Electric Drives
Laboratory
DII - UniPD

Azionamenti Elettrici

Slides delle lezioni

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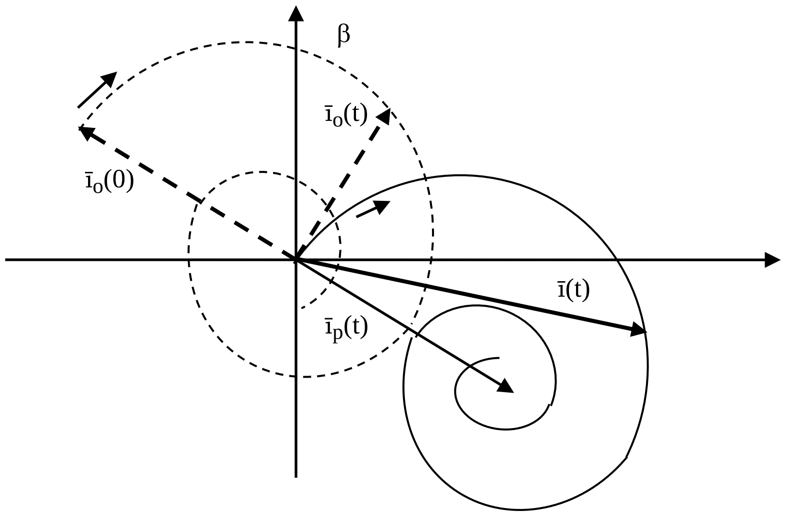
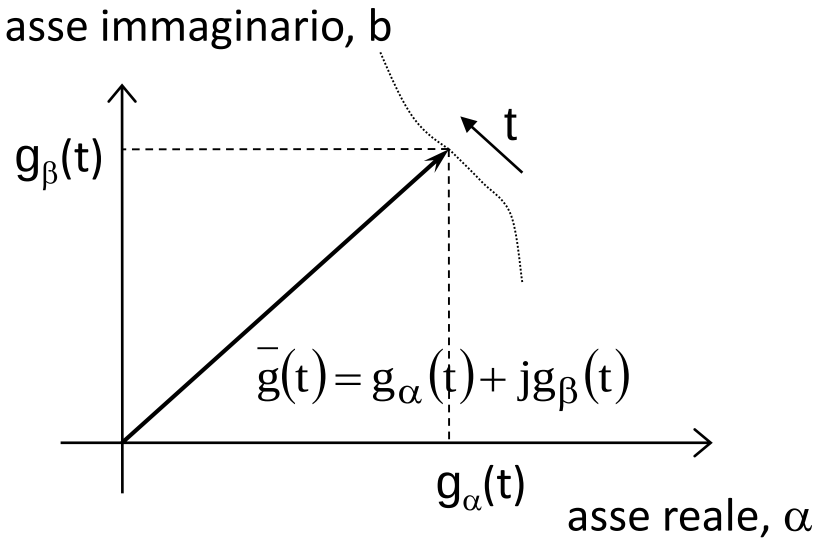
Vettori spaziali



Vettore spaziale - definizione



$$\overline{g}(t) = \frac{2}{3}\left[g_a(t) + g_b(t)e^{j2\pi/3} + g_c(t)e^{j4\pi/3}\right] = g_\alpha(t) + \mathrm{j}\, g_\beta(t)$$

$$Re[\overline{g}(t)] = g_\alpha(t) = \frac{2}{3}\left[g_a(t) - \frac{g_b(t)}{2} - \frac{g_c(t)}{2}\right]$$

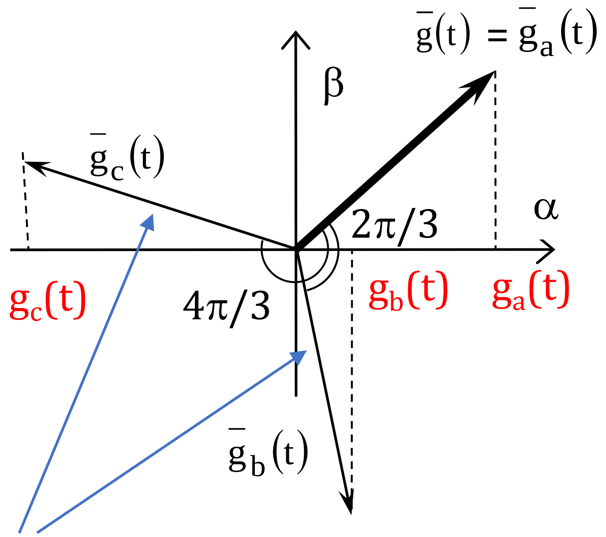
$$Im[\overline{g}(t)] = g_\beta(t) = \frac{2}{3}\left[g_b(t)\frac{\sqrt{3}}{2} - g_c(t)\frac{\sqrt{3}}{2}\right] = \frac{1}{\sqrt{3}}[g_b(t) - g_c(t)]$$

Grandezza «bi-dimensionale» associata ad una terna di grandezze trifase <u>di qualsiasi forma d'onda</u>

Vettore spaziale – terna bilanciata

$$g_o(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3} = 0$$

$$Re[\overline{g}(t)] = g_\alpha(t) = g_a(t)$$



$$\overline{g}_b(t) = \overline{g}(t)e^{-j2\pi/3} = \frac{2}{3}\left[g_a(t)e^{-j2\pi/3} + g_b(t) + g_c(t)e^{j2\pi/3}\right]$$

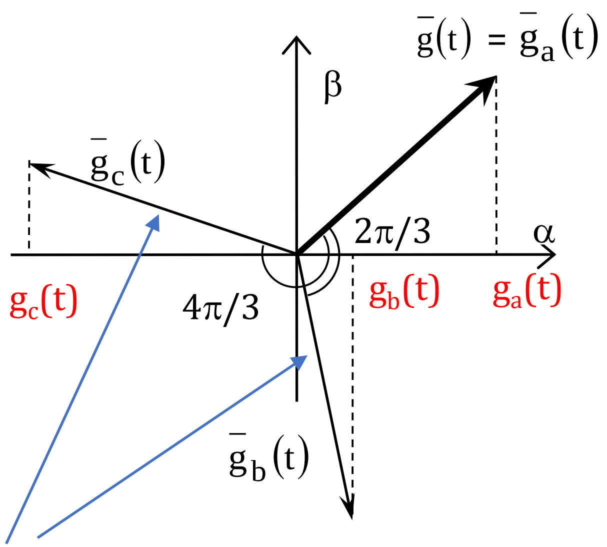
$$Re\left[\overline{g}_b(t)\right] = Re\left[\overline{g}(t)e^{-j2\pi/3}\right] = -g_\alpha(t)\frac{1}{2} + g_\beta(t)\frac{\sqrt{3}}{2} = g_b(t)$$

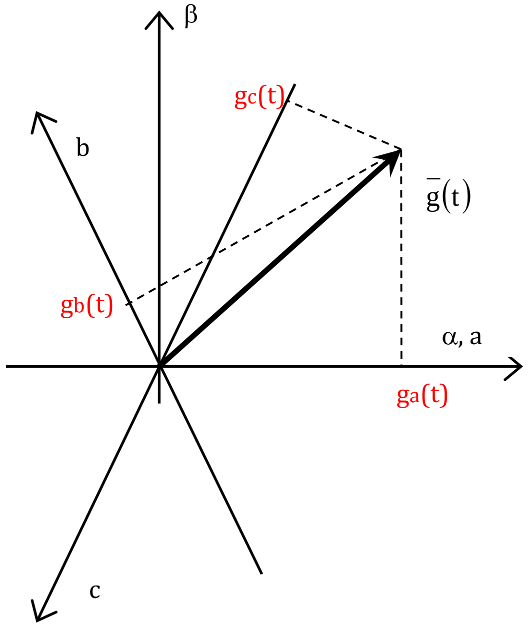
$$Re\left[\overline{g}_c(t)\right] = Re\left[\overline{g}(t)e^{-j4\pi/3}\right] = -g_\alpha(t)\frac{1}{2} - g_\beta(t)\frac{\sqrt{3}}{2} = g_c(t)$$

Riflessioni simmetriche: vettori ausiliari per esemplificare graficamente il calcolo di $\overline{g}_b(t)$ e $\overline{g}_c(t)$.

Vettore spaziale – terna bilanciata

$$g_o(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3} = 0$$





Riflessioni simmetriche: vettori ausiliari per esemplificare graficamente il calcolo di $\overline{g}_b(t)$ e $\overline{g}_c(t)$.

In alternativa alle proiezioni delle *Riflessioni simmetriche* sull'asse reale, si possono usare le proiezioni del vettore spaziale sugli assi *b* e *c.*

Vettore spaziale – terna non bilanciata

$$g_o(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3} \neq 0$$

$$\begin{cases} g_a(t) = g'_a(t) + g_o(t) \\ g_b(t) = g'_b(t) + g_o(t) \\ g_c(t) = g'_c(t) + g_o(t) \end{cases}$$

Si compone di una terna bilanciata e di una terna omopolare.

$$\overline{g}(t) = \frac{2}{3}\left[g_a(t) + g_b(t)e^{j2\pi/3} + g_c(t)e^{j4\pi/3}\right] =$$

$$= \frac{2}{3}\left[g'_a(t) + g'_b(t)e^{j2\pi/3} + g'_c(t)e^{j4\pi/3}\right]$$

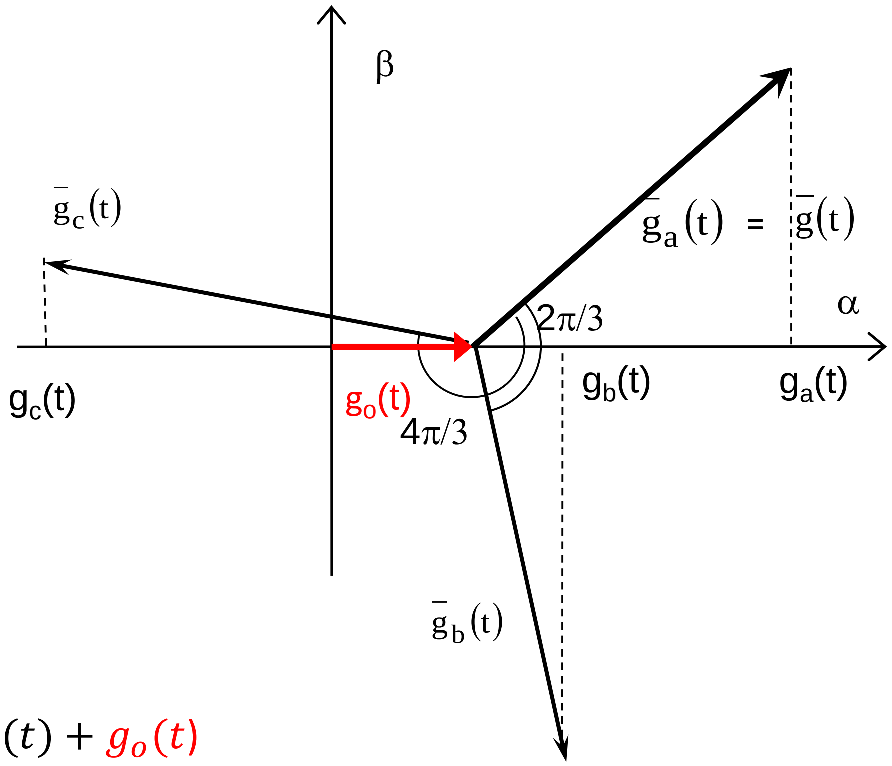
$\overline{g}(t)$ non dipende dalla componente omopolare!

$$g_\alpha(t) = \frac{2}{3}\left[g_a(t) - \frac{g_b(t)}{2} - \frac{g_c(t)}{2}\right]$$

$$g_\beta(t) = \frac{1}{\sqrt{3}}\left[g_b(t) - g_c(t)\right]$$

Vettore spaziale – terna non bilanciata

$$g_o(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3} \neq 0$$



$$g_a(t) = Re[\overline{g}(t)] + g_o(t) = Re[\overline{g}_a(t)] + g_o(t) = g_\alpha(t) + g_o(t)$$

$$g_b(t) = Re[\overline{g}(t)e^{-j2\pi/3}] + g_o(t) = Re[\overline{g}_b(t)] + g_o(t) = -\frac{1}{2}g_\alpha(t) + \frac{\sqrt{3}}{2}g_\beta(t) + g_o(t)$$

$$g_c(t) = Re[\overline{g}(t)e^{-j4\pi/3}] + g_o(t) = Re[\overline{g}_c(t)] + g_o(t) = -\frac{1}{2}g_\alpha(t) - \frac{\sqrt{3}}{2}g_\beta(t) + g_o(t)$$

Espressioni matriciali

$$\begin{bmatrix} g_\alpha \\ g_\beta \\ g_o \end{bmatrix} = \underline{g}_{\alpha\beta o} = \underline{T}_{abc\rightarrow\alpha\beta o}\ \underline{g}_{abc} = \underline{T}_{abc\rightarrow\alpha\beta o} \begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix}$$

$$\underline{g}_{abc} = \underline{T}_{\alpha\beta o\rightarrow abc}\,\underline{g}_{\alpha\beta o}$$

$$\underline{T}_{abc\rightarrow\alpha\beta o} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$$\underline{T}_{\alpha\beta o\rightarrow abc} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \end{bmatrix} = \left(\underline{T}_{abc\rightarrow\alpha\beta o}\right)^{-1}$$

$$g_\alpha(t) = \frac{2}{3}\left[g_a(t) - \frac{g_b(t)}{2} - \frac{g_c(t)}{2}\right]$$

$$g_\beta(t) = \frac{1}{\sqrt{3}}\left[g_b(t) - g_c(t)\right]$$

$$g_o(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3}$$

$$g_a(t) = g_\alpha(t) + g_o(t)$$

$$g_b(t) = -\frac{1}{2}g_\alpha(t) + \frac{\sqrt{3}}{2}g_\beta(t) + g_o(t)$$

$$g_c(t) = -\frac{1}{2}g_\alpha(t) - \frac{\sqrt{3}}{2}g_\beta(t) + g_o(t)$$

Espressioni matriciali *(nel caso in cui non sia di interesse o sia nulla la componente omopolare)*

$$\begin{bmatrix} g_\alpha \\ g_\beta \end{bmatrix} = \underline{g}_{\alpha\beta} = \underline{T}_{abc\to\alpha\beta}\ \underline{g}_{abc} = \underline{T}_{abc\to\alpha\beta} \begin{bmatrix} g_a \\ g_b \\ g_c \end{bmatrix}$$

$$\underline{g}_{abc} = \underline{T}_{\alpha\beta o\to abc}\ \underline{g}_{\alpha\beta o}$$

$$\underline{T}_{abc\to\alpha\beta} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$

$$\underline{T}_{\alpha\beta\to abc} = \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix}$$

$$g_\alpha(t) = \frac{2}{3}\left[g_a(t) - \frac{g_b(t)}{2} - \frac{g_c(t)}{2}\right]$$

$$g_\beta(t) = \frac{1}{\sqrt{3}}\left[g_b(t) - g_c(t)\right]$$

$$g_o(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3}$$

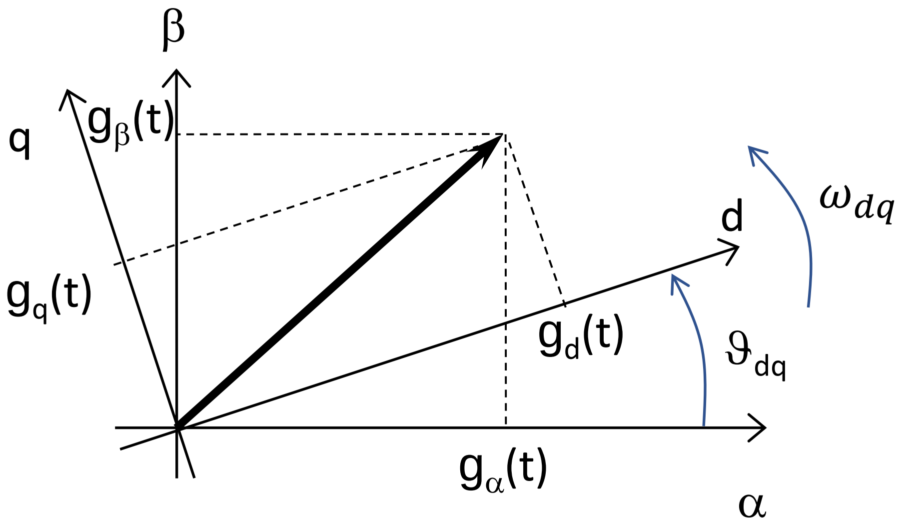
$$g'_a(t) = g_\alpha(t) \triangleq g_a(t)$$

$$g'_b(t) = -\frac{1}{2}g_\alpha(t) + \frac{\sqrt{3}}{2}g_\beta(t) \triangleq g_b(t)$$

$$g'_c(t) = -\frac{1}{2}g_\alpha(t) - \frac{\sqrt{3}}{2}g_\beta(t) \triangleq g_c(t)$$

Vettore spaziale nel sistema di riferimento rotante (ruotato): assi d-q

$$\vartheta_{dq}(t) = \int_0^t \omega_{dq}(t)dt + \vartheta_{dq}(0)$$



Da dq ad α–β

$$\bar{g}_{\alpha\beta} = e^{j\vartheta_{dq}}\,\bar{g}_{dq}$$

$$(g_\alpha + jg_\beta) = (cos\vartheta_{dq} + jsin\vartheta_{dq})\,(g_d + jg_q)$$

Da α–β a dq

$$\bar{g}_{dq} = e^{-j\vartheta_{dq}}\,\bar{g}_{\alpha\beta}$$

$$(g_d + jg_q) = (cos\vartheta_{dq} - jsin\vartheta_{dq})(g_\alpha + jg_\beta)$$

Espressioni matriciali

$$\begin{bmatrix} g_\alpha \\ g_\beta \end{bmatrix} = \underline{g}_{\alpha\beta} = \underline{T}_{dq\rightarrow\alpha\beta} \; \underline{g}_{dq} = \underline{T}_{dq\rightarrow\alpha\beta} \begin{bmatrix} g_d \\ g_q \end{bmatrix}$$

$$\underline{g}_{dq} = \underline{T}_{\alpha\beta\rightarrow dq} \; \underline{g}_{\alpha\beta}$$

$$\underline{T}_{dq\rightarrow\alpha\beta} = \begin{bmatrix} cos\vartheta_{dq} & -sin\vartheta_{dq} \\ sin\vartheta_{dq} & cos\vartheta_{dq} \end{bmatrix}$$

$$\underline{T}_{\alpha\beta\rightarrow dq} = \begin{bmatrix} cos\vartheta_{dq} & sin\vartheta_{dq} \\ -sin\vartheta_{dq} & cos\vartheta_{dq} \end{bmatrix} = (\underline{T}_{dq\rightarrow\alpha\beta})^{-1}$$

$$(\underline{T}_{dq\rightarrow\alpha\beta})^{-1} = (\underline{T}_{dq\rightarrow\alpha\beta})^{t}$$

Da dq ad α–β

$$\bar{g}_{\alpha\beta} = e^{j\vartheta_{dq}} \; \bar{g}_{dq}$$

$$(g_\alpha + jg_\beta) = (cos\vartheta_{dq} + jsin\vartheta_{dq})\,(g_d + jg_q)$$

Da α–β a dq

$$\bar{g}_{dq} = e^{-j\vartheta_{dq}} \; \bar{g}_{\alpha\beta}$$

$$(g_d + jg_q) = (cos\vartheta_{dq} - jsin\vartheta_{dq})(g_\alpha + jg_\beta)$$

Espressioni matriciali : considerazioni aggiuntive (1)

$$\underline{T}_{dq\to\alpha\beta} = \begin{bmatrix} cos\vartheta_{dq} & -sin\vartheta_{dq} \\ sin\vartheta_{dq} & cos\vartheta_{dq} \end{bmatrix} = e^{\underline{J}\vartheta_{dq}}$$

con $\underline{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Dimostrazione (usando la serie di Taylor)

$$e^{\underline{J}\vartheta_{dq}} = 1 + \underline{J}\vartheta_{dq} + (\underline{J}\vartheta_{dq})(\underline{J}\vartheta_{dq})/2 + \ldots = \begin{bmatrix} 1 - \vartheta_{dq}^2/2 + \vartheta_{dq}^4/24 - \cdots & -(\vartheta_{dq} - \vartheta_{dq}^3/6 + \vartheta_{dq}^5/120 - \cdots) \\ \vartheta_{dq} - \vartheta_{dq}^3/6 + \vartheta_{dq}^5/120 - \cdots & 1 - \vartheta_{dq}^2/2 + \vartheta_{dq}^4/24 - \cdots \end{bmatrix} = \begin{bmatrix} cos\vartheta_{dq} & -sin\vartheta_{dq} \\ sin\vartheta_{dq} & cos\vartheta_{dq} \end{bmatrix}$$

Proprietà

$e^{\underline{J}0} = \underline{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$e^{\underline{J}(\frac{\pi}{2})} = \underline{J}$

$$\frac{de^{\underline{J}\vartheta_{dq}(t)}}{dt} = e^{\underline{J}\vartheta_{dq}(t)}\,\underline{J}\omega_{dq}(t) = \omega_{dq}(t)\,\underline{J}\,e^{\underline{J}\vartheta_{dq}(t)}$$

Di norma non vale la proprietà commutativa, ma qui vale!

($e^{jo} = 1$ $\quad e^{j\pi/2} = j$ $\quad \frac{de^{j\vartheta_{dq}(t)}}{dt} = e^{j\vartheta_{dq}(t)}\mathrm{j}\,\omega_{dq}(t) = \omega_{dq}(t)\,je^{j\vartheta_{dq}(t)}$)

(Per confronto)

Espressioni matriciali : *considerazioni aggiuntive (2)*

espressioni matriciali

$$\underline{g}_{\alpha\beta} = e^{\underline{J}\vartheta_{dq}}\ \underline{g}_{dq}$$

$$\underline{g}_{dq} = e^{-\underline{J}\vartheta_{dq}}\ \underline{g}_{\alpha\beta}$$

Qui NON VALE la proprietà commutativa.

espressioni complesse

$$\bar{g}_{\alpha\beta} = e^{j\vartheta_{dq}}\ \bar{g}_{dq}$$

$$\bar{g}_{dq} = e^{-j\vartheta_{dq}}\ \bar{g}_{\alpha\beta}$$

Qui VALE la proprietà commutativa.

Vale inoltre la seguente proprietà

$da:\ (\underline{T}_{dq\rightarrow\alpha\beta})^{-1} = (\underline{T}_{dq\rightarrow\alpha\beta})^{t}$ → $\left(e^{\underline{J}\vartheta_{dq}}\right)^{t} = \left(e^{\underline{J}\vartheta_{dq}}\right)^{-1} = e^{-\underline{J}\vartheta_{dq}}$

Bilanci energetici *(potenza trifase, energia trifase) (1)*

In funzione delle grandezze trifase

$$p(t) = u_a(t)i_a(t) + u_b(t)i_b(t) + u_c(t)i_c(t)$$

$$p(t) = \underline{u}_{abc}^t \underline{i}_{abc}$$

che diventa

$$p(t) = (\underline{T}_{\alpha\beta o\to abc}\underline{u}_{\alpha\beta o})^t(\underline{T}_{\alpha\beta o\to abc}\underline{i}_{\alpha\beta o}) = (\underline{u}_{\alpha\beta o}^t\underline{T}_{\alpha\beta o\to abc}^t)(\underline{T}_{\alpha\beta o\to abc}\underline{i}_{\alpha\beta o}) = \underline{u}_{\alpha\beta o}^t\underbrace{(\underline{T}_{\alpha\beta o\to abc}^t\,\underline{T}_{\alpha\beta o\to abc})}_{\begin{bmatrix}3/2 & 0 & 0\\ 0 & 3/2 & 0\\ 0 & 0 & 3\end{bmatrix}}\underline{i}_{\alpha\beta o}$$

e dopo aver sviluppato i prodotti matriciali fra le matrici di trasformazione…….

Bilanci energetici (2)

…la potenza istantanea risulta

$$p(t) = \frac{3}{2}\left[u_\alpha(t)i_\alpha(t) + u_\beta(t)i_\beta(t)\right] + 3u_o(t)i_o(t) = \frac{3}{2}\left[\,\underline{u}_{\alpha\beta}^t(t)\underline{i}_{\alpha\beta}(t)\,\right] + 3u_o(t)i_o(t)$$

Il primo addendo (la parte non dipendente dalle componenti omopolare) vale anche

$$p(t) = \frac{3}{2}\left[u_\alpha(t)i_\alpha(t) + u_\beta(t)i_\beta(t)\right] = \frac{3}{2}\left[\,\underline{u}_{\alpha\beta}^t(t)\,\underline{i}_{\alpha\beta}(t)\,\right] = \frac{3}{2}\left[\underline{u}_{dq}^t(t)\left(\underline{T}_{dq\to\alpha\beta}\right)^t \underline{T}_{dq\to\alpha\beta}\,\underline{i}_{dq}(t)\,\right] =$$

$$= \frac{3}{2}\left[\underline{u}_{dq}^t(t)\left(\underline{T}_{dq\to\alpha\beta}\right)^{-1} \underline{T}_{dq\to\alpha\beta}\,\underline{i}_{dq}(t)\,\right] = \frac{3}{2}\left[\underline{u}_{dq}^t(t)\,\underline{i}_{dq}(t)\,\right]$$

Le trasformazioni da *abc* in $\alpha\beta$ e *dq* non *sono conservative per le potenze*