

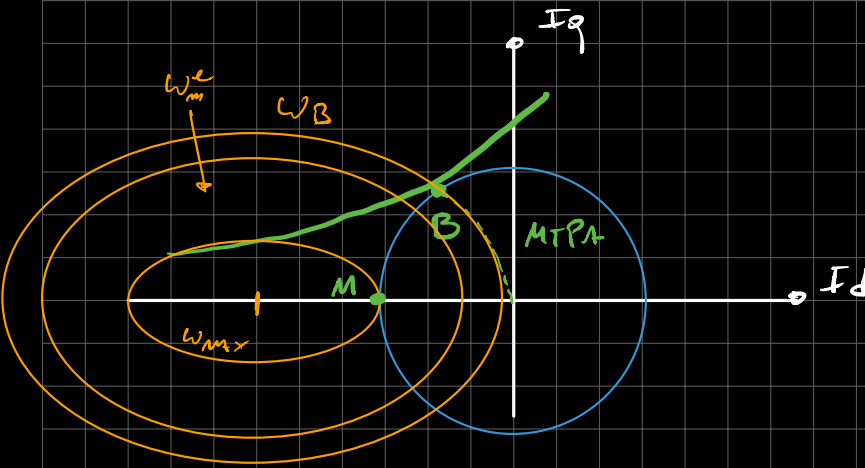
CALCOLO  $\omega_B$

$$\begin{cases} V_d = -\omega_B L_p = -\omega_B L_p I_q \\ V_q = \omega_B (\lambda_m + L_d I_d) \end{cases}$$

$$V_d^2 + V_q^2 = V_m^2$$

$$(\omega_B L_p I_q)^2 + \omega_B^2 (\lambda_m + L_d I_d)^2 = V_m^2$$

$$\omega_B = \frac{V_m}{\sqrt{(\lambda_m + L_d I_d)^2 + (L_p I_q)^2}}$$

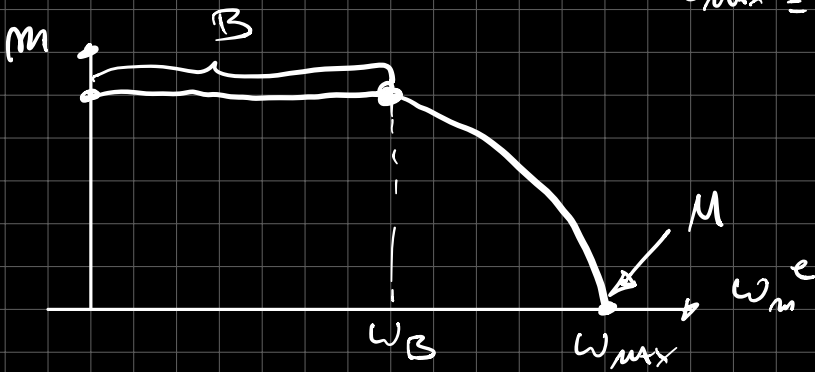


$$\omega_m^e = \omega_{max} \quad (\text{Punto M})$$

$$\begin{cases} V_d = \omega_{max} L_p \\ V_q = \omega_{max} \lambda_m \end{cases} \quad \begin{cases} I_d = -I_m \\ I_q = 0 \end{cases}$$

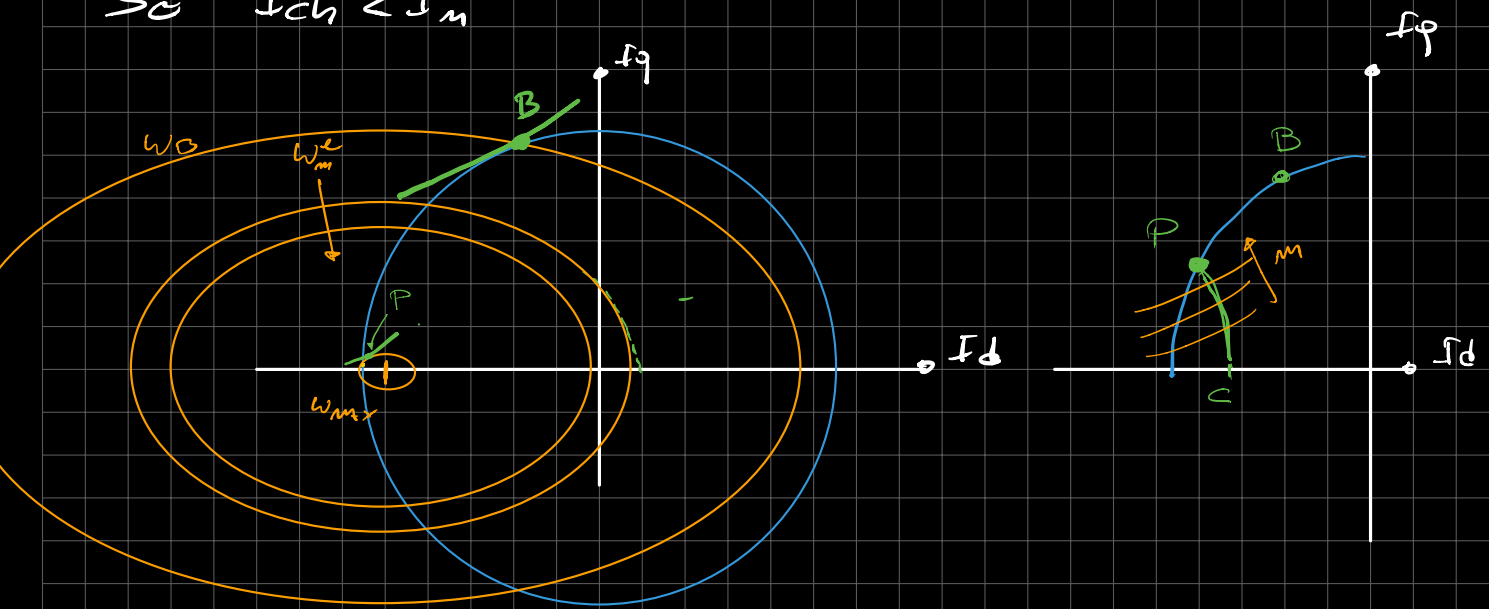
$$\begin{cases} V_d = 0 \\ V_q = \omega_{max} (\lambda_m - L_d I_m) \end{cases}$$

$$\omega_{max} = \frac{V_m}{\lambda_m - L_d I_m}$$

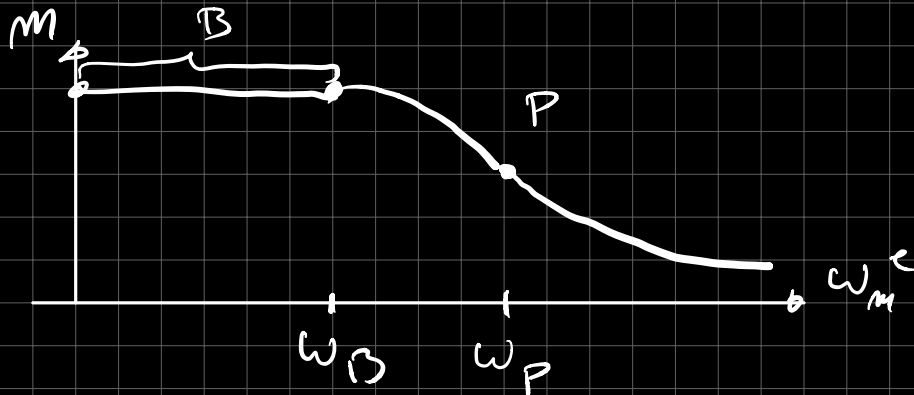


QUESTO SE  $\frac{\lambda_m}{L_d} > I_m$

Se  $I_{ch} < I_m$



Per  $\omega$  più elevate di  $\omega_p$  sopra MTPV

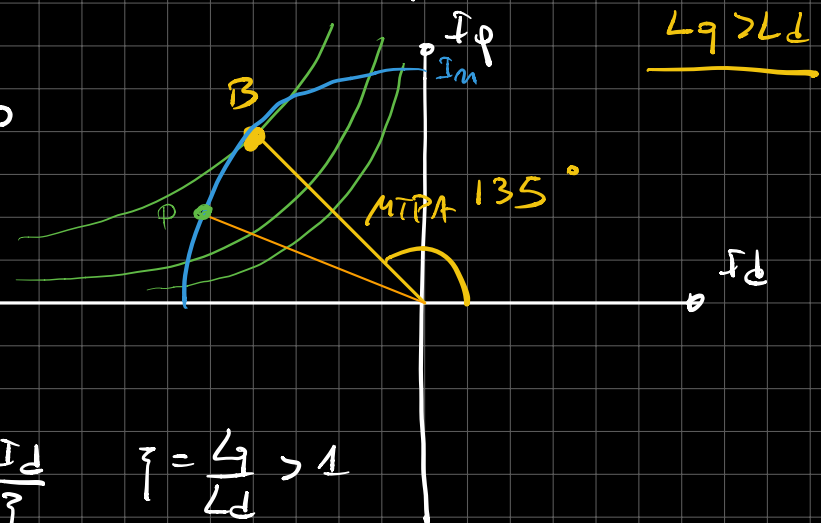


Si può dimostrare che il  $\omega_{op}$  MTPV risulta essere il seguente

$$I_q = \frac{L_d}{L_q} \sqrt{\frac{-(I_d + \frac{\lambda_m}{L_d}) [\lambda_m + (L_d - L_q) I_d]}{L_d - L_q}}$$

Nel caso REL  $\lambda_m = 0$

MTPA :  $\alpha_i = 135^\circ$   
 $I_d = -I_q = -\frac{I}{\sqrt{2}}$



MTPV :  $I_p = -\frac{L_d}{L_q} I_d = -\frac{I_d}{\lambda}$       $\lambda = \frac{L_q}{L_d} > 1$

# ESEMPIO ① MOTORE IPM

$$2p = 4$$

$$\lambda_m = 0,4 \text{ Vs}$$

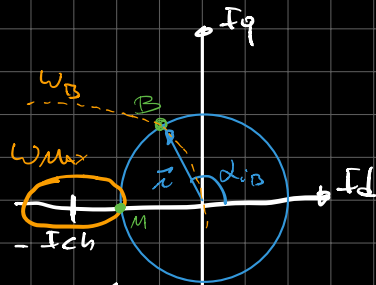
$$L_d = 16 \text{ mH}$$

$$L_q = 20 \text{ mH}$$

$$R \approx 0$$

$$U_m = 210 \text{ V}$$

$$I_m = 20 \text{ A}$$



$$I_{ch} = \frac{\lambda_m}{L_d} = \frac{0,4}{0,016} = 25 \text{ A} > I_m$$

a) Calcolare la massima coppia che il motore può sviluppare

$$\cos \alpha_{iB} = \frac{-\lambda_m + \sqrt{\lambda_m^2 + 8(L_d - L_q)^2 I_m^2}}{4(L_d - L_q) I_m}$$

$$= \dots = -0,186 \quad \alpha_{iB} = 103,72^\circ$$

$$I_{dB} = I_m \cos \alpha_{iB} = -3,72 \text{ A}$$

$$I_{pB} = I_m \sin \alpha_{iB} = 19,65 \text{ A}$$

$$m_B = \frac{3}{2} p \left[ \lambda_m I_{qB} + (L_d - L_q) I_{dB} I_{pB} \right]$$

$$= \dots = 23,55 \text{ Nm}$$

b) Calcolare la velocità base

$$V_d^2 + V_q^2 = U_m^2$$

$$\begin{cases} I_d = I_{dB} \\ I_q = I_{pB} \\ \omega_m = \omega_B \end{cases}$$

$$(\omega_B L_q)^2 + (\omega_B \lambda_d)^2 = U_m^2$$

$$\omega_B^2 \left[ (L_q I_{pB})^2 + (\lambda_m + L_d I_{dB})^2 \right] = U_m^2$$

$$\omega_B = \frac{U_m}{\sqrt{(L_q I_{pB})^2 + (\lambda_m + L_d I_{dB})^2}} = \frac{210}{\sqrt{(0,02 \cdot 19,65)^2 + (0,4 + 0,016 \cdot -3,72)^2}}$$

$$= 503,8 \frac{\text{rad}}{\text{s}}$$

$$\omega_{Bm} = \frac{\omega_B}{p} = 251,9 \frac{\text{rad}}{\text{s}} = 1929 \text{ rpm}$$

c) Calcolare la velocità massima  $\omega_{max}$

$$V_d^2 + V_q^2 = U_m^2$$

$$\omega_m^e = \omega_{max}$$

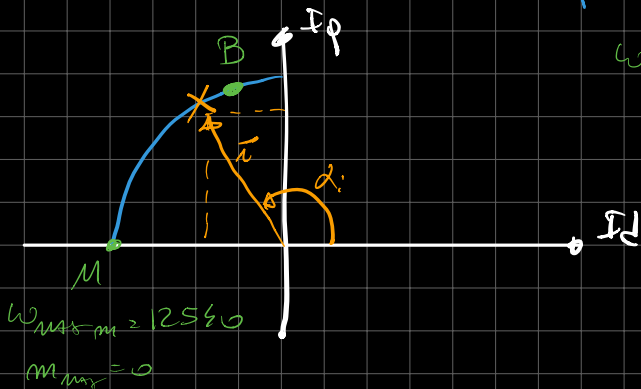
$$I_p = 0$$

$$I_d = -I_m$$

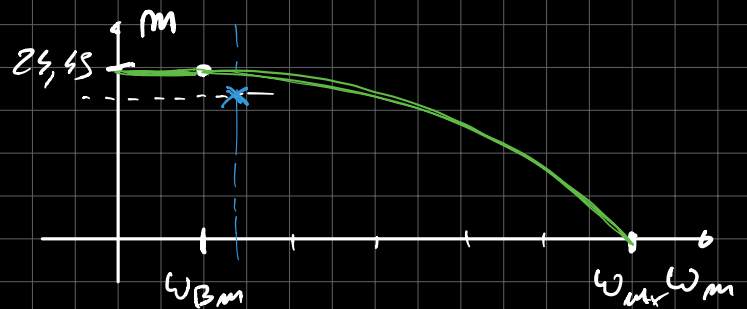
$$\omega_{max} = \frac{U_m}{\lambda_m - L_d I_m} = \frac{210}{0,4 - 0,016 \cdot 20} = 2625 \frac{\text{rad}}{\text{s}}$$

$$\omega_{max \text{ rpm}} = \frac{\omega_{max}}{p} = \frac{1312,5 \frac{\text{rad}}{\text{s}}}{5} = 12540 \text{ rpm}$$

d) Calcolare la massima coppia che si può ottenere alla velocità di 2200 rpm



$$\omega_{T2m} = 1929 \text{ rpm} \quad 25,55 \text{ Nm}$$



Nel punto X posso scrivere:

$$V_d^2 + V_q^2 = U_m^2$$

$$\begin{cases} \omega_m^e = 960 \frac{\text{rad}}{\text{s}} = p \cdot \omega_m = p \cdot \frac{2\pi \cdot 2200}{60} \\ I_d = I_{dx} \\ I_p = I_{px} \end{cases}$$

$$\left( \omega_m^e L_q I_{qx} \right)^2 + U_m^e^2 \left( \lambda_m + L_d I_{dx} \right)^2 = U_m^2$$

$$\left( L_q I_{qx} \right)^2 + \left( \lambda_m + L_d I_{dx} \right)^2 = \left( \frac{U_m}{\omega_m^e} \right)^2$$

$$\left( L_q I_m \sin \alpha_x \right)^2 + \left( \lambda_m + L_d I_m \cos \alpha_x \right)^2 = \left( \frac{U_m}{\omega_m^e} \right)^2$$

$$\left( 0,4 \sin \alpha_x \right)^2 + \left( 0,4 + 0,32 \cos \alpha_x \right)^2 = 0,2084$$

$$1 - \cos^2 \alpha_x$$

$$-0,0576 \text{ GS}^2 \alpha_i x + 0,256 \text{ GS} \alpha_i x + 0,1116 = 0$$

$$\text{GS} \alpha_i x = \begin{cases} -0,4 \\ 4,84 \end{cases}$$

$$I_{dx} = I_m \text{GS} \alpha_i x = 2(-0,4) = -0,8 \text{ A}$$

$$I_{px} = \sqrt{2^2 - 0,8^2} = 1,83 \text{ A}$$

$$m_x = \frac{3}{2} p \left[ \lambda_m I_{qx} + (L_d - L_q) I_{dx} I_{px} \right]$$

$$= 23,76 \text{ Nm}$$

ESEMPIO ② Motore REL

$$\lambda_m = 0$$

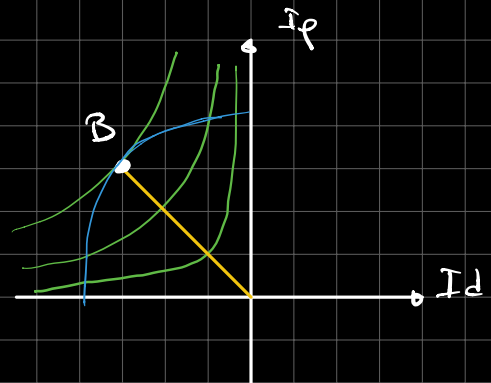
$$L_d = 100 \text{ mH}$$

$$L_q = 500 \text{ mH}$$

$$2p = 4$$

$$U_m = 180 \text{ V}$$

$$I_m = 19,85 \text{ A}$$



e+b: Determinare il punto base

$$\text{MTPA} : \alpha_i = 45 + 90 = 135^\circ$$

$$I_{dB} = -I_m / \sqrt{2} = -19,85 / \sqrt{2} = -14,0 \text{ A}$$

$$I_{qB} = I_m / \sqrt{2} = 19,85 / \sqrt{2} = 14,0 \text{ A}$$

$$m_B = \frac{3}{2} p (L_d - L_q) I_{dB} I_{qB} = \frac{3}{2} \cdot 2 \cdot (0,1 - 0,5) (-14,0) 14,0$$

$$= 70,59 \text{ Nm}$$

$$\lambda_d = L_d I_{dB} = -0,14 \text{ Vs}$$

$$\lambda_q = L_q I_{qB} = 0,5 \cdot 14,0 = 7,0 \text{ Vs}$$

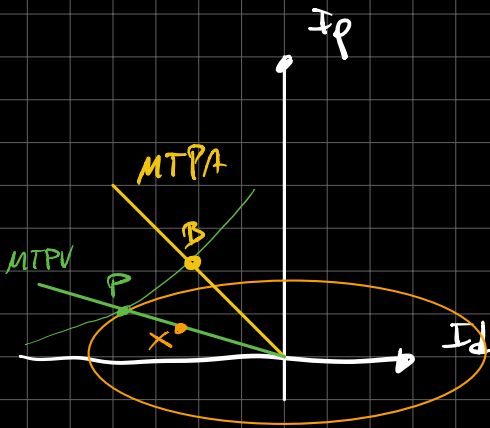
$$\omega_B = \frac{U_m}{\lambda} = \frac{U_m}{\sqrt{\lambda_d^2 + \lambda_q^2}}$$

$$= \frac{180}{\sqrt{0,14^2 + 7,0^2}}$$

$$= 25,6 \text{ rad/s}$$

$$\omega_{BM} = 2\pi \frac{220}{60} = 220 \text{ rpm}$$

c) Calcolare la torquenza MTPV



$$I_d' = \frac{L_d}{L_q} I_d$$

$$V_d^2 + V_q^2 = U_m^2$$

$$(L_q I_q)^2 + (L_d I_d)^2 = \left( \frac{U_m}{\omega_r} \right)^2 \quad \text{ELLISSE}$$

$$(L_q I_q)^2 + \left( L_d \frac{L_q I_d'}{L_d} \right)^2 = \left( \frac{U_m}{\omega_r} \right)^2$$

$$(L_q I_q)^2 + (L_q I_d')^2 = \left( \frac{U_m}{\omega_r} \right)^2 \quad \text{CERCHIO } I_d' - I_q$$

$$m = \frac{3}{2} p (L_d - L_q) \frac{L_q I_d' I_q}{L_d}$$

$$\text{MTPV} \quad I_d' = -I_q \quad \Leftrightarrow \quad \frac{L_d}{L_q} I_d = -I_q \quad I_d = -\frac{L_q}{L_d} I_q = -\frac{1}{\tau} I_q$$

$$\tan \delta_{ip} = \frac{I_q}{I_d} = -\frac{1}{\tau}$$

Nel nostro esempio  $\tan \delta_{ip} = -\frac{1}{\tau} = -\frac{1}{5} = -0.2$

$$\delta_{ip} = 168,7^\circ \quad I_{d_p} = I_m \cos \delta_{ip} = -19,63 \text{ A}$$

$$I_{q_p} = I_m \sin \delta_{ip} = 2,15 \text{ A}$$

La coppia in P vale:  $m_{mp} = \frac{3}{2} p (L_d - L_q) I_{d_p} I_{q_p}$   
 $= \dots = 27,15 \text{ Nm}$

$$\omega_{mp}^2 = \frac{U_m}{\sqrt{L_d^2 + L_q^2}} = \dots = 119,7 \frac{\text{rad}}{\text{s}}$$

$$\omega_{mp} = 59,85 \frac{\text{rad}}{\text{s}} = 575 \text{ rpm}$$

d) Calcolare la massima coppia che si ottiene a 750 rpm

Pole 750 rpm  $\Rightarrow \omega_{mf} = 575 \text{ rpm}$  mi modo sul MTPU

$$\omega_{mf} = \frac{2\pi \cdot 750}{60} = 78,5 \frac{\text{rad}}{\text{s}}$$

$$\omega_{mf}^e = \omega_{mf} \cdot p = 2 \cdot 78,5 = 157 \frac{\text{rad}}{\text{s}}$$

$$\lambda_d^2 + \lambda_q^2 = \left( \frac{U_m}{\omega_{mf}^e} \right)^2$$

$$\begin{aligned} I_d &= -3 I_q \\ &= -\frac{L_q}{L_d} I_q \end{aligned}$$

$$\begin{aligned} (L_d I_d)^2 + (L_q I_q)^2 &= \left( \frac{U_m}{\omega_{mf}^e} \right)^2 \\ \left( L_d \frac{L_q}{L_d} I_q \right)^2 + (L_q I_q)^2 &= \left( \frac{U_m}{\omega_{mf}^e} \right)^2 \end{aligned}$$

$$2(L_q I_q)^2 = \left( \frac{U_m}{\omega_{mf}^e} \right)^2 \Rightarrow I_q = \frac{U_m}{\omega_{mf}^e L_q} \frac{1}{\sqrt{2}} = \dots = 1,62 \text{ A}$$

$$I_d = -3 I_q = -5 \cdot 1,62 = -8,1 \text{ A}$$

$$I = \sqrt{I_d^2 + I_q^2} = 8,26 \text{ A} < I_m$$

$$m = \frac{3}{2} p (L_d - L_q) I_d I_q = \frac{3}{2} \cdot 2 \cdot 0,4 \cdot 1,62 \cdot 8,1 = 15,75 \text{ Nm}$$

