill-conditioned, and the conditioning tends to improve with the size of the shunt admittances (i.e. with the electrical connections between busbars and reference node) [6]. In the Z-matrix algorithm proposed by *Brown* et al. [1], fixed load impedances to ground were incorporated into the impedance matrix showing that this was "helpful in achieving rapid overall system convergence". In [7], which deals with Y and Z matrix methods, it is shown that the form of the network defining equations has a pronounced effect on the number of the iterations required for convergence, an hybrid form of the "transfer-ratio method" being the most convenient. The present method rationally combines and integrates the "good ideas" of the foregoing papers adding some completely new aspects. The method is based on the formal possibility to represent both loads and generators (except the slack-bus) by shunt elements which are included in a nodal admittance matrix. The main novelty, which will be demonstrated to be the key-factor for drastically reducing the number of iterations required for convergence with respect to the classic Y-matrix methods, is to represent the generators as shunt elements too. The voltage and power constraints are also catered for by these shunt admittances. The system may be thus thought as an "inert" system which is "excited" by the voltage phasor applied to the slack bus; the resulting nodal voltages determine absorbed (at load nodes) and injected (at generation nodes) complex powers depending upon the shunt admittances. A matrix iteration algorithm is then applied for adjusting the value of the shunt admittances in order to match voltage and power constrains. Alternatively, the adjustment is achieved by injecting in parallel with the shunt admittances suitable correction currents with no modification of the initial admittance values. In both methods the system is always solved in complex form. Conversion of generators and loads into shunt admittances makes the system matrix well conditioned and avoids numerical difficulties and singularity because of the presence of as many strong ties to ground as the number of load and generator buses.

3 Basic Procedure

A balanced and symmetrical three-phase power system is assumed so that the transmission system is represented only by its positive-phase-sequence network.

The steady-state regime of the system (see **Fig. 1**) is specified by the following scheduled power and voltage values at the network buses in per unit;

$$\left.\begin{array}{ll} \underline{u}_{\mathrm{a,r}} & \text{: slack bus reference voltage phasor:} \\ \left|\underline{u}_{\mathrm{b,r}}\right| \cdots \left|\underline{u}_{\mathrm{g,r}}\right| & \text{: voltage magnitude of generator buses b…g;} \\ p_{\mathrm{b,r}} \cdots p_{\mathrm{g,r}} & \text{: active power injected at generator buses b…g;} \\ p_{\mathrm{h,r}} + jq_{\mathrm{h,r}} \cdots p_{\mathrm{m,r}} + jq_{\mathrm{m,r}} & \text{: complex power (finite or null) absorbed by load buses h…m.} \end{array}\right\} \quad (1)$$

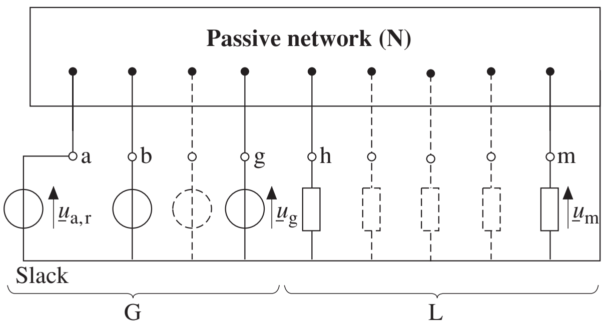

Fig. 1. Schematic representation of power system (G: generation nodes, L: load nodes)

The power system of **Fig. 1** can be formally splitted, as shown in **Fig. 2**, into the passive network N (interconnection and distribution network) and the set of shunt branches Sh, representing the generators G and loads L, considering the **slack generator** external, i.e. $\underline{i}_{\mathrm{aSh}} \equiv 0$.

In accordance with the symbols shown in **Fig. 2**, the matrix equation for the passive block N is given by

$$\underline{\boldsymbol{i}}_{\mathbf{N}} = \underline{\boldsymbol{Y}}_{\mathbf{N}} \, \underline{\boldsymbol{u}} \quad (2)$$

where:

$$\underline{\boldsymbol{i}}_N = \begin{bmatrix} \underline{i}_{\mathrm{aN}} & \underline{i}_{\mathrm{bN}} & \cdots & \underline{i}_{\mathrm{gN}} & \underline{i}_{\mathrm{hN}} & \cdots & \cdots & \cdots & \underline{i}_{\mathrm{mN}} \end{bmatrix}^t \quad (3)$$

$$\underline{\boldsymbol{u}} = \begin{bmatrix} \underline{u}_{\mathrm{a,r}} & \underline{u}_{\mathrm{b}} & \cdots & \underline{u}_{\mathrm{g}} & \underline{u}_{\mathrm{h}} & \cdots & \cdots & \cdots & \underline{u}_{\mathrm{m}} \end{bmatrix}^t \quad (4)$$

and $\underline{\boldsymbol{Y}}_{\mathbf{N}}$ is the $(\mathrm{n_G} + \mathrm{n_L}) \times (\mathrm{n_G} + \mathrm{n_L})$ passive network nodal admittance matrix.

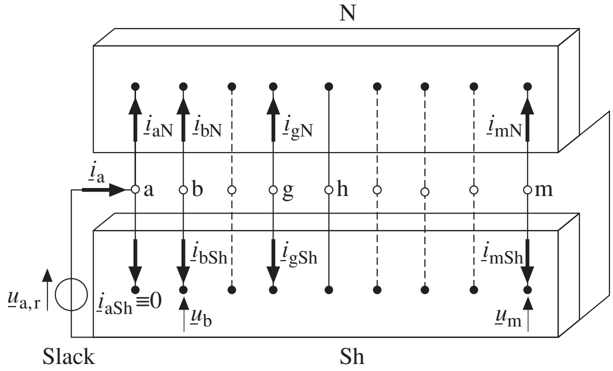

Fig. 2. Splitting of power system into passive network (N) and shunt branches (Sh)

For the Sh block the following matrix equation holds:

$$\underline{\boldsymbol{i}}_{\mathbf{Sh}} = \underline{\boldsymbol{Y}}_{\mathbf{Sh}} \, \underline{\boldsymbol{u}} \quad (5)$$

where:

$$\underline{\boldsymbol{i}}_{\mathbf{Sh}} = \begin{bmatrix} 0 & \underline{i}_{\mathrm{bSh}} & \cdots & \underline{i}_{\mathrm{gSh}} & \underline{i}_{\mathrm{hSh}} & \cdots & \cdots & \cdots & \underline{i}_{\mathrm{mSh}} \end{bmatrix}^t$$

and $\underline{\boldsymbol{Y}}_{\mathbf{Sh}}$ is the $(\mathrm{n_G} + \mathrm{n_L}) \times (\mathrm{n_G} + \mathrm{n_L})$ square diagonal matrix (see **Fig. 3**), whose admittance elements are defined below.

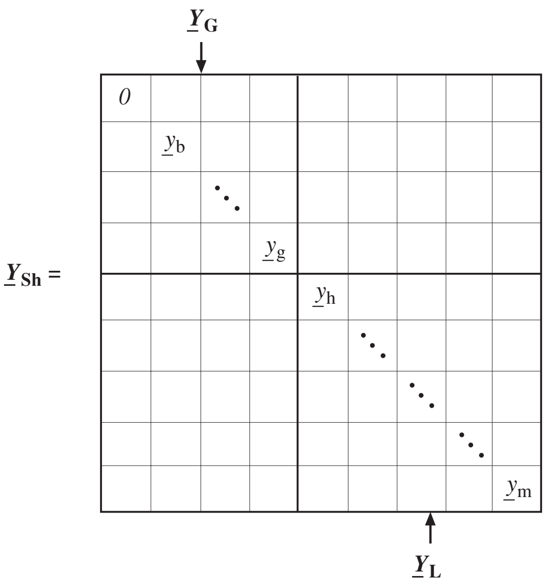

Fig. 3. Partitioned generator-load square diagonal matrix

Defining $\underline{S}_{\mathrm{m}} = p_{\mathrm{m}} + jq_{\mathrm{m}}$ as the complex power (finite or null) absorbed into the generic load bus m when the voltage $\underline{u}_{\mathrm{m}}$ is applied, it results

$$\underline{S}_{\mathrm{m}} = \underline{u}_{\mathrm{m}} \underline{i}^*_{\mathrm{mSh}}; \quad \underline{i}_{\mathrm{mSh}} = \frac{\underline{S}^*_{\mathrm{m}}}{\underline{u}^*_{\mathrm{m}}} = \underline{y}_{\mathrm{m}} \underline{u}_{\mathrm{m}}; \quad \underline{y}_{\mathrm{m}} = \frac{p_{\mathrm{m}} - jq_{\mathrm{m}}}{\left|\underline{u}_{\mathrm{m}}\right|^2}. \quad (6)$$

In a similar manner $\underline{S}_{\mathrm{g}} = p_{\mathrm{g}} + jq_{\mathrm{g}}$ is the complex power injected at the generic generator g bus with nodal voltage $\underline{u}_g$; consequently the generator also may be formally represented in steady-state regime by its "own admittance" $\underline{y}_g$ with applied voltage $\underline{u}_g$, i.e.

$$\underline{S}_g = -\underline{u}_g \, \underline{i}^*_{gSh}; \quad \underline{i}_{gSh} = -\frac{\underline{S}^*_g}{\underline{u}^*_g} = \underline{y}_g \, \underline{u}_g; \quad \underline{y}_g = \frac{-p_g + jq_g}{\left|\underline{u}_g\right|^2}. \tag{7}$$

Given that $\underline{i}_{aSh} \equiv 0$, it follows:

$$\underline{y}_a \equiv 0 . \tag{8}$$

As mentioned above, the basic concept which underlines the method is the following: the Sh block, composed by the above mentioned "inert" linear bipoles, is subject to both absorption and injection of complex power when, in steady-state regime, is "excited" by the application of the voltage phasor $\underline{u}_{a,r}$ applied to node a of the N transmission and distribution (passive) network. Hence, having determined $\underline{\boldsymbol{Y}}_{\mathbf{N}}$, through the usual topological procedures, and fixed $\underline{\boldsymbol{Y}}_{\mathbf{Sh}}$ (with the criteria described in the following), eq. (2) may be combined with eq. (5) to give

$$\underline{\boldsymbol{i}} = \underline{\boldsymbol{Y}} \, \underline{\boldsymbol{u}} \tag{9}$$

where $\underline{\boldsymbol{Y}} = \underline{\boldsymbol{Y}}_{\mathbf{N}} + \underline{\boldsymbol{Y}}_{\mathbf{Sh}}$ and $\underline{\boldsymbol{i}} = \begin{vmatrix} \underline{i}_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}^t$ is the column vector of currents injected at a . . . m system buses shown in **Fig. 2**.

$$\begin{array}{c} \\ \underline{i}_G \rightarrow \\ \\ \\ \\ \\ \underline{i}_L \rightarrow \\ \\ \\ \end{array}
\begin{array}{|c|} \hline \underline{i}_a \\ \hline 0 \\ \hline \vdots \\ \hline 0 \\ \hline 0 \\ \hline \vdots \\ \hline \vdots \\ \hline \vdots \\ \hline 0 \\ \hline \end{array}
=
\left[\begin{array}{c:c} \underline{Y}_{GG} & \underline{Y}_{GL} \\ \hdashline \underline{Y}_{LG} & \underline{Y}_{LL} \end{array}\right]
\begin{array}{|c|} \hline \underline{u}_{a,r} \\ \hline \underline{u}_b \\ \hline \dots \\ \hline \underline{u}_g \\ \hline \underline{u}_h \\ \hline \dots \\ \hline \dots \\ \hline \dots \\ \hline \underline{u}_m \\ \hline \end{array}
\begin{array}{c} \\ \leftarrow \underline{u}_G \\ \\ \\ \\ \\ \leftarrow \underline{u}_L \\ \\ \\ \end{array}$$

$$\underline{\boldsymbol{i}} \qquad \underline{Y} = \underline{Y}_N + \underline{Y}_{Sh} \qquad \underline{\boldsymbol{u}}$$

Fig. 4. Partitioned form of $\underline{\boldsymbol{i}} = \underline{\boldsymbol{Y}} \, \underline{\boldsymbol{u}}$

By introducing the partition shown in **Fig. 4**, it follows:

$$\underline{\boldsymbol{i}}_{\mathbf{G}} = \underline{\boldsymbol{Y}}_{\mathbf{GG}} \, \underline{\boldsymbol{u}}_{\mathbf{G}} + \underline{\boldsymbol{Y}}_{\mathbf{GL}} \, \underline{\boldsymbol{u}}_{\mathbf{L}} \tag{10}$$

$$\mathbf{0} = \underline{\boldsymbol{Y}}_{\mathbf{LG}} \, \underline{\boldsymbol{u}}_{\mathbf{G}} + \underline{\boldsymbol{Y}}_{\mathbf{LL}} \, \underline{\boldsymbol{u}}_{\mathbf{L}} . \tag{11}$$

Applying the standard matrix procedure for variable elimination, eq. (11) can be rewritten as

$$\underline{\boldsymbol{u}}_{\mathbf{L}} = -\underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL}} \, \underline{\boldsymbol{Y}}_{\mathbf{LG}} \, \underline{\boldsymbol{u}}_{\mathbf{G}} \tag{12}$$

substituting $\underline{\boldsymbol{u}}_{\mathbf{L}}$ in eq. (10) yields:

$$\underline{\boldsymbol{i}}_{\mathbf{G}} = \left[\underline{\boldsymbol{Y}}_{\mathbf{GG}} - \underline{\boldsymbol{Y}}_{\mathbf{GL}} \underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL}} \underline{\boldsymbol{Y}}_{\mathbf{LG}}\right] \underline{\boldsymbol{u}}_{\mathbf{G}} = \underline{\boldsymbol{Y}}_{\mathbf{Geq}} \underline{\boldsymbol{u}}_{\mathbf{G}} \tag{13}$$

where the square matrix $\underline{\boldsymbol{Y}}_{\mathbf{LL}}$ is generally nonsingular.

Eq. (13) may be further partitioned as **Fig. 5** clearly shows:

$$\underline{i}_G \rightarrow \begin{array}{|c|} \hline \underline{i}_a \\ \hdashline \underline{i}_x \\ \hline \end{array} = \overbrace{\begin{array}{|c|} \hline \underline{i}_a \\ \hdashline \boldsymbol{0} \\ \hline \end{array} = \begin{array}{|c|c|} \hline \underline{\boldsymbol{A}} & \underline{\boldsymbol{B}} \\ \hline \underline{\boldsymbol{C}} & \underline{\boldsymbol{D}} \\ \hline \end{array}}^{\underline{Y}_{Geq} = \underline{Y}_{GG} - \underline{Y}_{GL} \, \underline{Y}_{LL}^{-1} \underline{Y}_{LG}} \begin{array}{|c|} \hline \underline{u}_{a,r} \\ \hdashline \underline{\boldsymbol{u}}_x \\ \hline \end{array} \leftarrow \underline{\boldsymbol{u}}_G$$

Fig. 5. Partitioned form of $\underline{\boldsymbol{i}}_{\mathbf{G}} = \underline{\boldsymbol{Y}}_{\mathbf{Geq}} \, \underline{\boldsymbol{u}}_{\mathbf{G}}$

where $\underline{\boldsymbol{Y}}_{\mathbf{Geq}}$ completely characterizes the behavior of the system as seen at generation buses, having already synthesized the interactions between network, generators and loads.

Since $\underline{\boldsymbol{i}}_{\mathbf{x}} \equiv \mathbf{0}$ it follows that:

$$\mathbf{0} = \underline{\boldsymbol{C}} \, \underline{u}_{a,r} + \underline{\boldsymbol{D}} \, \underline{\boldsymbol{u}}_{\mathbf{x}} ; \tag{14}$$

hence the column vector $\underline{\boldsymbol{u}}_{\mathbf{x}}$ (phasor voltages at the buses b…g) is given by

$$\underline{\boldsymbol{u}}_{\mathbf{x}} = -\underline{\boldsymbol{D}}^{-1} \underline{\boldsymbol{C}} \, \underline{u}_{a,r} \tag{15}$$

where the square matrix $\underline{\boldsymbol{D}}$ is generally nonsingular.

Therefore, known the column vector $\underline{\boldsymbol{u}}_{\mathbf{G}}$, the column vector $\underline{\boldsymbol{u}}_{\mathbf{L}}$ is given by eq. (12) and the slack-bus current $\underline{i}_a$ is:

$$\underline{i}_a = \underline{\boldsymbol{A}} \, \underline{u}_{a,r} + \underline{\boldsymbol{B}} \, \underline{\boldsymbol{u}}_{\mathbf{x}} .$$

The steady-state regime due to the "excitation" of a with voltage $\underline{u}_{a,r}$, once fixed the matrix $\underline{\boldsymbol{Y}}_{\mathbf{Sh}}$, is thus completely defined.

4 Iteration Algorithm

To begin the iterative procedure, the complex shunt admittances are initially set equal to the following values:

– the equivalent (nominal) load admittances with initial $|\underline{u}_{h0}|\dots|\underline{u}_{m0}| = 1$ p.u. are computed as:

$$\begin{aligned} \underline{y}_{h1} &= \frac{p_{h,r}}{\left|\underline{u}_{h0}\right|^2} - j\frac{q_{h,r}}{\left|\underline{u}_{h0}\right|^2} \\ &\dots \\ \underline{y}_{m1} &= \frac{p_{m,r}}{\left|u_{m0}\right|^2} - j\frac{q_{m,r}}{\left|\underline{u}_{m0}\right|^2} ; \end{aligned} \tag{16}$$

– the equivalent generator initial admittances are estimated on the basis of scheduled active power and load and network reactive power approximated demand $q_{b1} \div q_{g1}$ (see **Appendix I**) as:

$$\begin{aligned} \underline{y}_{b1} &= -\frac{p_{b,r}}{\left|\underline{u}_{b,r}\right|^2} + j\frac{q_{b1}}{\left|\underline{u}_{b,r}\right|^2} \\ &\dots \\ \underline{y}_{g1} &= -\frac{p_{g,r}}{\left|u_{g,r}\right|^2} + j\frac{q_{g1}}{\left|\underline{u}_{g,r}\right|^2} ; \end{aligned} \tag{17}$$

in accordance to eq. (8)

$$\underline{y}_{\mathrm{a}} \equiv 0\,. \tag{18}$$

The initial matrix $\underline{\boldsymbol{Y}}_{\mathbf{Sh1}}$ (composed of the admittances eqs. (18), (17) and (16) as in **Fig. 3**) contributes with $\underline{\boldsymbol{Y}}_{\mathbf{N}}$ in determining, according to **Fig. 4** and **Fig. 5**, an equivalent matrix $\underline{\boldsymbol{Y}}_{\mathbf{Geq1}}$, (partitioned into the four submatrices $\underline{\boldsymbol{A}}_{\mathbf{1}}$, $\underline{\boldsymbol{B}}_{\mathbf{1}}$, $\underline{\boldsymbol{C}}_{\mathbf{1}}$, $\underline{\boldsymbol{D}}_{\mathbf{1}}$). Hence, in accordance with eq. (15), the first estimate of column vector $\underline{\boldsymbol{u}}_{\mathbf{x1}}$ of voltage phasors $\underline{u}_{\mathrm{b1}} \ldots \underline{u}_{\mathrm{g1}}$ is given by:

$$\underline{\boldsymbol{u}}_{\mathbf{x1}} = -\,\underline{\boldsymbol{D}}_{\mathbf{1}}^{-1}\,\underline{\boldsymbol{C}}_{\mathbf{1}}\,\underline{u}_{\mathrm{a,r}}\,; \tag{19}$$

being now $\underline{\boldsymbol{u}}_{\mathbf{G1}}$ completely determined, it is possible to calculate the first estimate of $\underline{\boldsymbol{u}}_{\mathbf{L1}}$ in accordance with (12)

$$\underline{\boldsymbol{u}}_{\mathbf{L1}} = -\,\underline{\boldsymbol{Y}}_{\mathbf{LL1}}^{-1}\,\underline{\boldsymbol{Y}}_{\mathbf{LG}}\,\underline{\boldsymbol{u}}_{\mathbf{G1}}\,. \tag{20}$$

The elements of vector $\underline{\boldsymbol{u}}_{\mathbf{x1}}$ are complex voltages $\underline{u}_{\mathrm{b1}} = u_{\mathrm{b1}}\, e^{j\delta_{\mathrm{b1}}} \ldots \underline{u}_{\mathrm{g1}} = u_{\mathrm{g1}}\, e^{j\delta_{\mathrm{g1}}}$ whose magnitudes will be generally different from the scheduled values $|\underline{u}_{\mathrm{b,r}}| \ldots |\underline{u}_{\mathrm{g,r}}|$ and, similarly, the elements of vector $\underline{\boldsymbol{u}}_{\mathbf{L1}}$ are complex voltages $\underline{u}_{\mathrm{h1}} \ldots \underline{u}_{\mathrm{m1}}$ whose magnitudes will be different from 1 p.u. initially assumed in eq. (16), resulting in active and reactive absorbed powers generally different from the scheduled values. The power flow problem is then solved by applying an iteration method which modulates the shunt admittances of both generator and load busses in order to satisfy all the scheduled voltage and power values (in the following named admittance matrix correction method, AMC). The AMC method has demonstrated to have excellent convergence properties (see Section 5 below); it requires, however, inversion of updated admittance matrices each iteration, which is a major drawback when solving large systems. This drawback has been elegantly overcame by the here named fringing current correction (FCC) method, described later, which conceptually applies the same kind of corrections as the AMC method (thus maintaining the same convergence properties) without the need of inverting the admittance matrices. The basic principles of the iterative procedure can be better understood by describing first the type of corrections performed in the AMC method, where the shunt admittances are actually updated; the FCC method achieves the same result by injecting a suitable set of correcting currents (maintaining the shunt admittances equal to their initial values). Both methods treat differently the generator and load busses.

4.1 AMC method

4.1.1 Generator admittance correction (updating $\underline{\boldsymbol{Y}}_{\mathbf{G}}$)

This correction consists of updating generator susceptances, initially calculated from eq. (17), considering that reactive power variations generally affect bus voltage magnitudes but negligibly phase angles. Given the first estimate of $\underline{\boldsymbol{u}}_{\mathbf{x1}}$ in accordance with eq. (19), in order to obtain a column vector $\underline{\boldsymbol{u}}_{\mathbf{x1,c}}$ (through a simple transformation $\boldsymbol{T}_{\mathbf{x}}$) having unmodified phase angle $\delta_{\mathrm{b1}} \ldots \delta_{\mathrm{g1}}$ but the scheduled voltage magnitudes $|\underline{u}_{\mathrm{b,r}}| \ldots |\underline{u}_{\mathrm{g,r}}|$, a current vector $\underline{\boldsymbol{\Delta i}}_{\mathbf{x1,c}}$ given by

$$\underline{\boldsymbol{\Delta i}}_{\mathbf{x1,c}} = \underline{\boldsymbol{C}}_{\mathbf{1}}\,\underline{u}_{\mathrm{a,r}} + \underline{\boldsymbol{D}}_{\mathbf{1}}\,\underline{\boldsymbol{u}}_{\mathbf{x1,c}} \tag{21}$$

should be injected at the generation buses b÷g. It involves the injection of incremental powers $\boldsymbol{\Delta p}_{\mathbf{x1,c}} + j\boldsymbol{\Delta q}_{\mathbf{x1,c}}$ (see **Fig. 6**).

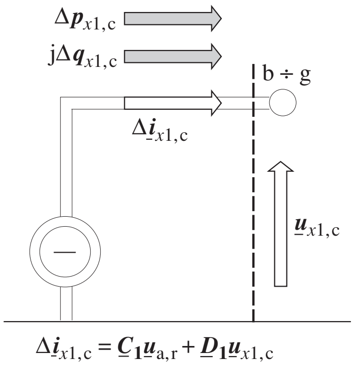

Fig. 6. Correction of generators b÷g

By considering the prevalent link between reactive powers and voltage magnitudes (and observing that eq. (17) clearly indicates each generator injects its own scheduled active power), the vector $j\boldsymbol{\Delta q}_{\mathbf{x1,c}}$ (see **Appendix II**) is applied for updating, according with eq. (17), the generator admittances

$$\begin{aligned}
\underline{y}_{\mathrm{b2}} &= -\frac{p_{\mathrm{b,r}}}{\left|\underline{u}_{\mathrm{b,r}}\right|^2} + j\,\frac{q_{\mathrm{b1}} + \Delta q_{\mathrm{b1,c}}}{\left|\underline{u}_{\mathrm{b,r}}\right|^2}\,,\\
&\ldots\\
\underline{y}_{\mathrm{g2}} &= -\frac{p_{\mathrm{g,r}}}{\left|u_{\mathrm{g,r}}\right|^2} + j\,\frac{q_{\mathrm{g1}} + \Delta q_{\mathrm{g1,c}}}{\left|\underline{u}_{\mathrm{g,r}}\right|^2}
\end{aligned} \tag{22}$$

holding $\underline{y}_{\mathrm{a}} \equiv 0$. Thus the new matrix $\underline{\boldsymbol{Y}}_{\mathbf{G2}}$ (diagonal) can be constructed. It should be noted that the generator susceptance correction is substantially related to the sensitiveness (expressed by eq. (21)) of the passive system (including loads) around the considered regime seen by generators.

4.1.2 Load admittance correction (updating $\underline{\boldsymbol{Y}}_{\mathrm{L}}$)

The imposition of $\underline{\boldsymbol{u}}_{\mathbf{x1,c}}$ determines completely the vector $\underline{\boldsymbol{u}}_{\mathbf{G1,c}}$ as in (23)

$$\underline{\boldsymbol{u}}_{\mathbf{G1,c}} = \begin{bmatrix} \underline{u}_{\mathrm{a,r}} \\ \hline \underline{\mathbf{u}}_{\mathbf{x1,c}} \end{bmatrix} \tag{23}$$

which allows to calculate, through (20), the following load voltage vector:

$$\underline{\boldsymbol{u}}_{\mathbf{L1,c}} = -\underline{\boldsymbol{Y}}_{\mathbf{LL1}}^{-1}\,\underline{\boldsymbol{Y}}_{\mathbf{LG}}\,\underline{\boldsymbol{u}}_{\mathbf{G1,c}} \tag{24}$$

The elements $\underline{u}_{h1,c} \ldots \underline{u}_{m1,c}$ composing the vector $\underline{\boldsymbol{u}}_{\mathbf{L1,c}}$ give the possibility to update the load admittances

$$\underline{y}_{h2} = \frac{p_{h,r}}{\left|\underline{u}_{h1,c}\right|^2} - j\frac{q_{h,r}}{\left|\underline{u}_{h1,c}\right|^2},$$

$$\ldots \tag{25}$$

$$\underline{y}_{m2} = \frac{p_{m,r}}{\left|\underline{u}_{m1,c}\right|^2} - j\frac{q_{m,r}}{\left|\underline{u}_{m1,c}\right|^2};$$

thus an updated matrix $\underline{\boldsymbol{Y}}_{\mathbf{L2}}$ (diagonal) is computed. The iterative cycle must be completed with the calculation of $\underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL2}}$ and the new matrix $\underline{\boldsymbol{Y}}_{\mathbf{Geq2}} = [\underline{\boldsymbol{Y}}_{\mathbf{GG2}} - \underline{\boldsymbol{Y}}_{\mathbf{GL}}\underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL2}}\underline{\boldsymbol{Y}}_{\mathbf{LG}}]$ (it permits $\underline{\boldsymbol{A}}_2$, $\underline{\boldsymbol{B}}_2$, $\underline{\boldsymbol{C}}_2$, $\underline{\boldsymbol{D}}_2$ to be computed) required to begin the next iterative cycle. A clearer representation of the AMC iteration procedure is given in the flow-chart of **Fig. 7**.

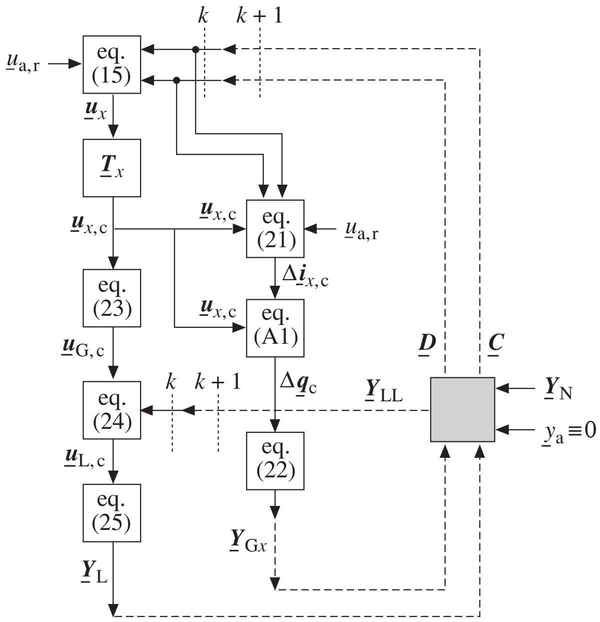

Fig. 7. Flow-chart of AMC

4.2 FCC method

The possibility of avoiding the updating of $\underline{\boldsymbol{Y}}_{\mathbf{G}}$, $\underline{\boldsymbol{Y}}_{\mathbf{L}}$ and the inversions of $\underline{\boldsymbol{Y}}_{\mathbf{LL}}$ and $\underline{\boldsymbol{D}}$ in the iterative cycle is based on assuming for the system a constant model equal to that of the starting procedure of AMC (characterized by $\underline{\boldsymbol{Y}}_{\mathbf{N}}$, $\underline{\boldsymbol{Y}}_{\mathbf{G1}}$, $\underline{\boldsymbol{Y}}_{\mathbf{L1}}$, $\underline{\boldsymbol{A}}_1$, $\underline{\boldsymbol{B}}_1$, $\underline{\boldsymbol{C}}_1$, $\underline{\boldsymbol{D}}_1$) and considering the suitable correction current vectors $\Delta\underline{\boldsymbol{i}}_{\mathbf{xq,c}}$ (at generator buses) and $\Delta\underline{\boldsymbol{i}}_{\mathbf{L,c}}$ (at load buses). To reach convergence the following formulas (see **App. II÷IV**) are applied sequentially as depicted in the flow-chart of **Fig. 8**.

$$\underline{\boldsymbol{u}}_{\mathbf{x}} = -\underline{\boldsymbol{D}}_{\mathbf{1}}^{-1}\underline{\boldsymbol{C}}_{\mathbf{1}}\underline{u}_{a,r} + \underline{\boldsymbol{D}}_{\mathbf{1}}^{-1}\left(\Delta\underline{\boldsymbol{i}}_{\mathbf{x,c}} - \underline{\boldsymbol{L}}_{\mathbf{xL,c}}\Delta\underline{\boldsymbol{i}}_{\mathbf{L,c}}\right) \tag{A8}$$

$$\underline{\boldsymbol{u}}_{\mathbf{L,c}} = -\underline{\boldsymbol{Y}}_{\mathbf{LL1}}^{-1}\underline{\boldsymbol{Y}}_{\mathbf{LG}}\underline{\boldsymbol{u}}_{\mathbf{G,c}} + \underline{\boldsymbol{Y}}_{\mathbf{LL1}}^{-1}\Delta\underline{\boldsymbol{i}}_{\mathbf{L,c}} \tag{A5}$$

$$\Delta\underline{\boldsymbol{i}}_{\mathbf{L,c}} = \text{as defined in } \textbf{Appendix IV} \tag{A9}$$

$$\Delta\underline{\boldsymbol{i}}_{\mathbf{x}} = \underline{\boldsymbol{C}}_{\mathbf{1}}\underline{u}_{a,r} + \underline{\boldsymbol{D}}_{\mathbf{1}}\underline{\boldsymbol{u}}_{\mathbf{x}} + \underline{\boldsymbol{L}}_{\mathbf{xL}}\Delta\underline{\boldsymbol{i}}_{\mathbf{L}} \tag{A7}$$

$$\Delta\underline{\boldsymbol{i}}_{\mathbf{xq,c}} = \text{as defined in } \textbf{Appendix II} \tag{A2}$$

The beginning of first cycle ($k = 1$), regarding with the calculation of vector $\Delta\underline{\boldsymbol{i}}_{\mathbf{L}}$ (load correction currents), needs to impose $\Delta\underline{\boldsymbol{i}}_{\mathbf{x}} \equiv \mathbf{0}$ and $\Delta\underline{\boldsymbol{i}}_{\mathbf{L}} \equiv \mathbf{0}$, thus giving same $\underline{\boldsymbol{u}}_{\mathbf{x1}}$ (as in eq. (19)) $\underline{\boldsymbol{u}}_{\mathbf{x1,c}}$, $\underline{\boldsymbol{u}}_{\mathbf{G1,c}}$ (as in eq. (23)), $\underline{\boldsymbol{u}}_{\mathbf{L1,c}}$ (as in eq. (24)); from load voltage vector $\underline{\boldsymbol{u}}_{\mathbf{L1,c}}$ the correction currents $\Delta\underline{\boldsymbol{i}}_{\mathbf{L2,c}}$ can be computed, so that, as **Appendix IV** shows, the scheduled complex power is absorbed in each load bus.

As concerns the b ÷ g generator corrections, eq. (A7) gives $\Delta\underline{\boldsymbol{i}}_{\mathbf{x2,c}}$ (conditioned from $\underline{\boldsymbol{u}}_{\mathbf{x1,c}}$ and from load updating $\Delta\underline{\boldsymbol{i}}_{\mathbf{L2,c}}$) and then the quadrature component current vector $\Delta\underline{\boldsymbol{i}}_{\mathbf{xq2,c}}$ must be extracted through eq. (A2). The so calculated vectors $\Delta\underline{\boldsymbol{i}}_{\mathbf{L2,c}}$ and $\Delta\underline{\boldsymbol{i}}_{\mathbf{xq2,c}}$ will be injected in the second cycle. The procedure is iterated until convergence. Since the meaning of fringing current correction is absolutely equivalent to the above of admittance correction (but without modifying the system model i.e. constant admittances and $\underline{\boldsymbol{A}}_1$, $\underline{\boldsymbol{B}}_1$, $\underline{\boldsymbol{C}}_1$, $\underline{\boldsymbol{D}}_1$ unchanged), the power flow solution is achieved with exactly the same number of iteration but the time per iteration is drastically reduced proportionally with the size of $\underline{\boldsymbol{Y}}_{\mathbf{LL}}$. In each iteration until convergence, slack, load and generator complex powers are calculated with eqs. (A12), (A10), (A11) respectively (see **App. V**).

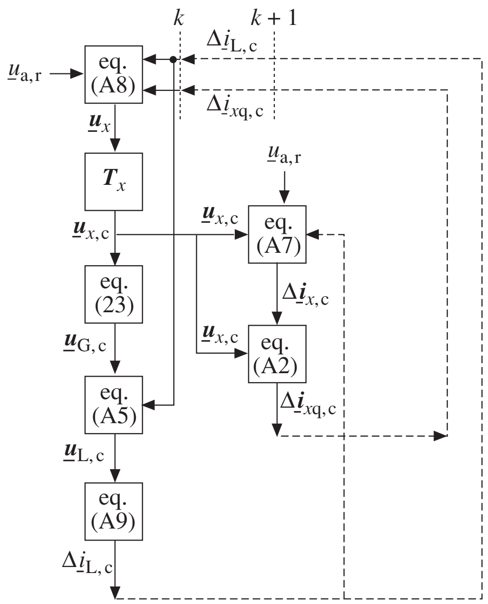

Fig. 8. Flow-chart of FCC

5 Convergence Performance using a 500 MHz Pentium (128 MB RAM)

The procedure has been tested on a number of case systems and compared with different commercial packages based on the Newton-Raphson method. Particular attention has been given to ill-conditioned cases, where the procedure has shown to have better convergence properties than other methods. It is known that the main drawback of the classic Y-matrix and Z-matrix methods is the rather slow convergence (i.e. high number of iterations), whereas the Newton-Raphson and derived methods usually achieve convergence within very few iterations. Similarly to the latter, the present procedure can reach convergence with very few iterations, when applying either the AMC or the FCC correction method, and with a cpu-time comparable or even less then that of Newton-Raphson methods when the much faster FCC method is applied. Such a good convergence performance derives from having converted the generators, as well as the loads, into shunt admittances included in the Y-matrix: increasing the number of shunt elements makes the system matrix better conditioned and decreases the risk of numerical difficulties and singularities. This is well demonstrated in **Tab. 1** below that reports the number of iterations and cpu-times required for a 42-bus system as a function of the percentage of generator power represented as fixed shunt admittances included in the Y-matrix, the remaining portion being catered for by fringing currents (loads are fully represented as shunt admittances). If only load admittances are included in the Y-matrix (first column in **Tab. 1**), convergence is reached very slowly. It must be emphasized that this is the case adopted in classical impedance matrix old methods [1]. It results clearly that the execution times drastically reduce increasing the percentage of generator admittances considered and become particularly small with the total inclusion of load and generators admittances.

% gen. adm.	0	20	40	60	80	100
no. of iterat.	38	29	21	12	7	3
cpu-times [s]	0,203	0,155	0,112	0,064	0,037	0,016

Tab. 1. Number of iterations and cpu times for a 42-bus system within 10^{-3} MW/Mvar maximum bus mismatches

It is well known that a good first estimate is of importance for achieving the convergence: in this approach it has been observed that a good first estimate of the generator susceptance may be realised with a simplified procedure for assessing the generator reactive power described in **App. I**. The benefits of such estimate are shown in **Tab. 2**, reporting the number of iterations (consequently the cpu-times) required for convergence on a number of test networks, in case of assuming an initial estimate of generated reactive power equal to zero or evaluated as in **App. I**.

System	Number of iterations with $q_b \div q_g = 0$	Number of iterations with $q_b \div q_g$ estimated as in App. I
18-bus	5	3
21-bus	4	2
42-bus	5	3
60-bus	NO CONVERGENCE	4
105-bus	7	4
150-bus	7	5
252-bus	NO CONVERGENCE	3
450-bus	12	5

Tab. 2. Number of iterations with different initial $q_b \div q_g$ within 10^{-3} MW/Mvar maximum bus mismatches

System	Total cpu-time [s] with AMC	Total cpu-time [s] with FCC
18-bus	0,047	0,003
21-bus	0,047	0,003
42-bus	0,047	0,003
60-bus	0,156	0,011
105-bus	0,25	0,017
150-bus	0,89	0,047
252-bus	1,25	0,063
450-bus	13,24	0,89

Tab. 3. Total cpu-times with two different methods within 10^{-3} MW/Mvar maximum bus mismatches

The execution times (most of all with FCC) obtained in the study of up to 450-bus network seems to be very encouraging and comparable with other power flow methods. An attempt of graphical extrapolation would indicate for the FCC a cpu time of about $3 \div 4$ s to solve a 1000-bus network. As mentioned in the introduction, the method has shown to be capable of achieving an extremely high degree of accuracy (up to 10^{-14} MW/Mvar maximum bus mismatches) yielding the solution with cpu-times not much higher than those reported in **Tab. 3**. Moreover it should be noted that the proposed method can achieve convergence also in those systems with high r/x ratio in some lines and in the situations close to voltage collapse, where the Newton-Raphson methods may have numerical instabilities.

6 Conclusions

The present work demonstrates that the major drawbacks of the classical admittance matrix approach, namely poor convergence properties, can be efficiently overcome by considering both loads and generators as shunt admittances to be included into the admittance matrix. The basic procedure is straightforward and, coupled with the appropriate criteria, has shown to have excellent convergence performance. In addition, the simplicity of the approach and the conciseness of the matrix algebra applied, allows to rapidly implement the procedure on any commercially available math-packages such as "Mathematica" or "Matlab". The presented method allows to calculate the power flow regime either in very large multiarea network without using the typical diakoptic formulas or in unbalanced three-phase power systems showing not negligible negative and zero components.

7 List of Symbols and Abbreviations

7.1 Symbols

$\underline{u}$	complex voltage
$\lvert\underline{u}\rvert$	voltage magnitude
$\underline{u}_{a,r}$	scheduled slack bus voltage phasor
$\underline{i}$	complex nodal current
$\underline{y}$	complex admittance
$\underline{\boldsymbol{Y}}$	complex admittance matrix
$\underline{S}$	complex power

p	active power
q	reactive power
$\Delta\underline{\boldsymbol{i}}, \Delta\boldsymbol{p}, \Delta\boldsymbol{q}$	incremental correction vectors
$\delta_b \ldots \delta_g$	phase angles of generator bus voltages b÷g
÷	from … to …
≡	is identically equal to
⊗	element-by-element array multiplication
/	element-by-element array division
diag($\underline{\boldsymbol{X}}$)	main diagonal vector of $\underline{\boldsymbol{X}}$
$\boldsymbol{T}_{\mathbf{x}}$	transformation matrix
n_G	number of generator buses
n_L	number of load buses
Im	imaginary part of a complex quantity

7.2 Subscripts

r	scheduled value
c	corrected value
G	generator buses a÷g
L	load buses h÷m
a	slack-bus
x	generator buses b÷g
N	passive network
Sh	shunt branches
1,2, … k	first, second, …, k-th iteration
0	initial estimate
q	quadrature component

7.3 Superscripts

t	transposition
*	complex conjugate
–1	matrix inversion

7.4 Abbreviations

AMC	Admittance Matrix Correction
FCC	Fringing Current Correction

Appendix I

An efficient method for evaluating approximately the initial reactive power $q_{b1} \div q_{g1}$ of generators (useful in eq. (17) to first estimate generator admittances) is briefly presented. The network is thought as ideal or without losses so that $\underline{\boldsymbol{Y}}_{\mathbf{N}}$ has only its imaginary part. The generators are not incorporated in $\underline{\boldsymbol{Y}}_{\mathbf{Sh}}$ whereas the loads are represented with their nominal conductance and susceptance as in eq. (16). The generator voltage phasors (including slack bus) are assumed as laying on the real axis and of magnitude equal to scheduled values. Eq. (13) gives the generators currents $\underline{\boldsymbol{i}}_{\mathbf{G}}$, and hence the desired reactive power first estimate.

Appendix II

Denoting with ⊗ element by element array multiplication the reactive power vector is given:

$$j\Delta\boldsymbol{q}_{\mathbf{x1,c}} = j\left[\mathrm{Im}\left(\underline{\boldsymbol{u}}_{\mathbf{x1,c}} \otimes \Delta\underline{\boldsymbol{i}}^{*}_{\mathbf{x1,c}}\right)\right]. \tag{A1}$$

The vector $\Delta\underline{\boldsymbol{i}}_{\mathbf{xq1,c}}$ (quadrature component current vector) that springs out of the same injection of reactive power, must also satisfy

$$\underline{\boldsymbol{u}}_{\mathbf{x1,c}} \otimes \Delta\underline{\boldsymbol{i}}^{*}_{\mathbf{xq1,c}} = j\left[\mathrm{Im}\left(\underline{\boldsymbol{u}}_{\mathbf{x1,c}} \otimes \Delta\underline{\boldsymbol{i}}^{*}_{\mathbf{x1,c}}\right)\right]$$

that yields obviously,

$$\Delta\underline{\boldsymbol{i}}_{\mathbf{xq1,c}} = -j\left[\mathrm{Im}\left(\underline{\boldsymbol{u}}_{\mathbf{x1,c}} \otimes \Delta\underline{\boldsymbol{i}}^{*}_{\mathbf{x1,c}}\right)\right] \Big/ \underline{\boldsymbol{u}}^{*}_{\mathbf{x1,c}} \tag{A2}$$

where / is element by element array division.

Appendix III

Generalizing eqs. (10) and (11) yields:

$$\underline{\boldsymbol{i}}_{\mathbf{G}} = \underline{\boldsymbol{Y}}_{\mathbf{GG1}}\,\underline{\boldsymbol{u}}_{\mathbf{G}} + \underline{\boldsymbol{Y}}_{\mathbf{GL}}\,\underline{\boldsymbol{u}}_{\mathbf{L}} \tag{A3}$$

$$\Delta\underline{\boldsymbol{i}}_{\mathbf{L}} = \underline{\boldsymbol{Y}}_{\mathbf{LG}}\,\underline{\boldsymbol{u}}_{\mathbf{G}} + \underline{\boldsymbol{Y}}_{\mathbf{LL1}}\,\underline{\boldsymbol{u}}_{\mathbf{L}}\,. \tag{A4}$$

Solving eq. (A4) for $\underline{\boldsymbol{u}}_{\mathbf{L}}$ and substituting it in eq. (A3) yields:

$$\underline{\boldsymbol{u}}_{\mathbf{L}} = -\underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL1}}\underline{\boldsymbol{Y}}_{\mathbf{LG}}\underline{\boldsymbol{u}}_{\mathbf{G}} + \underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL1}}\Delta\underline{\boldsymbol{i}}_{\mathbf{L}} \tag{A5}$$

$$\underline{\boldsymbol{i}}_{\mathbf{G}} = \left(\underline{\boldsymbol{Y}}_{\mathbf{GG1}} - \underline{\boldsymbol{Y}}_{\mathbf{GL}}\underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL1}}\underline{\boldsymbol{Y}}_{\mathbf{LG}}\right)\underline{\boldsymbol{u}}_{\mathbf{G}} + \underline{\boldsymbol{Y}}_{\mathbf{GL}}\underline{\boldsymbol{Y}}^{-1}_{\mathbf{LL1}}\Delta\underline{\boldsymbol{i}}_{\mathbf{L}} \tag{A6}$$

Eq. (A6) can be partitioned as follows:

$$\underbrace{\begin{bmatrix} \underline{i}_{\mathrm{af}} \\ \Delta\underline{i}_{x} \end{bmatrix}}_{\underline{i}_{\mathrm{G}}} = \underbrace{\overbrace{\begin{bmatrix} \underline{A}_1 & \underline{B}_1 \\ \underline{C}_1 & \underline{D}_1 \end{bmatrix}}^{\underline{Y}_{\mathrm{GG1}} = \underline{Y}_{\mathrm{GL}}\,\underline{Y}^{-1}_{\mathrm{LL1}}\,\underline{Y}_{\mathrm{LG}}}}_{\underline{Y}_{\mathrm{Geq1}}} \underbrace{\begin{bmatrix} \underline{u}_{\mathrm{a,r}} \\ \underline{u}_{x} \end{bmatrix}}_{\underline{u}_{\mathrm{G}}} + \underbrace{\overbrace{\begin{bmatrix} \underline{L}_{\mathrm{aL}} \\ \underline{L}_{x\mathrm{L}} \end{bmatrix}}^{\underline{Y}_{\mathrm{GL}}\,\underline{Y}^{-1}_{\mathrm{LL1}}}}_{\underline{L}} \underbrace{\begin{bmatrix} \\ \end{bmatrix}}_{\Delta\underline{i}_{\mathrm{L}}}$$

It can be derived in particular that:

$$\Delta\underline{\boldsymbol{i}}_{\mathbf{x}} = \underline{\boldsymbol{C}}_{\mathbf{1}}\underline{u}_{\mathrm{a,r}} + \underline{\boldsymbol{D}}_{\mathbf{1}}\underline{\boldsymbol{u}}_{\mathbf{x}} + \underline{\boldsymbol{L}}_{\mathbf{xL}}\Delta\underline{\boldsymbol{i}}_{\mathbf{L}} \tag{A7}$$

Eq. (7A) can be solved for $\underline{\boldsymbol{u}}_{\mathbf{x}}$ as follows:

$$\underline{\boldsymbol{u}}_{\mathbf{x}} = -\underline{\boldsymbol{D}}^{-1}_{\mathbf{1}}\underline{\boldsymbol{C}}_{1}\underline{u}_{\mathrm{a,r}} + \underline{\boldsymbol{D}}^{-1}_{\mathbf{1}}\left(\Delta\underline{\boldsymbol{i}}_{\mathbf{x}} - \underline{\boldsymbol{L}}_{\mathbf{xL}}\Delta\underline{\boldsymbol{i}}_{\mathbf{L}}\right). \tag{A8}$$

It should be noted that $\underline{\boldsymbol{Y}}_{\mathbf{LG}}$ and $\underline{\boldsymbol{Y}}_{\mathbf{GL}}$ only depend upon N network.

Appendix IV

By observing **Fig. A1**, application of the scheduled complex power $\underline{S}_{\mathrm{m,r}}$ yields

$$\Delta\underline{S}_{\mathrm{m}} = \underline{S}_{\mathrm{m1}} - \underline{S}_{\mathrm{m,r}}\,.$$

The foregoing equation can be written as (see eq. (16)):

$$\underline{u}_{\mathrm{m}}\,\underline{\Delta i}_{\mathrm{m}}^{*} = \left|\underline{u}_{\mathrm{m}}\right|^{2}\cdot \underline{y}_{\mathrm{m1}}^{*} - I^{2}\cdot \underline{y}_{\mathrm{m1}}^{*}$$

hence

$$\underline{\Delta i}_{\mathrm{m}} = \underline{y}_{\mathrm{m1}}\cdot\frac{\left|\underline{u}_{\mathrm{m}}\right|^{2}-1}{\underline{u}_{\mathrm{m}}^{*}}.$$

By using vector operations (i.e. element by element array division and multiplication) it may be rewritten in vectorial form as in (A9).

$$\underline{\boldsymbol{\Delta i}}_{\mathbf{L}} = \left(\mathrm{diag}\left(\underline{\boldsymbol{Y}}_{\mathbf{L}}\right)/\underline{\boldsymbol{u}}_{\mathbf{L,c}}^{*}\right)\otimes\left(\left|\underline{\boldsymbol{u}}_{\mathbf{L,c}}\right|^{2}-\boldsymbol{1}\right). \tag{A9}$$

where **1** is an identity column vector and diag($\underline{\boldsymbol{Y}}_{\mathbf{L}}$) is the main diagonal vector of $\underline{\boldsymbol{Y}}_{\mathbf{L}}$.

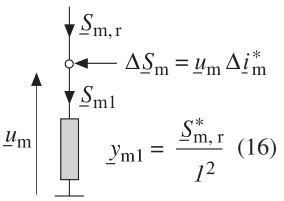

Fig. A1. Typical m load bus characterized by nominal admittancey $\underline{y}_{\mathrm{m1}}$ and scheduled $\underline{S}_{\mathrm{m,r}}$

Appendix V

By observing **Fig. A2**, in each iteration until convergence, the generator complex power is computed by

$$\underline{S}_{\mathrm{g}} = \underline{u}_{\mathrm{g}}\,\underline{\Delta i}_{\mathrm{g}}^{*} - \underline{u}_{\mathrm{g}}\,\underline{u}_{\mathrm{g}}^{*}\,\underline{y}_{\mathrm{g1}}^{*} = \underline{u}_{\mathrm{g}}\left(\underline{\Delta \mathrm{i}}_{\mathrm{g}} - \underline{u}_{\mathrm{g}}\,\underline{y}_{\mathrm{g1}}\right)^{*} \tag{A10}$$

whereas the load complex power is given by

$$\underline{S}_{\mathrm{m}} = \underline{u}_{\mathrm{m}}\,\underline{u}_{\mathrm{m}}^{*}\,\underline{y}_{\mathrm{m1}}^{*} - \underline{u}_{\mathrm{m}}\,\underline{\Delta i}_{\mathrm{m}}^{*} = \underline{u}_{\mathrm{m}}\left(-\underline{\Delta i}_{\mathrm{m}} + \underline{u}_{\mathrm{m}}\,\underline{y}_{\mathrm{m1}}\right)^{*}. \tag{A11}$$

Finally, the slack-bus complex power is:

$$\underline{S}_{\mathrm{a}} = \underline{u}_{a,r}\,\underline{i}_{\mathrm{af}}^{*} = \underline{u}_{a,r}\left(\underline{\boldsymbol{A}}_{\mathbf{1}}\,\underline{u}_{a,r} + \underline{\boldsymbol{B}}_{\mathbf{1}}\,\underline{\boldsymbol{u}}_{\mathbf{x}} + \underline{\boldsymbol{L}}_{\mathbf{aL}}\,\underline{\boldsymbol{\Delta i}}_{\mathbf{L}}\right)^{*}. \tag{A12}$$

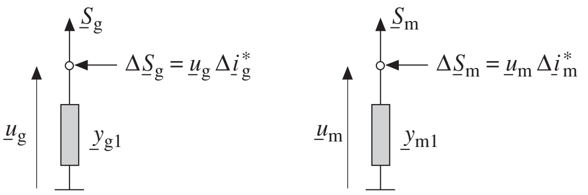

Fig A2. Complex powers in generator and load busses

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