Heat transfer in building elements in steady state conditions

Basics of heat transfer

Thermal conduction

The heat transfer conduction is the exchange of energy between two zones at different temperatures in a solid medium, a liquid or a gas with negligible transfer of material. In each part of the medium interested by this phenomenon the temperature of each element is a function of its position and time instant considered. For an orthogonal co-ordinate system, it can be written:

$$t = f(x, y, z, \tau) \tag{1}$$

In the following work the body will be assumed as continuous, isotropic and with physical characteristics unchangeable with respect to time and temperature. The problem of the calculation of the conduction heat transfer is related to the determination of temperatures in the considered body.

The determination of the function (1) can be obtained by resorting to the principle of energy conservation with respect to a system with a generic volume V (Figure 1). In this way, in each time step $d\tau$ the equation of conservation energy can be written as:

$$dQ_e + dQ_i = dU$$

where dQ_e is the heat exchanged, dQ_i is the heat generation inside the system and dU is the variation of internal energy of the system.

(2)



Figure 1: Heat conduction in a general medium

The heat exchanged dQ_e conducted into the volume *Vol* across the section *S* can be calculated by means of the Fourier equation. In this way the rate at which heat is conducted across a surface, per unit area, in the direction of \vec{n} is given by:

$$q^*_d = -k(gradt) \cdot \vec{n} = -k \frac{\partial t}{\partial n}$$
(3)

where k is the conductivity of the medium.

Considering the medium as isotropic, in case of no internal heat generation the Fourier equation (3) can be written as:

$$\frac{\partial t}{\partial \tau} = a \nabla^2 t \tag{4}$$

where $a = k/(\rho c_{\rho})$ is the thermal diffusivity, ρ is the density and c_{ρ} the specific heat capacity. The specific heat flux (3.4) can be written as:

 $q^* = -k\nabla t \tag{5}$

If the heat flux is one dimensional the equations become:

$$\frac{\partial t}{\partial \tau} = a \frac{\partial^2 t}{\partial x^2} \tag{6}$$

$$q^* = -\lambda \frac{\partial t}{\partial x} \tag{7}$$

where *x* is the axe parallel to the direction of the propagating heat flux.

Thermal conduction in steady state conditions in a linear element

Let us consider a piece of material with parallel surfaces as usual happens in a wall section. As shown in Figure 2, considering steady state conditions, integrating equation (7) leads to the following equation:

$$q^* = \frac{k}{\Delta x} (t_{si} - t_{se}) \tag{8}$$

As can be seen, it is evident an analogy between the heat transfer problem and the electrical Ohm equation. In equation (8) the ratio $k / \Delta x = \Gamma$, named thermal conductance; the same problem can be solved by using the thermal resistance $R = \Delta x/k$. When a wall is made of different material layers, equation (8) can be written as:

$$q^* = \frac{t_{si} - t_{se}}{\sum_j R_j} \tag{9}$$

Sometimes constructions may not be built with uniform and homogeneous materials; in these cases the thermal resistance has to be evaluated by means of laboratory tests and/or with calculations. In any case, once determined the thermal resistance or conductance of the building element and the thickness Δx , the equivalent conductivity k^* of the building element can be used.

In Figure 3 some examples of non-homogeneous wall structires are shown. Typically bricks are materials where the thermal conductivity is provided as an equivalent value, since the resistance of the wall is defined based on specific calculations and/or laboratory tests. Also

structural slabs with lighting filling to decrease the weight are commonly non-homogeneous structures.



Figure 2: Heat conduction through a building element from inner surface to outer surface



Figure 3: Some examples of non-homogeneous materials: bricks with different grade of porosity (from a to c) and horizontal slab (from d to f)

Thermal conductivity of different materials

Thermal conductivity is a material property. Usualy it is named k or λ and it is expressed in [W/(m K)]. The thermal conductivity of a material depends on its temperature, density and moisture content. The thermal conductivity normally found in tables is the value valid for normal room temperature. This value will not differ much between 0°C and 70° C. When high temperatures are involved, in ovens for instance, the influence of temperature has to be taken into account.

Generally light materials are better insulators than heavy materials, because light materials often contain air enclosures. Dry still air has a very low conductivity. A layer of air will not

always be a good insulator though, because heat is easily transferred by radiation and convection, as shown later.

When a material, for instance insulating material, becomes wet, the air enclosures fill with water and, because water is a better conductor than air, the conductivity of the material increases. That is why it is very important to install insulation materials when they are dry and take care that they remain dry.

The thermal conductivity is a function of the density. This can be seen in Figure 4 for concrete-based materials and in Figure 5 for building materials with thermal conductivity below 1 W/(m K). As can be seen, insulants are supposed to have low density.



Concrete-based materials

Figure 4: Thermal conductivity vs. density in concrete based materials



Materials with $\lambda < 1$ W/(m K)

Figure 5: Thermal conductivity vs. density in materials with k < 1 W/(m K)

In Table 1 a list of some building materials is shown with their thermal conductivity. The values reported have to be considered as qualitative, since for most of them the thermal conductivity depends on the density and different values may be found in commerce. Usually it is recommended to increase the thermal conductivity by about 10% to consider the vapour content in operating conditions in reality (the declared values have been established in laboratory with low content of humidity).

Beside the thermal conductivity, usually the density and the vapour permeability are also usual parameters for building materials which are required for the building energy calculations

Material	Thermal conductivity [W/(m K)]		
Light concrete	0.40		
Reinforced concrete	2.00		
Glass	1.00		
Stone	2.50		
Bricks	0.50		
Hollow bricks	0.40		
Internal plaster	0.50		
External plaster	0.90		
Gypsum board	0.12		
Rock wool	0.045		
Glass wool	0.05		
Expanded polystyrene foam (EPS)	0.04		
Extruded polystyrene foam (EPS)	0.03		
Polyurethane	0.035		
Glass foam	0.052		
Cork boards	0.045		
PVC	0.19		

Table 1: Typical bvalues of thermal conductivity of the different materials

Heat transfer on internal and external surfaces

In general on a certain surface we have to consider the incoming heat fluxes; if we do not consider the fluxes due to solar radiation or other sources (as lighting) the heat fluxes on a surface are the conduction, convection and infrared radiation, as shown in Figure 6.



Figure 6: Heat flows to be considered on a generic surface without solar radiation and other radiant loads (e.g. lighting)

As far as the conduction is concerned, the heat flow can be calculated under steady state or under unsteady state conditions. The 1-D conduction heat flux under steady state conditions can be calculated as already reported in equation (9) in terms of specific heat flux. If we are interested in the heat flux, the following equation can be written, explicitating the generic surface S:

$$q_d = S (t_{s,i} - t_{s,o})/R$$
(10)

where *S* is the area of the considered surface, *R* is the thermal resistance of the wall between internal surface at temperature $(t_{s,i})$ and the outer surface temperature $(t_{s,o})$, which could be external or another indoor environment at the same temperature or at

different temperature.

The convective heat flow between the surface *S* (with uniform temperature t_s) and the air (with uniform temperature t_a) can be expressed as:

$$q_c = S h_c \left(t_s - t_a \right) \tag{11}$$

where h_c is the convective heat flow between surface and air.

As for the radiant thermal flow between two surfaces (with absolute temperatures $T_{s,1}$ and $T_{s,2}$) the hypothesis of radiantly grey surfaces with emissivity close to one can be assumed, which leads to:

$$q_r = \sigma S \left(T^4{}_{s1} - T^4{}_{s2} \right) / b \tag{12}$$

where

$$b = (1/\varepsilon_1 + 1/\varepsilon_2 - 1) \tag{13}$$

and depends on the emissivity factors of each surface (ε_1 and ε_2), σ is the Stefan-Boltzman constant equal to 5,67x10⁻⁸ W/(m² K⁴). Under the hypothesis of radiantly grey surfaces with emissivity close to one ($b \cong 1$), considering the usual range of temperatures, it is possible to simplify the equation as:

$$q_r = 4 \sigma T^3_{s,m} S [(T_{s1} - T_{s2})]$$
(14)

where and mean value $T_{s,m}$. This equation can be written as:

$$q_r = h_r S [(T_{s1} - T_{s2})]$$
(15)

where h_r is the radiant heat exchange coefficient of the surface, which can be assumed to be 5.5 W/(m² K).

For the indoor environment, considering the operative temperature of the room ti, which takes into account the radiant and the convective heat fluxes, the following equation can be written for the internal surface heat exchange coefficient:

$$h_{si} = h_{ci} + h_{ri} = 1/R_{si}$$
(16)

Similarly for the external surface heat exchange coefficient:

$$h_{se} = h_{ce} + h_{re} = 1/R_{se} \tag{17}$$

The ambient temperature is considered in the calculations as boundary condition outside. Depending on the calculation and/or program used the heat transfer coefficient h_i and h_e , expressed in [W/(m² K)] or the reciprocal thermal resistance R_{si} and R_{se} expressed in [m² K / W].

Position of the Surface	Rsi	Rse
	[m2 K / W]	[m2 K / W]
Vertical or inclination α of the surface to the horizontal	0.13	0.04
such that $90^{\circ} \ge \beta \ge 60^{\circ}$ (heat flow direction $\pm 30^{\circ}$ from the		
horizontal plane)		

Table 2: Surface thermal resistances used in calculations.

Vertical or inclination α of the surface to the horizontal	0.10	0.04
such that $60^{\circ} \ge \beta \ge 0^{\circ}$ (heat flow direction more than 30°		
from the horizontal plane)		

Heat transfer in cavities

In cavities the heat transfer is quite complex. First of all in the cavity there are two surfaces with a view factor equal to one. Hence the radiant heat exchange is a function of the emissivity of the two surfaces, namely the coefficient b expressed in equation (13). Depending on the emissivity of one surface the coefficient b changes and hence the thermal resistance can vary.

As can be seen in Figure 7, the thickness of the cavity plays an important role in the overall thermal resistance of the cavity. If we consider a cavity of few millimetres we can assume that due to the limited thickness of the cavity, the two main processes of heat transfer are the infrared radiation between the surfaces and the thermal conduction of the air inside the cavity. As a matter of fact if the thickness of the cavity is limited, the air inside acts as insulant material; an increase of thickness has as effect a rise in the thermal resistance, according to the equation (8). The linear trend of the thermal resistance of a cavity can be seen up to between $0.5 \div 1.0$ cm (Figure 7). If we increase the distance between the two surfaces then convection starts and the linear trend starts a decrease in the incline, reaching a maximum around 2 cm and then decreasing again down to a fixed value at around 4 cm. The difference between the continuous line and the dotted lines is related to the emissivity coefficient of the surfaces: b = 0.82 means that both surfaces have $\varepsilon_1 = \varepsilon_2 = 0.9$, while if one of the surface has lower emissivity b is lower and the thermal resistance of the cavity will be higher due to the higher resistance in the infrared heat exchange.



VERTICAL CAVITY

Figure 7: Thermal resistance of a vertical cavity as a function of b, see equation (13)

The chart of Figure 8 refers to a vertical cavity, when dealing with horizontal cavities, the diagrams of Figure 8.A for ascendant heat flux and Figure 8.B for descendent heat flux apply.



Figure 8: Thermal resistance of a horizontal cavity as a function of b, see equation (13): ascendant flux (A) and descendent flux (B).

Thermal transmittance of a building component

In the problems of heating/cooling buildings usually the indoor temperature t_i and the outdoor temperature t_{amb} are used. This means that the overall balance of the inner surface has to deal with both convection with the air and infrared radiation with the other surfaces (Figure 9). Looking at the heat flow through a building element in steady state conditions means that the indoor temperature is fixed and the outdoor temperature is constant and neither solar radiation nor internal gains are present. In this case the overall surface balance can be estimated by an overall surface heat exchange coefficient h_{si} (or via the reciprocal internal resistance R_{si}):

$$q^* = h_{Si}(t_i - t_{Si}) = \frac{t_i - t_{Si}}{R_{Si}}$$
(18)

The same problem has to be solved on the external surface of the wall. In this case there are two possible ways. In the simplest model the infrared radiation exchanged between the surface and the sky is neglected, thus leading to an overall external surface heat exchange coefficient h_{se} (or the reciprocal external resistance R_{se}):

$$q^* = h_{se}(t_{se} - t_{amb}) = \frac{t_{se} - t_{amb}}{R_{se}}$$
(19)

In this way a wall can be schematized as a series of thermal resistances and the heat flux can be estimated as:

$$q^* = \frac{t_i - t_{amb}}{R_{si} + \sum_j R_j + R_{se}} = \frac{t_i - t_{amb}}{R_{tot}}$$
(20)

The reciprocal of the overall thermal resistance is named transmittance of the wall, known also as U-value: $U = 1 / R_{tot}$, therefore the heat flux through a linear wall can be expressed as:

$$q^* = U(t_i - t_{amb}) \tag{21}$$

As already said we are interested in the heat flux, hence the following equation is commonly used:

$$q = U S (t_i - t_{amb}) \tag{22}$$



Figure 9: Heat conduction through a building element: from indoor temperature to outdoor temperature

Example 1: U-value of an opaque wall of a building stock



Example 2: U-value of an opaque wall of with insulation

In ythis example the same materials as the example 1 are used. Instead of the hollow brick there is an insulation layer (yellow layer).



Thermal transmittance of windows

A window is an opening in a wall or roof that allows the passage of light and sometimes air. Windows (Figure 10) are usually composed by a glazing fixed on a sash (which is the movable part of the window), connected by hinges to the frame (fixed part) which is then fixed on the wall or on the roof. Most of glazed windows may be opened (operable windows). Usually the frame and the sash are made of wood, plastic, or metal (usually aluminium), but recently there are also combination of materials (Figure 11).



Figure 10: Main definition of a window and its components: example of sash opening (A, Source: Trompe l'oeil) and definitiuon according to EN ISO 10077 (B, Source: [1])



Figure 11: Examples of sash and frame: PVC (A), wood (B), aluminium (C).

The frame (frame and sash) have a reference surface

The glazed area A_g of a window is the smaller of the visible areas seen form both sides (see Figure 12); any overlapping of gaskets is ignored. This definition can be given also to the opaque panel area (A_p) in case of doors.

The total visible perimeter of the glazing l_g is the sum of the visible perimeter of the glass panes in the window. If the perimeters are different on either side of the pane, then the larger of the two shall be used (see again Figure 12). The same definition can be applied to doors for defining the total perimeter of the opaque panel l_p .



Figure 12: Explication of the glazing area (A_g) and of the total perimeter of the glazing I_g .

The internal projected frame area $(A_{f,i})$ is the area of the projection of the internal frame, including sashes, if present, on a plane parallel to the glazing panel. The external projected frame area $(A_{f,e})$ is the area of the projected of the external frame, including sashes if present on a plane parallel to the glazing panel. The frame area is the larger of the two projected areas seen from both sides. Referring to Figure 13, the frame area is the maximum value between $A_{f,i}$ and $A_{f,e}$. Referring to the figure, $A_{f,i} = A_1 + A_3$ and $A_{f,e} = A_5 + A_6$. The area of the window (Aw) is the sum of the frame area and the glazing area:

$$A_w = A_g + A_f \tag{23}$$



Figure 13: Explication of the frame area A_f.

Thermal transmission of glazing

The glass present a relatively high thermal conductivity $k = 1 \text{ W}/(\text{m}^2 \text{ K})$. As a result, a single glass of 4 mm has a transmittance of about 6 W / (m²K), as shown in the example 1.

Example 1: single glass

$$s = 4 \text{ mm} = 0,004 \text{ m}$$

$$\lambda = 1 \text{ W/(m K)}$$

$$R_v = \frac{s}{\lambda} = \frac{0,004 \text{ m}}{1 \text{ W/(m K)}} = 0,004 \text{ m}^2 \text{ K / W}$$

$$R_i = 1/h_i = 1/8 = 0,125 \text{ m}^2 \text{ K / W}$$

$$R_e = 1/h_e = 1/25 = 0,04 \text{ m}^2 \text{ K / W}$$

$$R_{tot} = R_e + R_i + R_v = 0,04 + 0,004 + 0,125 = 0,169 \text{ m}^2 \text{ K / W}$$

$$U_g = 1/R_{tot} = 1 / 0,169 = 5,92 \text{ W/(m}^2 \text{ K)}$$

If we want to increase the efficiency of the glazing we need to consider a double double glazing separated by a space as shown in Example 2.



The performance can further improve by measn of a low-emissivity coating on one of the two surfaces of the double glass, as shown in the Example 3.

Example 3: double glass with low emissivity coating



In Table 3 it is possible to see the resistances which can be achieved with 2 or 3 glasses and qith or without the low-emissivity coating.

Thickness			Thermal res <i>R</i> s m ² ·K/V	istance V	
of air space	One side coated with a normal emissivity of				Both sides uncoated
mm	0,1	0,2	0,4	0,8	
6	0,211	0,191	0,163	0,132	0,127
9	0,299	0,259	0,211	0,162	0,154
12	0,377	0,316	0,247	0,182	0,173
15	0,447	0,364	0,276	0,197	0, <mark>186</mark>
50	0,406	0,336	0,260	0,189	0,179

Table 3: Specific thermal resistance of different air spaces possibilities in a vertical window

An additional improvement in the performance of the Ug can be obtained by filling the air space with a gas which may allow to achieve a lower thermal conductivity value with respect to the air. The gases used are mainly krypton, argon, SF_6 and xenon. The Figure 14 shows the impact of argon compared to gas, while in Table 4 a complete overview of the different combinations of gas and glazing systems can be seen.



Figure 14: Effect of filling with argon the air space compared to air

Glazing			Thermal transmittance for different types of gas space a $U_{\rm g}$					
Type	Glass	Normal emissivity	Dimensions mm	Air	Argon	Krypton	SF6 ^b	Xenon
		0,89	<mark>4-6</mark> -4	3,3	3, <mark>0</mark>	2,8	3,0	2,6
	Uncoated		4-8-4	3,1	2,9	2,7	3,1	2,6
	glass (normal		4-12-4	2,8	2,7	2,6	3,1	2,6
	glass)		4-16-4	2,7	2,6	2,6	3,1	2,6
	<i>K</i>	8	4-20-4	2,7	2,6	2,6	3,1	2,6
	S	(4-6-4	2,7	2,3	1,9	2,3	1,6
	0		4-8-4	2,4	2,1	1,7	2,4	1,6
	Coated class	≤ 0,2	4-12-4	2,0	1,8	1,6	2,4	1,6
	coated glass	A BAREY CALL OF	4-16-4	1,8	1,6	1,6	2,5	1,6
			4-20-4	1,8	1,7	1,6	2,5	1,7
		≤ 0,15	4-6-4	2,6	2,3	1,8	2,2	1,5
Berlin			4-8-4	2,3	2,0	1,6	2,3	1,4
Double	Coated class		4-12-4	1,9	1,6	1,5	2,3	1,5
giazing	oodica glass		4-16-4	1,7	1,5	1,5	2,4	1,5
			4-20-4	1,7	1,5	1,5	2,4	1,5
		≤ 0,1	4-6-4	2,6	2,2	1,7	2,1	1,4
			4-8-4	2,2	1,9	1,4	2,2	1,3
	One pane		4-12-4	1,8	1,5	1,3	2,3	1,3
	coated glass		4-16-4	1,6	1,4	1,3	2,3	1,4
			4-20-4	1,6	1,4	1,4	2,3	1,4
		≼ 0,05	4-6-4	2,5	2,1	1,5	2,0	1,2
			4-8-4	2,1	1,7	1,3	2,1	1,1
	Coated glass		4-12-4	1,7	1,3	1,1	2,1	1,2
	obared glass		4-16-4	1,4	1,2	1,2	2,2	1,2
			4-20-4	1,5	1,2	1,2	2,2	1,2

Table 4: Thermal resistance of double and triple glazing fille with different gases forvertical glazing

Wooden frame windows

The window frame U-value of a window made of wood can be calculated in a simplified way, by means of the diagram of Figure 15. Depending if the wood is soft or hard two curves may be chosen. The parameter on the horizontal axes is the thickness of the frame df. This is calculated as the average between the thicnkesses of the sash and the frame as shown in Figure 16. As can be seen, in Figure 16.a it is shown a section of the wall with the window. In Figure 16.b different possible combinations of wooden frame windows can be seen. Based on the evaluated d1 and d2 the average has to be evaluated:

$$d_f = 0.5 \cdot (d_1 + d_2)$$

(24)



Figure 15: Simplified evaluation of the Uf of a wooden frame window



Figure 16: Explication on how to define the thickness d^{*f*} *of the frame for a wooden frame window*

Metal frame windows

For metal frames without thermal break a value of $U_{f0} = 5.9 \text{ W/(m}^2 \cdot \text{K})$ has to be used. For metallic aluminum frames with thermal breaks use U_{f0} value has to be evaluated from the Figure 17 (the part highlighted in black). In this case the minimum distance between opposite sides of the frame have to be considered.

Plastic frame windows

Plastic frame transmittance depends on materials and on the dimensions of the cells constituting the structure of frame and sash. The typical values for the most diffused materials are shown in Table 5.



Figure 17: Value of U_f for a metal frame window with thermal breaks. X is the minimum distance between indoor and outdoor environment in [mm]; Y is the $U_f[W/(m^2 K)]$

Frame material	Frame type	U _f W/(m²⋅K)
Polyurethane	with metal core thickness of PUR ≥ 5 mm	2,8
DVC bellow coeffice a	two hollow chambers external internal	2,2
PVC-hollow profiles ^a	three hollow chambers external internal	2,0

Table 5: Thermal transmittances for plastic frames with metal reinforcements

Combined effect of frame, window and spacer

The connection between the sash and the glass is a critical point because of geometrical discontinuity and due to the non-homogeneous coupling of materials, as shown in Figure 18.a. This discontinuity leads to an additional heat flow which leads to a local decrease of temperature (see potential surface condensation problems in Figure 18.b), a discontinuous temperature profile (Figure 18.c) and an additional heat flux due to the local thermal bridge.

The transmittance of the glazing U_g can be applied to the central area of the window (only glass, not including spacers or frame. Frame transmittance (U_f) is calculated without the glazing element.

The combined effect of the frame, window and spacer is considered in the thermal linear loss Ψ_g depending on the material of the spacer. This is a linear heat loss expressed in

[W/(m K)], hence the overall heat flux due to the spacer is evaluated as $\Psi_g \ge l_g$ being lg the whole perimeter of the window as previously defined. In Table 6 different typical values of the heat loss coefficient of the spacer are reported.



Figure 18: Sketch of the spacer (a), potential problem of surface condensation (b) and possible analysis of the whole window with a detailed 2-D analysis (c)

Table 6: Values of linear thermal transmittance for common types of glazing spacer bars(e.g. aluminium or steel)

	Linear thermal transmittance for different types of glazing $arPsi_{ extsf{g}}$			
Frame type	Double or triple glazing uncoated glass air- or gas-filled	Double ^a or triple ^b glazing low-emissivity glass air- or gas-filled		
Wood or PVC	0,06	0,08		
Metal with a thermal break	0,08	0,11		
Metal without a thermal break	0,02	0,05		
 a One pane coated for double glazed b Two panes coated for triple glazed 	1.			

Overall U-value of a window

The thermal transmittance of a window U_w is represented by the overall heat flow outgoing from the window and the overall surface of the window. The outgoing heat flow is the sum of the heat loss through the glass ($U_g A_g$) and the heat loss of the frame ($U_f A_f$) including the heat loss through the spacer ($\Psi_g I_g$). The overall surface of the window is $A_g + A_f$. Hence, the thermal transmittance of a window U_w shall be calculated using the following equation:

$$U_{w} = (U_{g} A_{g} + U_{f} A_{f} + I_{g} \Psi_{g}) / (A_{g} + A_{f})$$
(25)

References

- [1] EN ISO 10077-1. Thermal performance of windows, doors and shutters Calculation of thermal transmittance Part 1: General. 2017
- [2] EN ISO 10077-2. Thermal performance of windows, doors and shutters Calculation of thermal transmittance Part 2: Numerical method for frames. 2007.