



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA



Dipartimento  
di Ingegneria Industriale

# Energy and Buildings

## Building Energy Models

### Content

- (1) Buildings as dynamic systems
- (2) Overview on building energy models
- (3) Degree-days methods
- (4) Quasi steady-state models
- (5) Lumped-capacitance models
- (6) Transfer functions methods

# Buildings as dynamic systems

## STATIC SYSTEM

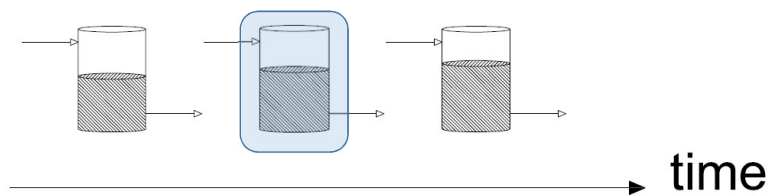
The system can be described by direct, instantaneous links between the variables without any influence by their earlier values.

## DYNAMIC SYSTEM

The system evolves over time. The evolution can be described by a law that links current and past (or future) values of the system variables.

# Buildings as dynamic systems

## Example: mass balance of a water tank

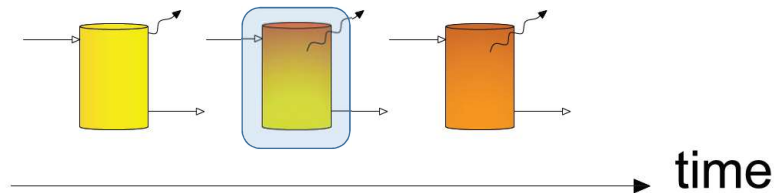


$$\rho \frac{dV}{dt} = \sum_i \dot{m} - \sum_o \dot{m}$$

$$\rho S \frac{dz}{dt} = \sum_i \dot{m} - \sum_o \dot{m}$$

# Buildings as dynamic systems

Example: energy balance of a thermal storage system



$$\rho V c_p \frac{d\theta}{dt} = \sum_i \dot{m}_i c_p \theta_i - \sum_o \dot{m}_o c_p \theta - UA(\theta - \theta_0)$$

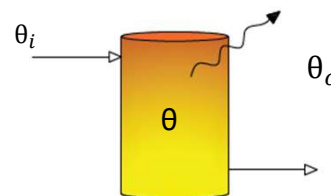
# Buildings as dynamic systems

Example: energy balance of a thermal storage system

$$\rho V c_p \frac{d\theta}{dt} = \sum_i \dot{m}_i c_p \theta_i - \sum_o \dot{m}_o c_p \theta - UA(\theta - \theta_0)$$

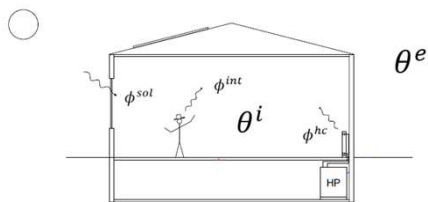
$$M \dot{\theta} = K \theta + f$$

M is the mass of water  
 $\theta$  is the water temperature (state of the system)  
 $\theta_0$  is the temperature of the environment



# Buildings as dynamic systems

Example: energy balance of a building

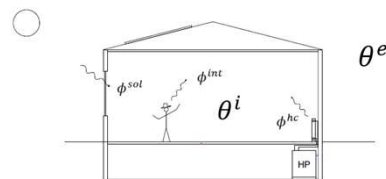


$$M\dot{\theta} = K\theta + f$$



# Buildings as dynamic systems

Example: energy balance of a building

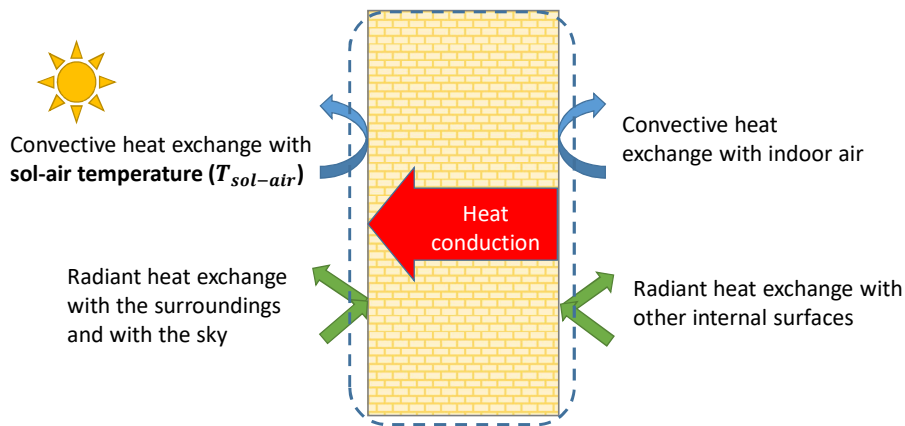


The building is a dynamic system with:

- **complex structure:** three dimensional, multi-layered envelope with opaque and glazed components, different thermal zones etc
- **many overlapping physical phenomena:** emission of long-wave radiation to the sky, absorption of short-wave solar radiation, transient heat conduction through the envelope, heat exchange by convection and radiation between internal wall surface of the walls, internal internal generation of heat and vapour etc

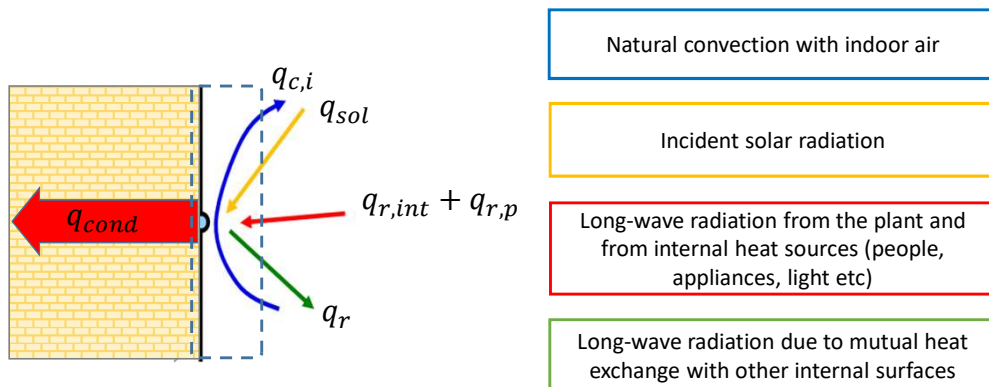
# Buildings as dynamic systems

## Energy balance of a room



# Buildings as dynamic systems

## Energy balance of a room



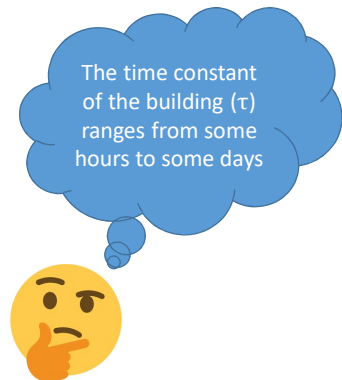
## Building energy models: overview

There are different modelling approaches, depending on the level of detail needed.

Level of detail	Time-step (model resolution)
Annual energy needs	1 year
Monthly energy needs	1 month
Daily energy needs	1 day
Hourly heating/cooling load profile	1 hour
Internal temperature calculation including the HVAC system	< 1 hour

>>  $\tau$

<  $\tau$



## Building energy models: overview

There are different modelling approaches, depending on the level of detail needed.

Level of detail	Time-step (model resolution)
Annual energy needs	1 year
Monthly energy needs	1 month
Daily energy needs	1 day
Hourly heating/cooling load profile	1 hour
Internal temperature calculation including the HVAC system	< 1 hour

>>  $\tau$

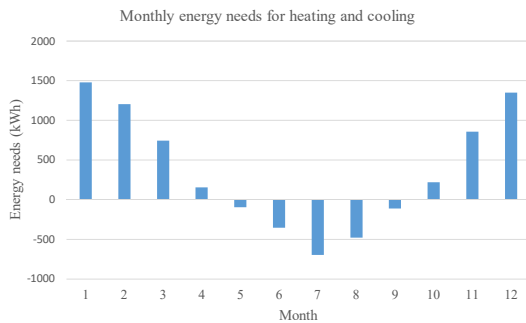
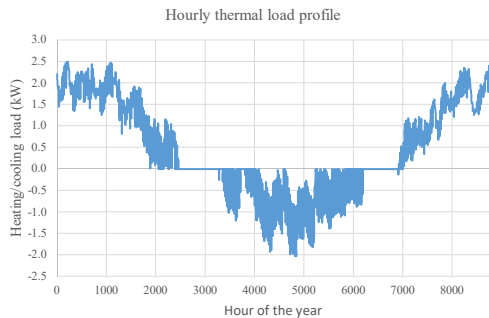
We may consider using simpler models!

<  $\tau$

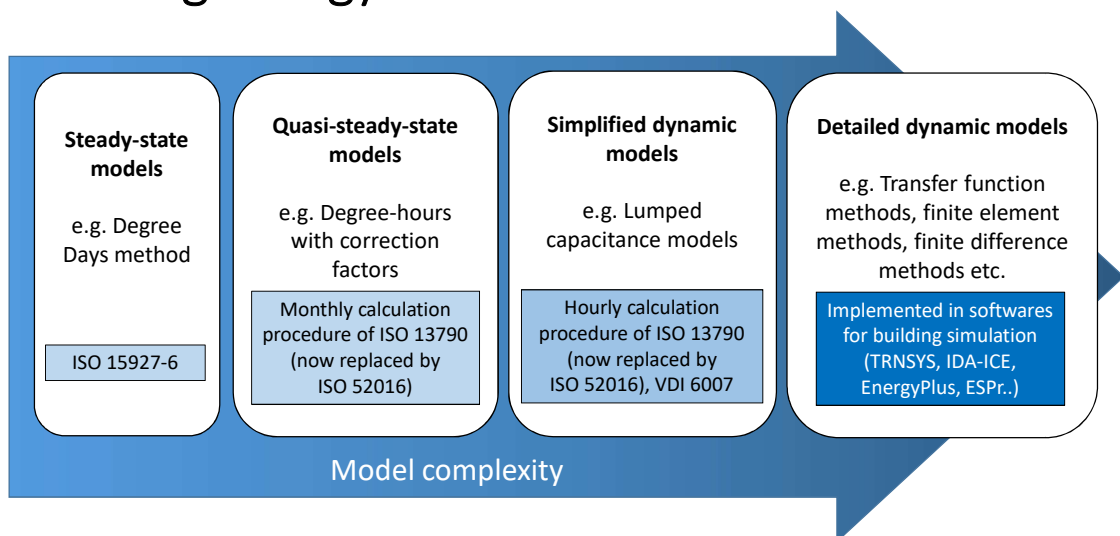
Considering the transient behaviour of the building is necessary!

# Building energy models: overview

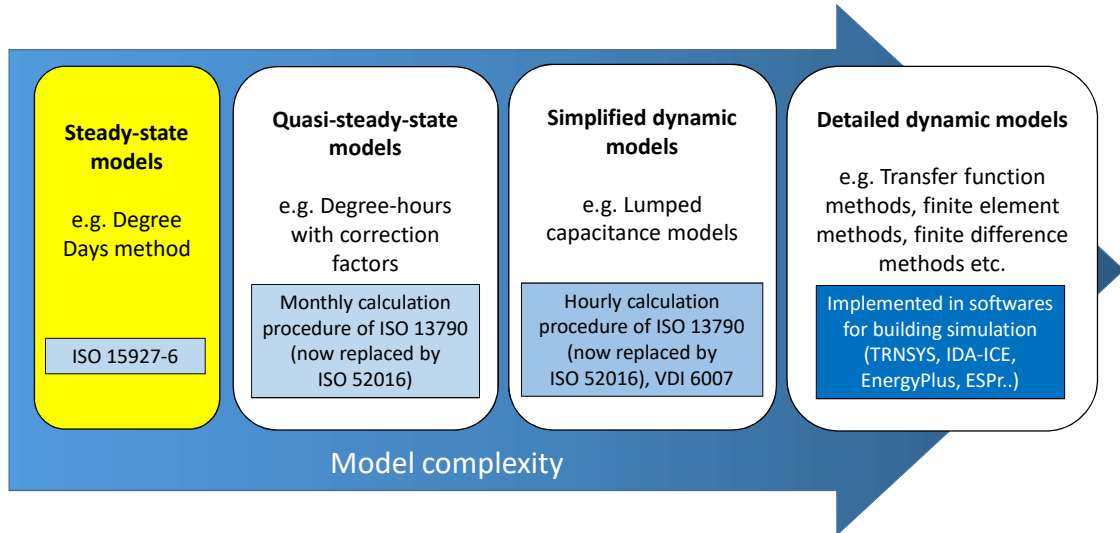
There are different modelling approaches, depending on the level of detail needed.



# Building energy models: overview



# Degree-days method

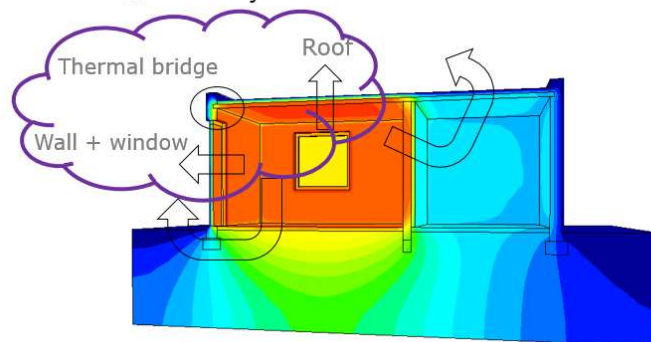


# Degree-days method

## Transmission heat losses

$$H_{T,ie} = \sum_k S_k U_k + \sum_j \psi_j l_j$$

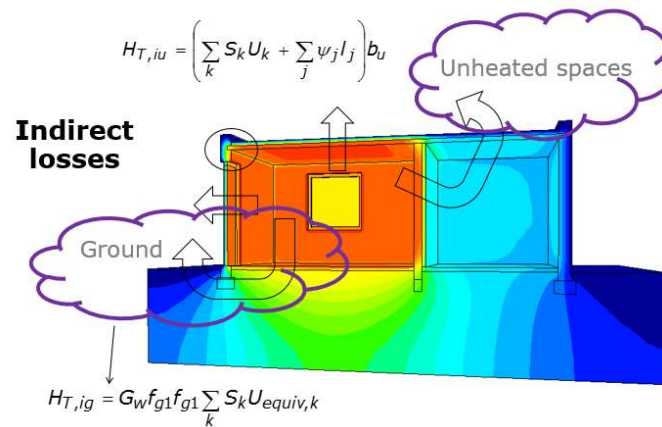
**Direct losses**





# Degree-days method

## Transmission heat losses



# Degree-days method

## Transmission heat losses

Transmission heat transfer coefficient  $H_T$  [W/K]

$$q_T = H_T (T_i - T_e) = (H_{T,ie} + H_{T,iu} + H_{T,ig}) (T_i - T_e)$$

$$H_{T,ie} = \sum_k S_k U_k + \sum_j \psi_j l_j$$

$$H_{T,iu} = \left( \sum_k S_k U_k + \sum_j \psi_j l_j \right) b_u$$

$$H_{T,ig} = G_w f_{g1} f_{g1} \sum_k S_k U_{equiv,k}$$

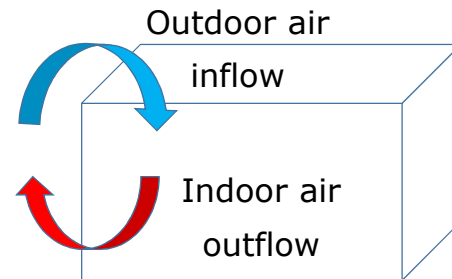
## Degree-days method

### Ventilation/infiltration heat losses

Ventilation heat transfer coefficient  $H_V$  [W/K]

$$q_V = H_V(t_i - t_{amb})$$

$$H_V = ACR c_{p,a} \rho V \left( \frac{n_h}{24} \right)$$



## Degree-days method

### Heating energy demand

$$Q_{sh} = (H_T + H_V) DD \frac{24}{1000} \left[ \frac{kWh}{year} \right]$$

$$DD = \sum_{d=1}^{365} (T_b - T_{e,d}) \text{ when } T_{e,d} \leq T_{e,lim}$$

# Degree-days method

## Heating energy demand

It could be used for estimating the standard consumption of a building.

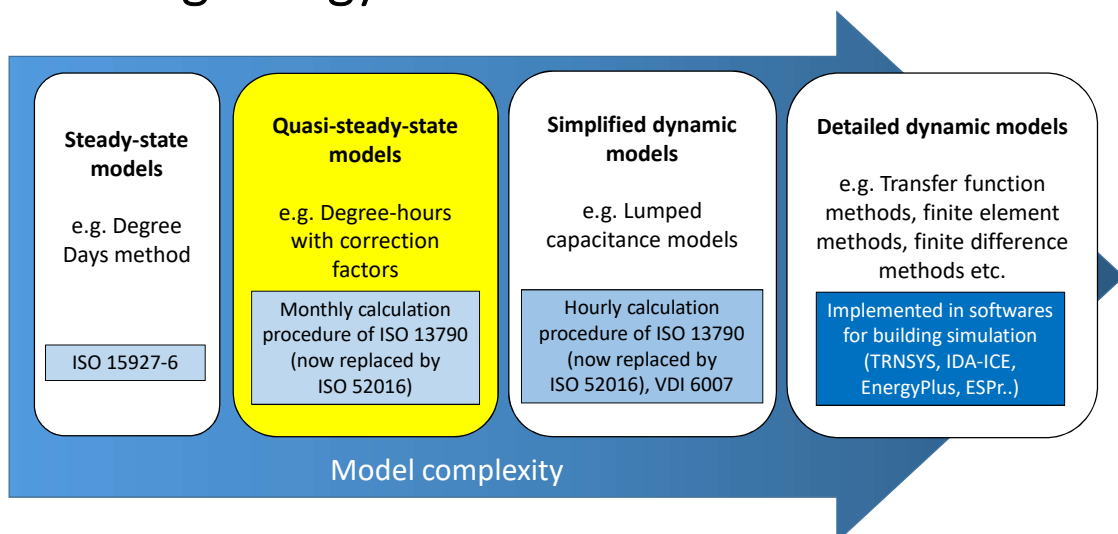
If you know the consumption of a heating system in a certain period (e.g. generic year i), you can **calculate a standard consumption** referred to the DD of the considered place.

Example

A building in Padova (DD = 2383 K) consumed  $C=4500 \text{ m}^3$  of natural gas during year Y. Year Y had only  $DD_Y = 2260$ . So the standard consumption of that building is

$$C = C_Y \frac{DD}{DD_Y} = 4500 \frac{2383}{2260} = 4744 \text{ m}^3$$

## Building energy models: overview



# Quasi-steady-state model

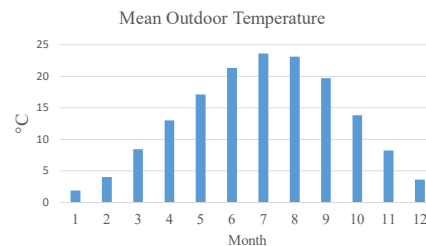
## Monthly calculation procedure

Transmission heat transfer coefficient [W/K]  $H_{tr} = \sum UA + \sum l \Psi$

Ventilation heat transfer coefficient [W/K]  $H_{ve} = \rho_a c_{p,a} n V$

$$Q_{tr} = H_{tr} (T_{set,H} - T_{e,m}) \frac{24}{1000} n_{d,m}$$

$$Q_{ve} = H_{ve} (T_{set,H} - T_{e,m}) \frac{24}{1000} n_{d,m}$$



# Quasi-steady-state model

## Monthly calculation procedure

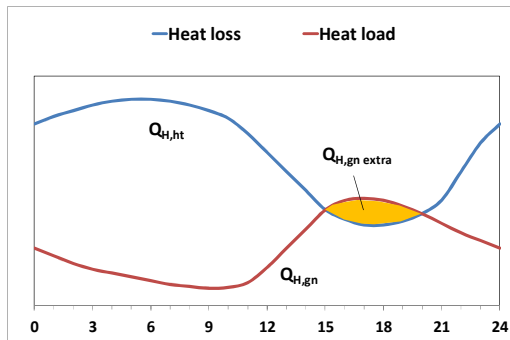
The dynamic effects are taken into account by introducing the **gain utilization factor** for heating  $\eta_{H,gn}$  and the **loss utilization factor** for cooling  $\eta_{C,ls}$ .

$$Q_{H,nd} = (Q_{tr} + Q_{ve}) - \eta_{H,gn} (Q_{int} + Q_{sol})$$

$$Q_{C,nd} = (Q_{int} + Q_{sol}) - \eta_{C,ls} (Q_{tr} + Q_{ve})$$

# Quasi-steady-state model

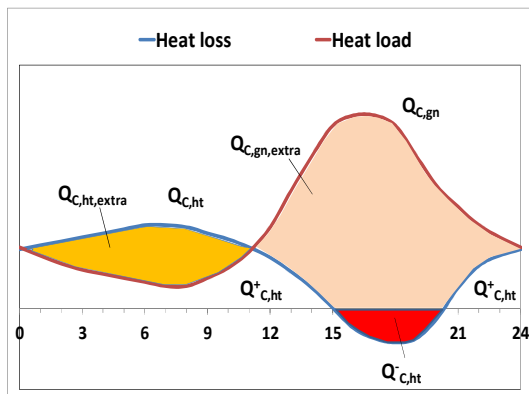
## Monthly calculation procedure



$$\eta_{H,gn} = \frac{Q_{H,gn} - Q_{H,gn,extra}}{Q_{H,gn}}$$

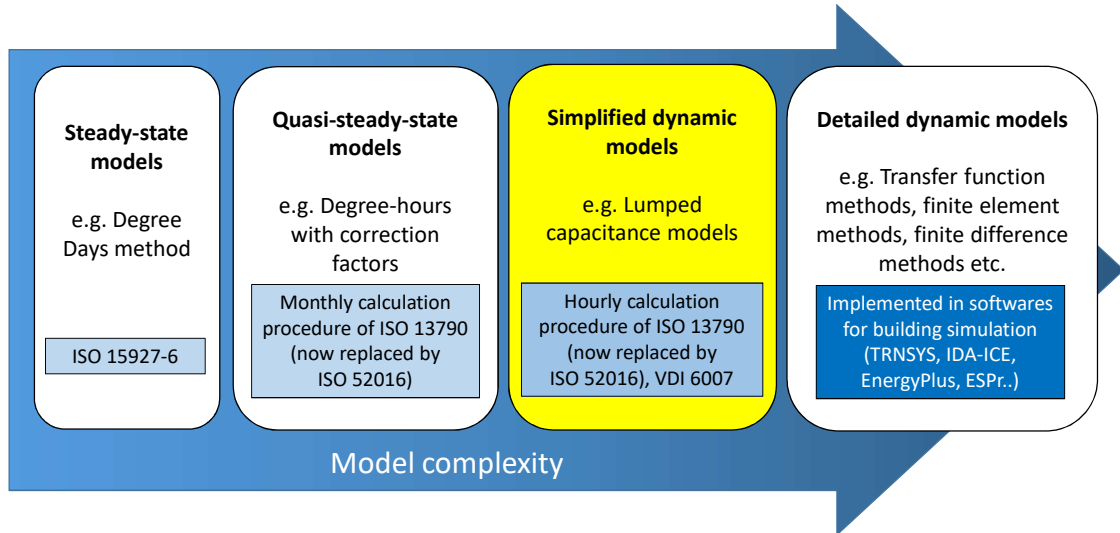
# Quasi-steady-state model

## Monthly calculation procedure



$$\eta_{C,ht} = \frac{Q_{C,ht} - Q_{C,ht,extra} - |Q_{C,ht}^-|}{Q_{C,ht} - |Q_{C,ht}^-|}$$

# Building energy models



## Simplified dynamic models

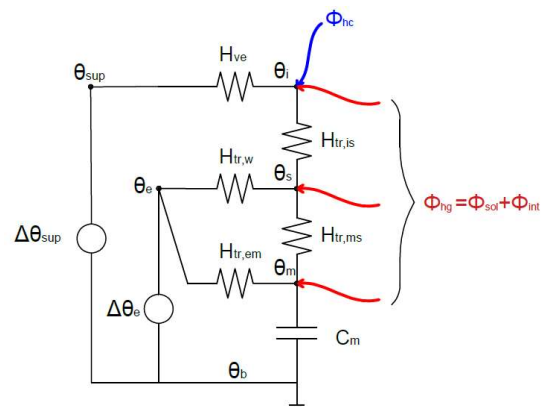
### Simple hourly method

5R1C model proposed by ISO 13790

A time-step of 1 hour (or lower) can be used

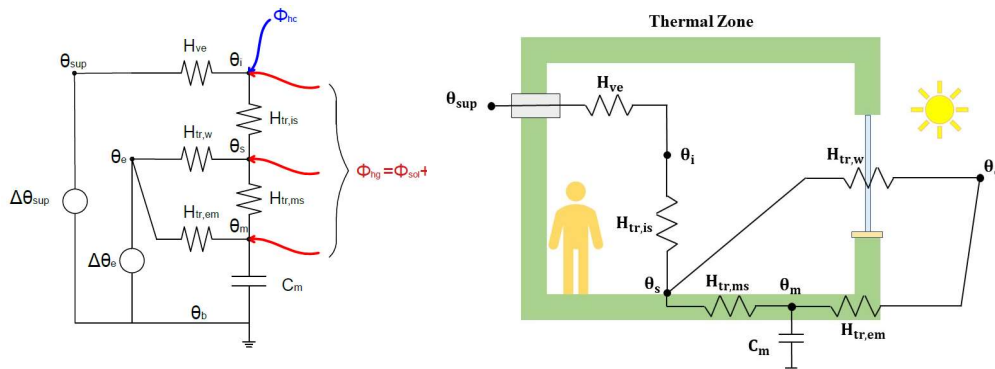
Both heating and cooling energy needs can be assessed

Several lumped-capacitance models based on the electrical analogy were proposed in the technical and scientific literature



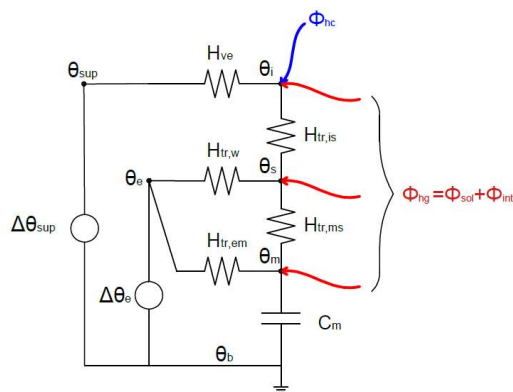
# Simplified dynamic models

## Simple hourly method



# Simplified dynamic models

## Simple hourly method



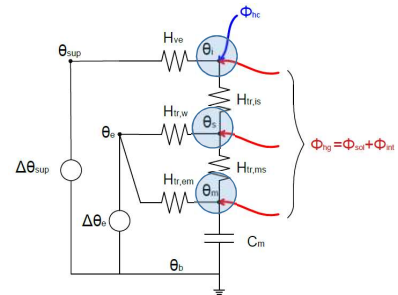
$\Phi_{sol}$  includes:

- Short-wave radiation transmitted through glazed building components (+);
- Short-wave radiation absorbed by opaque building components (+);
- Long-wave radiation to the sky and to the ground (-)

Heat gains  $\Phi_{sol}$  and  $\Phi_{int}$  are distributed to the 3 temperature nodes according to coefficients defined in the Standard

# Simplified dynamic models

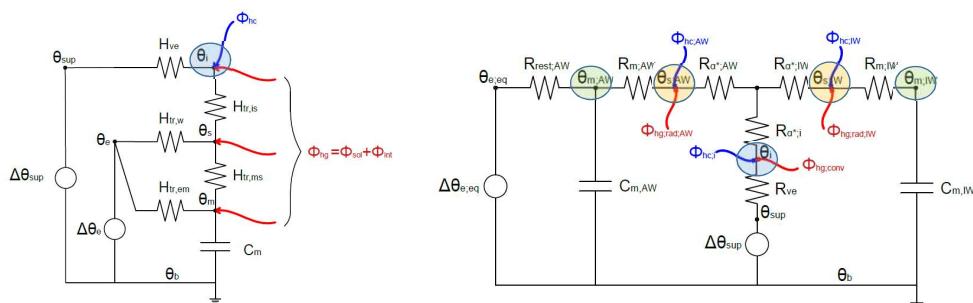
## Simple hourly method



$$\begin{cases} H_{ve}(\theta_{sup} - \theta_i) + H_{tr, is}(\theta_s - \theta_i) + \Phi_{hg, i} + \Phi_{hc} = 0 \\ H_{tr, w}(\theta_e - \theta_s) + H_{tr, is}(\theta_i - \theta_s) + H_{tr, ms}(\theta_m - \theta_s) + \Phi_{hg, s} + \Phi_{hc} = 0 \\ H_{tr, em}(\theta_e - \theta_m) + H_{tr, ms}(\theta_s - \theta_m) + \Phi_{hg, m} = \frac{C_m}{\Delta t} (\theta_m - \theta_m^0) \end{cases}$$

# Simplified dynamic models

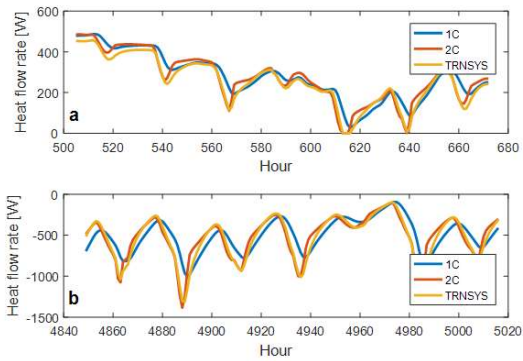
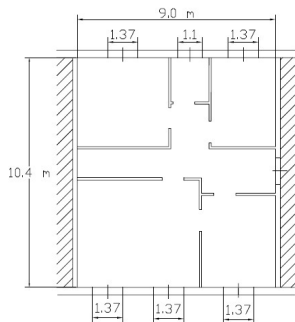
Comparison between different lumped-capacitance models:  
5R1C (EN ISO 13790) vs 7R2C (VDI 6007)





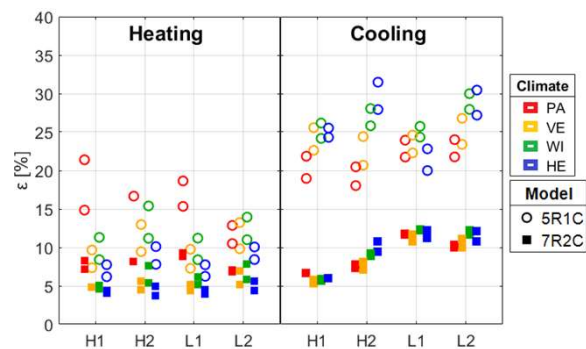
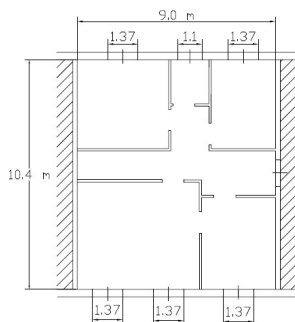
# Simplified dynamic models

Comparison between different lumped-capacitance models:  
5R1C (EN ISO 13790) vs 7R2C (VDI 6007)

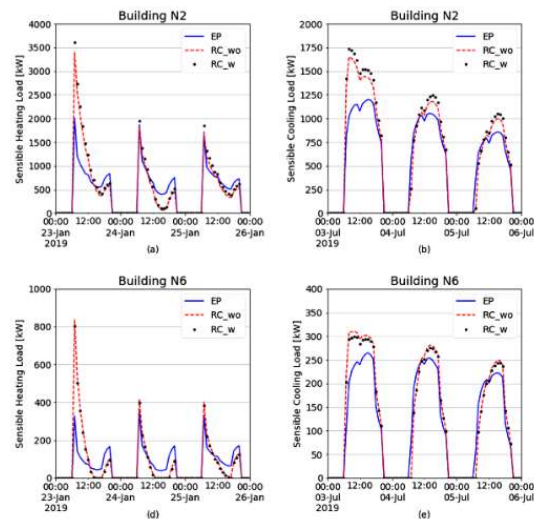


# Simplified dynamic models

Comparison between different lumped-capacitance models:  
5R1C (EN ISO 13790) vs 7R2C (VDI 6007)



## Simplified dynamic models

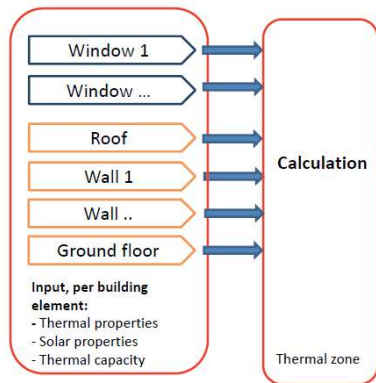


## Simplified dynamic models

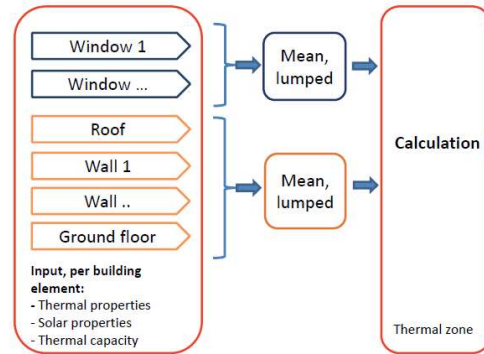
### Simple hourly method

- Simple Hourly Method of EN ISO 13790:2008 predicts annual energy needs for space heating with fair accuracy
- The thermal load profile in the heating season is quite good, but it fails predicting the peak load after long shut down periods (e.g. weekends)
- It is not suitable for the calculation of thermal load profile and the indoor air temperature profile in the cooling season due to high diurnal fluctuations of the heat gains

# Simplified dynamic models



a) Improved hourly method (and similar for monthly method) in EN ISO 52016-1



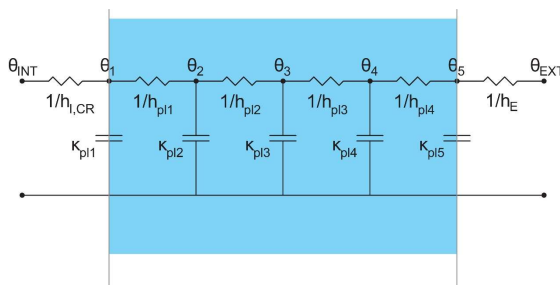
b) Simplified hourly method in EN ISO 13790:2008

VJ1

# Simplified dynamic models

## Improved hourly method

Resistance-capacitance model of ISO 52016:2017



In the new Standard (ISO 52016) each **building element** is modelled as a resistance-capacitance network with 5 thermal capacitances  $k_{pl1}$

## Diapositiva 38

---

VJ1 Vivian Jacopo; 10/11/2020

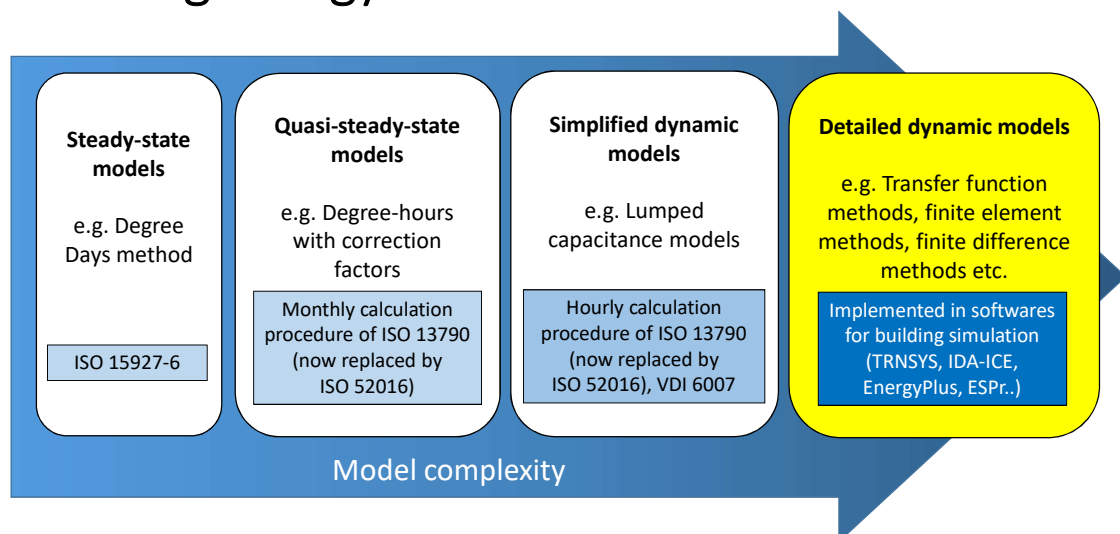
# Simplified dynamic models

## Improved hourly method

Improvements of the hourly method of EN ISO 52016-1:2017 compared to the previous Standard EN ISO 13790:2008 include:

- calculation of free-floating internal air temperatures, e.g. under summer conditions without cooling or winter conditions without heating;
- calculation of design heating or cooling load;
- latent energy needs for (de)humidification;
- calculation of heat exchange with the ground considering the thermal inertia of the ground according to ISO 13370

# Building energy models



# Thermal conduction with transfer functions

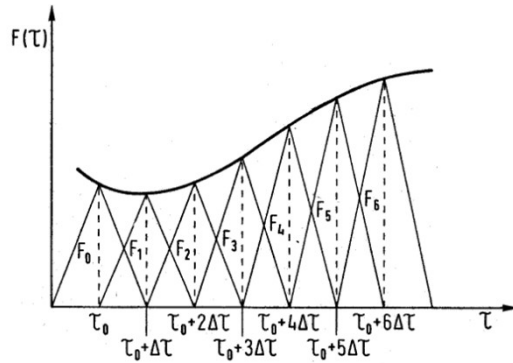
Transfer functions  $D$ : it combines the solicitation  $\Omega(\tau)$  applied to a physical system in the time  $\tau$  with the response  $O(\tau)$  according to the equation:

$$O(\tau) = D * \Omega(\tau)$$

RESPONSE      TRANSFER FUNCTION      SOLICITATION

generally  $O(\tau)$  and  $\Omega(\tau)$  are continuous functions in time domain. It is easier to treat them as functions of time

By choosing properly  $\Delta\tau$ , it is possible to approximate the original curve



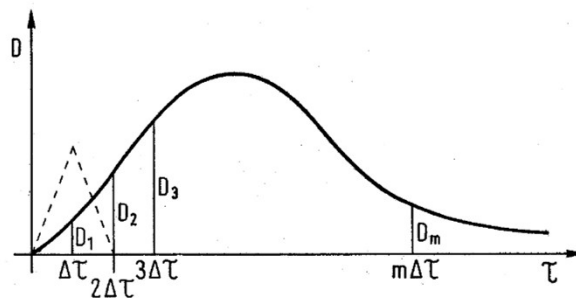
41

# Thermal conduction with transfer functions

$D$  response to an unitary impulse

$\Omega_u(\tau) \rightarrow$  triangular impulse with unitary amplitude

$D \rightarrow D_j, j = 1 \dots \infty$



42

q-th term: represents the system response to a triangular impulse with unitary amplitude applied to the system at a time step that precedes  $q\Delta\tau$  the one considered:


$$O_1 = D_1\Omega_1$$

$$O_2 = D_1\Omega_2 + D_2\Omega_1$$

$$O_3 = D_1\Omega_3 + D_2\Omega_2 + D_3\Omega_1$$

$$O_4 = D_1\Omega_4 + D_2\Omega_3 + D_3\Omega_2 + D_4\Omega_1$$

$$O_k = \sum_{j=1, \infty} D_j \Omega_{k-j+1}$$



43

$$O_k = D_1\Omega_k + D_2\Omega_{k-1} + \dots + D_N\Omega_{k-N+1} +$$

$$+ D_{N+1}\Omega_{k-N} + D_{N+2}\Omega_{k-N-1} + \dots$$

It is always possible to evaluate a number  $N$ , so that for each term greater than  $N$  the ratio between two consecutive terms can be considered constant, i.e.

$$\frac{D_{j+1}}{D_j} \approx c_R$$

$$O_k = D_1\Omega_k + D_2\Omega_{k-1} + \dots + D_N\Omega_{k-N+1} +$$

$$+ c_R D_N\Omega_{k-N} + c_R^2 D_N\Omega_{k-N-1} + \dots$$

$$O_{k-1} = D_1\Omega_{k-1} + D_2\Omega_{k-2} + \dots + D_N\Omega_{k-N} +$$

$$+ c_R D_N\Omega_{k-N-1} + c_R^2 D_N\Omega_{k-N-2} + \dots$$

44

$O_k - c_R O_{k-1}$  gives:

$$O_k = D_1 \Omega_k + (D_2 - c_R D_1) \Omega_{k-1} + \dots + (D_N - c_R D_{N-1}) \Omega_{k-N-1} + c_R O_{k-1}$$

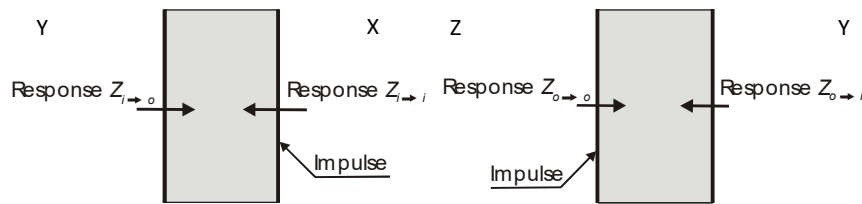
$$O_k = \sum_{j=1, N} D'_j \Omega_{k-j+1} + c_R O_{k-1}$$

$$D'_1 = D_1 \quad j = 1$$

$$D'_j = D_j - c_R D_{j-1} \quad j = 2, \dots, N$$

45

## Transfer functions for walls



$$(q^*_i)_k = \sum_{j=1, \infty} Z_{i \to i, j}(t_{si})_{k-j+1} + \sum_{j=1, \infty} Z_{o \to i, j}(t_{so})_{k-j+1}$$

$$(q^*_o)_k = \sum_{j=1, \infty} Z_{i \to o, j}(t_{si})_{k-j+1} + \sum_{j=1, \infty} Z_{o \to o, j}(t_{so})_{k-j+1}$$

$$(q^*_i)_k = \sum_{j=1, N} Z'_{i \to i, j}(t_{si})_{k-j+1} + \sum_{j=1, N} Z'_{o \to i, j}(t_{so})_{k-j+1} + c_R (q^*_i)_{k-1}$$

$$(q^*_o)_k = \sum_{j=1, N} Z'_{i \to o, j}(t_{si})_{k-j+1} + \sum_{j=1, N} Z'_{o \to o, j}(t_{so})_{k-j} + c_R (q^*_o)_{k-1}$$

46



## Triangular impulse

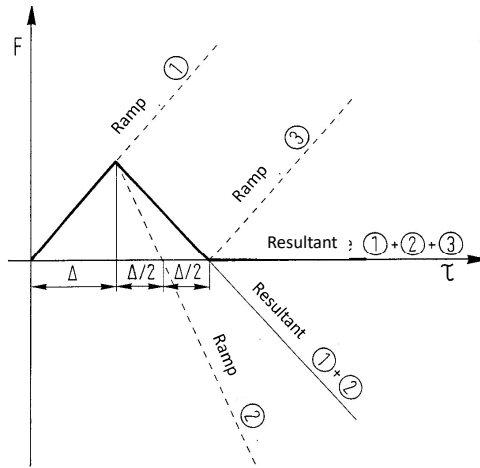
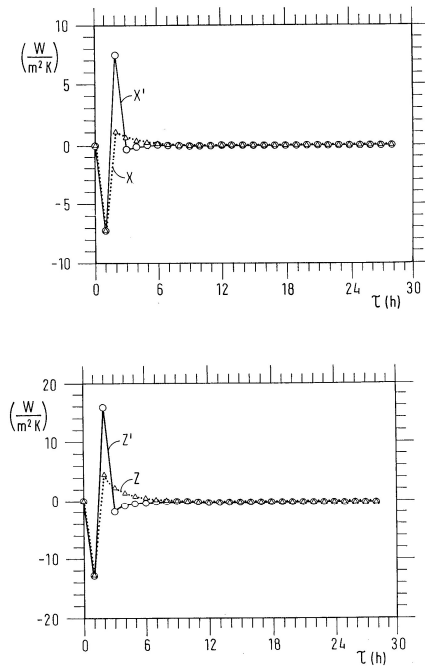
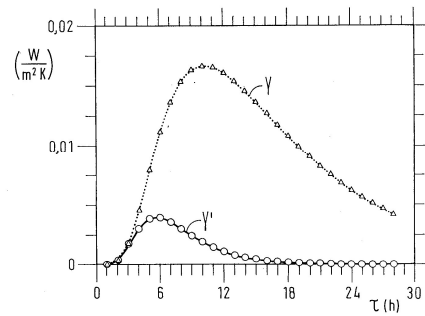


Fig. 5.16

47



## Response factors



48

# Transfer Function Methods

## **Conduction Transfer Functions with Laplace transform**



Round-off and truncation errors for short timesteps and massive structures!

# Transfer Function Methods

## **Conduction Transfer Functions with state-space formulation**

PROs

- Ability to obtain CTF for shorter timesteps;
- Ability to obtain CTF for 2D and 3D heat conduction problems.

CONs

- More time-consuming to find CTF

# Transfer Function Methods

## Conduction Transfer Functions with Finite Difference approximations

$$C_p \rho \Delta x \frac{T_i^{j+1} - T_i^j}{\Delta t} = \frac{1}{2} \left( k_W \frac{T_{i+1}^{j+1} - T_i^{j+1}}{\Delta x} + k_E \frac{T_{i-1}^{j+1} - T_i^{j+1}}{\Delta x} + k_W \frac{T_{i+1}^j - T_i^j}{\Delta x} + k_E \frac{T_{i-1}^j - T_i^j}{\Delta x} \right) \quad (3.14)$$

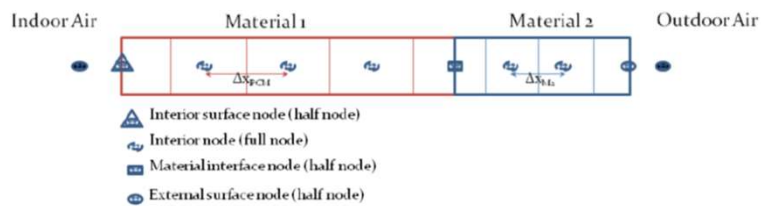
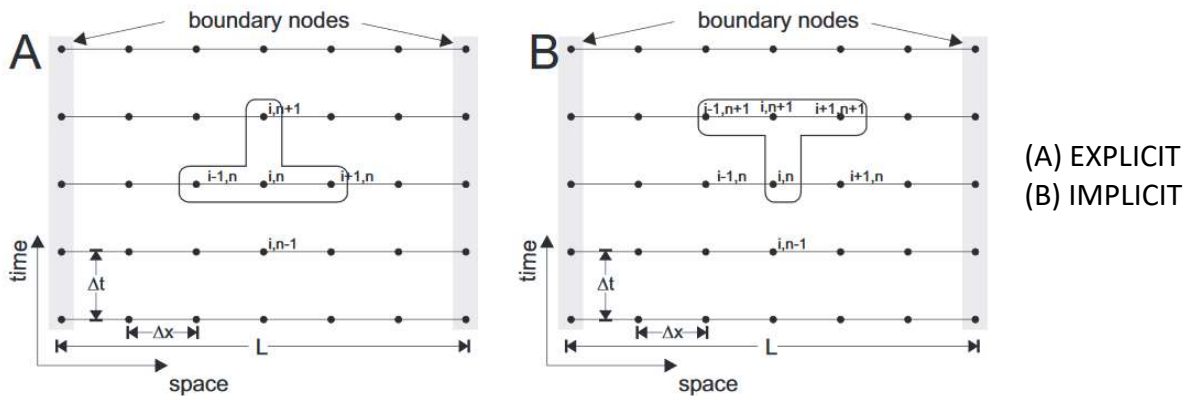


Figure 3.6: Node depiction for Conduction Finite Difference Model

# Transfer Function Methods

## Conduction Transfer Functions with Finite Difference approximations



# Transfer Function Methods

## EnergyPlus

EnergyPlus uses CTF calculated with the state-space formulation as a default method. Alternatively, the finite difference approximations based on an implicit scheme may be used in particular cases:

- Very low timestep
- Materials with variable thermal conductivity or PCM

It is able to run building energy simulations of massive buildings with very low timesteps (down to 1 minute)!

# Transfer Function Methods

## EnergyPlus

EnergyPlus is an integrated simulation environment! It finds a solution for three domains simultaneously: thermal zone (building), distribution system and heating/cooling production plant.

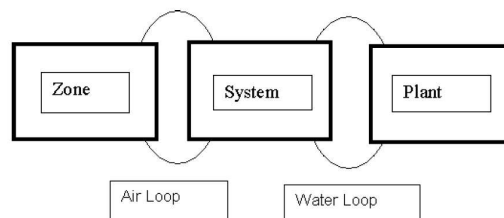


Figure 2.2: Schematic of Simultaneous Solution Scheme

# References

- EN 12831-1:2017. Energy performance of buildings – Method for calculation of the design heat load.
- EN ISO 13790:2008. Energy performance of buildings - Calculation of energy use for space heating and cooling.
- EN ISO 52016-1:2017. Energy performance of buildings – Energy needs for heating and cooling, internal temperatures and sensible and latent heat loads. Part 1: Calculation procedures.
- EnergyPlus™ v9.3.0 Documentation – Engineering Reference. U.S. Department of Energy. Available at: <https://energyplus.net/documentation>
- Underwood C.P., Yik F.W.H. Modelling methods for energy in buildings. Blackwell Science. 2004. Available online at <https://onlinelibrary.wiley.com/doi/pdf/10.1002/9780470758533>
- Vivian J, Zarrella A, Emmi G, De Carli. An evaluation of the suitability of lumped-capacitance models in calculating energy needs and thermal behaviour of buildings. Energy and Buildings 150 (2017) 447-465. <https://doi.org/10.1016/j.enbuild.2017.06.021>
- Mazzarella L, Scoccia R, Colombo P, Motta M. Improvement to EN ISO 52016-1:2017 hourly heat transfer through a wall assessment: the Italian National Annex. Energy and Buildings 210 (2020) 109758. <https://doi.org/10.1016/j.enbuild.2020.109758>
- Van Dijk D, Spiekman M, van Hoeffelen LH. EPB standard EN ISO 52016: Calculation of the building's energy needs for heating and cooling, internal temperatures and heating and cooling load. REHVA Journal – May 2016.