MODELS FOR THE THERMAL BALANCE OF A ROOM

1 Steady state model

The steady state model is usually used for determining the design heating capacity of a heating system. This approach has been adopted in the standard EN 12831 [1]. It is based on the assumption that the heat flow depends on the transmission (q_T) and ventilation (q_V) losses:

$$q = q_T + q_V \qquad (1)$$

Such losses usuallly refer to the temperature difference between the indoor temperature t_i and the outdoor temperature t_{amb}, even if they are losses related to environments at different temperatures, such as the ground ($q_{T,ig}$), an unheated room ($q_{T,iue}$) and with a room at significantly different temperature ($q_{T,ij}$). For this purpose, heat loss coefficients are usually used, named H [W/K], which allow to write the following equation:

$$q_T = (H_{T,ie} + H_{T,iue} + H_{T,ig} + H_{T,ij})(t_i - t_{amb}) \qquad (2)$$

The heat loss coefficient for the direct transmission between the indoor and the outdoor environment is defined through the transmittance of the external walls U_k (with surface S_k) and the correspondent thermal bridges ψ_j (with length l_j):

$$H_{T,ie} = \sum_k S_k U_k + \sum_j \psi_j l_j \qquad (3)$$

The heat loss coefficient for the indirect transmission between the indoor and the outdoor environment through an unheated space is defined through the transmittance of the walls dividing the two adjacent spaces U_k and the correspondent thermal bridges ψ_j via a coupling factor b_u:

$$H_{T,iu} = \left(\sum_k S_k U_k + \sum_j \psi_j l_j\right) b_u \qquad (4)$$

The coefficient b_u is a reduction factor in order to take into account the difference between the temperature of the unheated space t_u and the outdoor environment t_{amb}. The coupling coefficient b_u can be calculated through the equation (5) or (6), depending if the temperature t_u is known or unknown respectively:

$$b_u = \frac{t_i - t_u}{t_i - t_{amb}} \qquad (5)$$

$$b_u = \frac{H_{ue}}{H_{iu} - H_{ue}} \qquad (6)$$

where H_{iu} and H_{ue} take into account both the transmission and ventilation losses respectively from the indoor environment to the unheated space and from the unheated space to the outdoor environment.
The heat loss coefficient for the transmission losses between the indoor and the outdoor environment through the ground is defined as:

$$H_{T,ig} = G_w f_{g1} f_{g1} \sum_k S_k U_{equiv,k} \quad (7)$$

where G_w is a factor considering the aquifer below the ground (G_w =1 if the aquifer and the floor slab is greater than 1 m, else G_w = 1.15), f_{g1} is a coefficient which considers the annual variation of outdoor temperature (usually f_{g1} = 1.45), f_{g2} is a coefficient which takes into account the change betwen the yearly average outdoor temperature ($t_{m,amb}$) and the design temperature (t_{amb}):

$$f_{g2} = \frac{t_i - t_{m,amb}}{t_i - t_{amb}} \quad (8)$$

The parameter $U_{equiv,k}$ is a corrected transmittance for considering that the heat losses through the ground are mainly located in the perimetral zone. For considering this aspect the parameter B' is defined as the ratio between the area of the floor (S_g) and the semiperimeter of the external walls:

$$B' = \frac{S_g}{0.5 \cdot P} \quad (9)$$

As an example, in figure 4.1 on the left side a building with all external walls is shown; in this case S_g = 150 m^2, P=50 m and B' = 6 m. In figure 4.1 on the right side, the external walls are only two, hence S_g= 75 m^2, P = 15 m and B' = 10 m. Based on B' value, the Figure 4.2 shows how to evaluate the corrected $U_{equiv,g}$ value, depending on the original U value calculated for the structure on the ground.
A wider number of cases considering the slab below or above the ground can be found in EN12831.

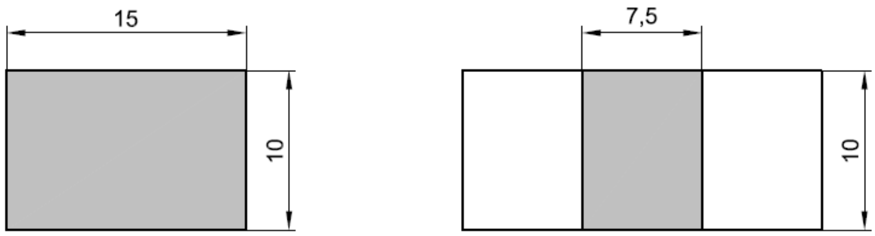


Figure 1: Different possibilities of heat losses through the ground of the considered building (in grey)

Source: [1]

For walls dividing rooms at different temperature (t_i and t_j respectively) the following equation can be written:

$$H_{T,iu} = f_{ij} \sum_k S_k U_k \quad (10)$$

where f_{ij} is calculated as:

$$f_{ij} = \frac{t_i - t_j}{t_i - t_{amb}} \quad (11)$$

The losses due to ventilation can be calculated as:

$$q_V = H_V(t_i - t_{amb}) \tag{12}$$

where the ventilation loss coefficient H_V is:

$$H_V = \dot{V}ol\rho c_p(1 - E) \tag{13}$$

where E is the efficiency of the heat recovery system, if any.

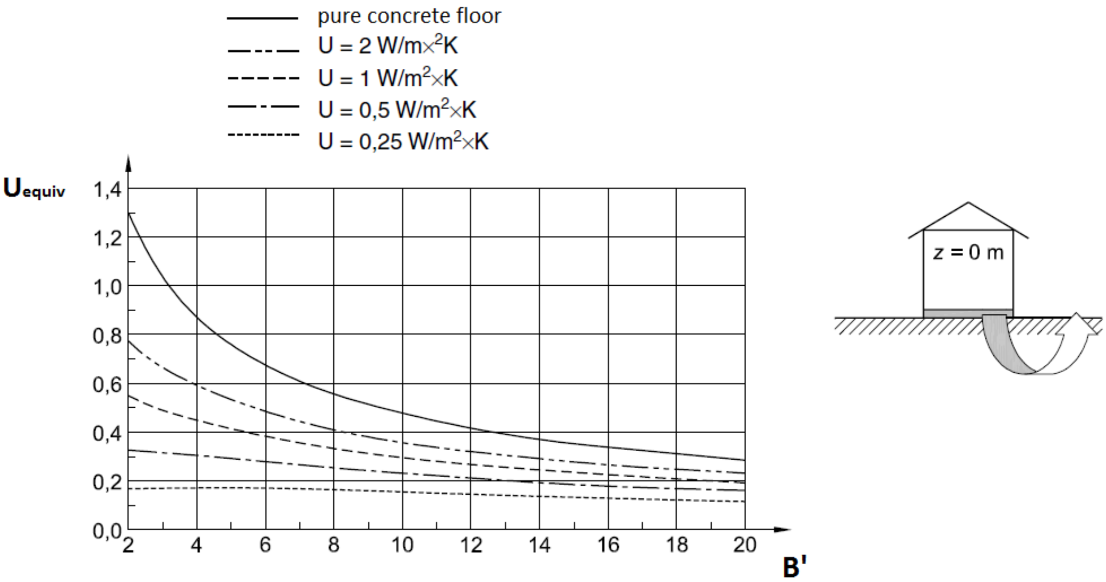


Figure 2: Correction of the U-value of a structure with heat loss through the ground as a function of the parameter B' calculated via equation (9)

Source: [1]

2 External resistance with internal capacitance

The most simple model for considering dynamic behaviour of structures is the model with an external resistance with internal capacitance. This model can be used for determining the start up or the shut down times for a heating system. This model is also used as a basis for the quasi-steady state model, for determining the interactions of solar and internal gains and the structures, as shown afterward [2].

Let us consider a room with one external wall as a system "of the first order" (Figure 3) constituted by a concentrated mass with negligible internal thermal resistance and a surface thermal resistance.

This assumption is possible if we consider the heat loss through the building envelope and due to the air infiltration rate and with a thermal mass of the whole walls. Such a system has therefore the following global resistance R:

$$\frac{1}{R} = H_T + H_V \tag{14}$$

The equivalent heat capacity C of the system can be written in this way:

$$C = \sum_j C_j = \sum_j \left[\frac{\sum_k \left(\rho_k \cdot c_{p,k} \cdot s_k \right)}{s_j} \cdot Vol_j \right] \quad (15)$$

where C_j is the heat capacity of each building element (having volume Vol_j and thickness s_j), $c_{p,k}$, ρ_k and s_k are respectively specific heat at constant pressure, the density and the thickness of each layer of the building element.

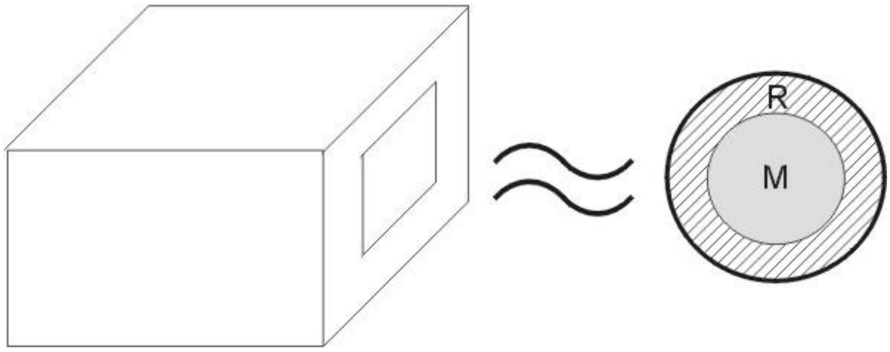


Figure 3: Simplified assumption in the resistance-capacity model for a simplified system which considers the room and the envelope

Let us suppose that this system is in thermal equilibrium with an initial uniform temperature t_i. Making the assumption that the system is subjected to an external constant temperature t_{amb} from the time instant $\tau_0 = 0$ (step wise in boundary conditions), the temperature change in time τ can be calculated in this way:

$$\frac{(t_{amb} - t)d\tau}{R} = Cdt \quad (16)$$

If the thermal characteristics are constant in the considered temperature range, the equation (16) can be expressed as:

$$\frac{dt}{t_{amb} - t} = \frac{1}{RC} d\tau \quad (17)$$

By integrating equation (17) and taking into account the initial condition $t = t_i$ for $\tau = 0$, it follows that:

$$t_{amb} - t = (t_{amb} - t_i) \cdot e^{-\frac{1}{RC} \cdot \tau} \quad (18)$$

which can be written also as:

$$t_{amb} - t = (t_{amb} - t_i) \cdot e^{-\tau / \tau_0} \quad (19)$$

where

$$\tau_0 = RC \quad (20)$$

is the time constant, which represents the time required to get a temperature change equal to 63.2% of the global temperature change (t_i - t_e) or the time required to obtain the

temperature change (t_i - t_{amb}) if the rate of change would be constant at its initial value.
As an example, in order to underline the effect of the building envelope, three cases are shown considering a generic room both for residential and office uses. The considered room has one external wall (the other ones are considered adiabatic). Two different window dimensions are considered: one 3 by 1,2 m (Figure 4) the other one 6 by 1,5 m, i.e. the same width of the external wall (Figure 5).
Three different types of building envelopes are considered: the first one poorly insulated (Table 1), the second one insulated (Table 2) and the third one well insulated (Table 3). In the third case a mechanical ventilation with 0.8 efficiency is considered as well. The cases refer to an initial indoor temperature of 20°C and an outdoor temperature of -10°C. As it can be observed (Figure 6), in the case of small window the poorly insulated room (τ_0 = 51 h, i.e. 2.1 days) after 12 hours the indoor temperature is 13.7°C, while in the insulated room (τ_0 = 107 h, i.e. 4.5 days) the indoor temperature is 16.8°C and in the well insulated room (τ_0 = 384 h, i.e. 16 days) the indoor temperature is 19.5°C. As it can be observed (Figure 7), for a large window the poorly insulated room (τ_0 = 36 h, i.e. 1.5 days) after 12 hours the indoor temperature is 11.5°C, while in the insulated room (τ_0 = 71 h, i.e. 3.0 days) the indoor temperature is 15.3°C and in the well insulated room (τ_0 = 254 h, i.e. 10.6 days) the indoor temperature is 18.6°C.
The results shows how strong the impact of thermal resistances is in new buildings and how the intermittent operation of the plants has reduced relevance. In fact it has to be observed that the energy saving due to night set-back is proportional to the lowering of internal temperature and the night time period (about 8 hours) involves a very limited temperature fall for well insulated buildings.

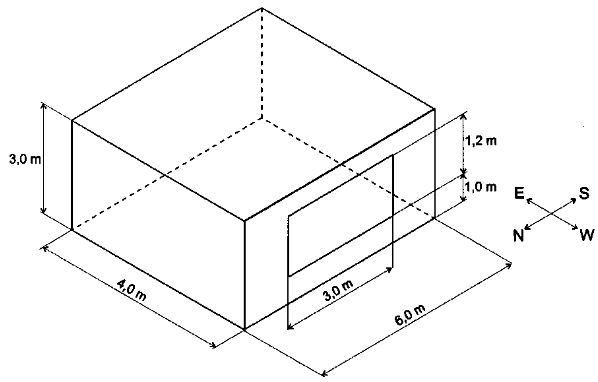


Figure 4: Room with small window

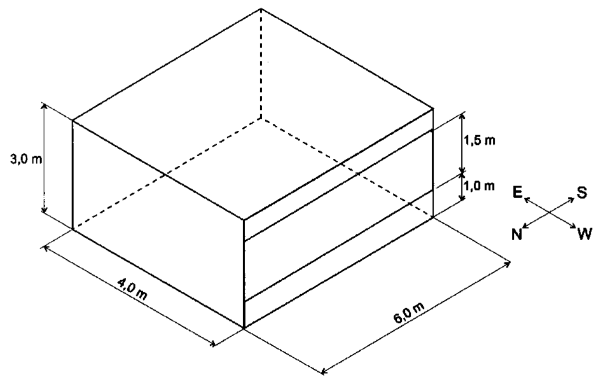


Figure 5: Room with large window

Table 1: Characteristics of a poorly insulated room

Wall type	**Materials**	***s*** [m]	λ [W/(m K)]	ρ [kg/m³]	c_p [J/(kg K)]	***U*** [W/(m² K)]
Internal wall	Gypsum board	0.01	0.21	900	840	0.83
	Insulation	0.05	0.05	150	840	
	Gypsum board	0.01	0.21	900	840	
Ceiling and floor	Concrete	0.07	1.20	2500	840	0.91
	Insulation	0.05	0.05	150	840	
	Cement blocks	0.18	0.54	800	840	
External wall	Light concrete panels	0.26	0.80	1700	840	2.10
Window	Single glass					6.0

Table 2: Characteristics of an insulated room

Wall type	**Materials**	***s*** [m]	λ [W/(m K)]	ρ [kg/m³]	c_p [J/(kg K)]	***U*** [W/(m² K)]
Internal wall	Gypsum board	0.01	0.21	900	840	0.83
	Insulation	0.05	0.05	150	840	
	Gypsum board	0.01	0.21	900	840	
Ceiling and floor	Concrete	0.07	1.20	2500	840	0.91
	Insulation	0.05	0.05	150	840	
	Cement block	0.18	0.54	800	840	
External wall	Light concrete	0.26	0.80	1700	840	0.56
	Insulation	0.06	0.045	150	840	
Window	Double glass					3.0

Table 3: Characteristics of a well insulated room

Wall type	**Materials**	***s*** [m]	λ [W/(m K)]	ρ [kg/m³]	c_p [J/(kg K)]	***U*** [W/(m² K)]
Internal wall	Gypsum board	0.01	0.21	900	840	0.83
	Insulation	0.05	0.05	150	840	
	Gypsum board	0.01	0.21	900	840	
Ceiling and floor	Concrete	0.07	1.20	2500	840	0.91
	Insulation	0.05	0.05	150	840	
	Cement block	0.18	0.54	800	840	
External wall	Light concrete	0.26	0.80	1700	840	0.21
	Insulation	0.15	0.035	150	840	
Window	Triple glass					0.9

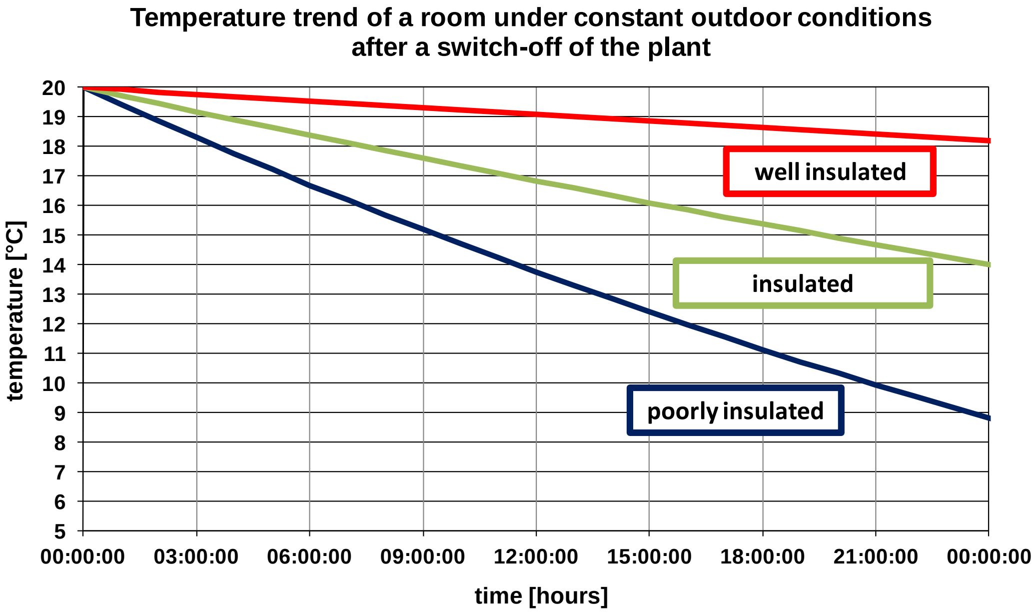


Figure 6: Temperature drop in the case of a room with a small window

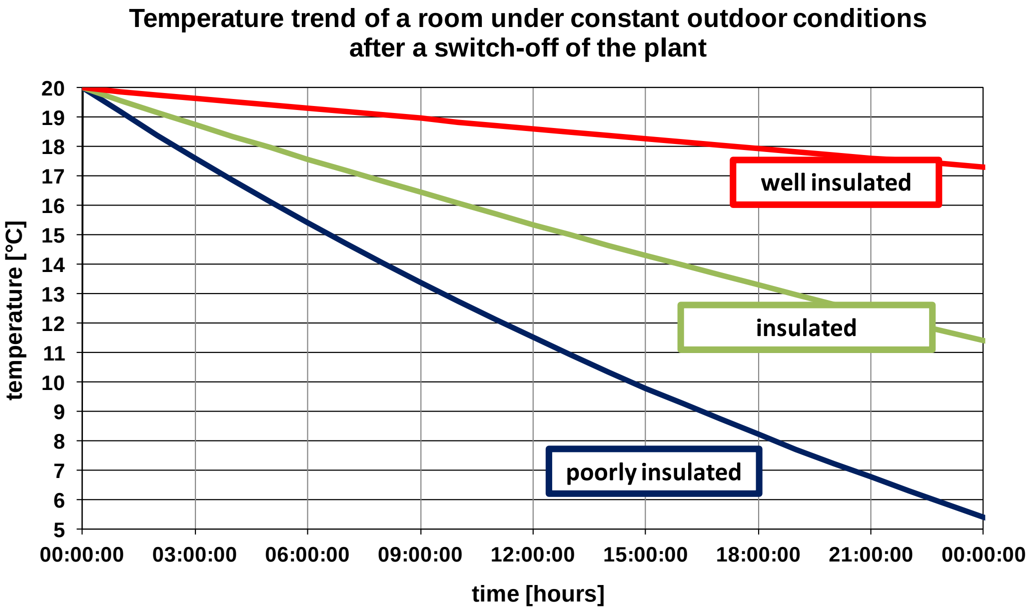


Figure 7: Temperature drop in the case of a room with a large window

3 Quasi steady state model

The quasi-steady state model is the basis for the standard ISO EN 13790 [3], for determining sensible heating and cooling demand. The reference calculation step could be monthly or seasonal.
The building energy need for space heating and cooling in the reference period for a room with this method is based on a single equation, respectively equation (21) for the heating period and equation (23) for the cooling period. The average values of the weather

conditions are considered (average solar energy and mean outdoor temperature). As shown in detail hereafter, due to the use of a parameter which considers intrinsically the thermal behaviour of the structures via the time constant of the room, the method is called quasi steady state.
This method is the basis of the energy certifications in Europe and it is the most widely used. It is more accurate for determining the energy demand in heating season rather than in cooling season, mainly due to the difference between indoor and outdoor temperatures.

3.1 Building energy demand for space heating

The building energy need for space heating in the reference period for a room ($Q_{H,nd}$) for conditions of continuous heating, is calculated as given by the following equation:

$$Q_{H,nd} = Q_{H,ht} - \eta_{H,gn} Q_{H,gn} \tag{21}$$

where $Q_{H,ht}$ is the total heat transfer for the heating mode, $Q_{H,gn}$ gives the total heat loads (solar radiation and internal loads) for the heating mode and $\eta_{H,gn}$ is the dimensionless gain utilization factor.
This last coefficient considers the fact that not all the heat loads can be fruitfully used. As shown in Figure 8, during a typical day in heating season the building has losses, which are typically higher in night time (lower outdoor temperatures) and lower during the day (higher temperatures). As for heat loads, they have usually a peak during the day, due to the solar radiation. When a heat gain occurs it needs some time before it appears clearly and, at the same time, when it stops it needs some time in order to disappear. This delay time is due to the thermal inertia of the room, i.e. by the time constant of the room, as defined by equation (20), since the heat gain has to be first partially stored and then released by the structures.
Considering again Figure 8, the heat load $Q_{H,gn}$ is useful if it is lower than the amount of heat loss, therefore the heating energy demand is represented by the area between the two profiles. When the heat load exceeds the heat loss the surplus of heat gain $Q_{H,gn,extra}$ is not useful, hence it cannot be included in the calculations. Therefore the efficiency in the use of internal gains can be calculated as:

$$\eta_{H,gn} = \frac{Q_{H,gn} - Q_{H,gn,extra}}{Q_{H,gn}} \tag{22}$$

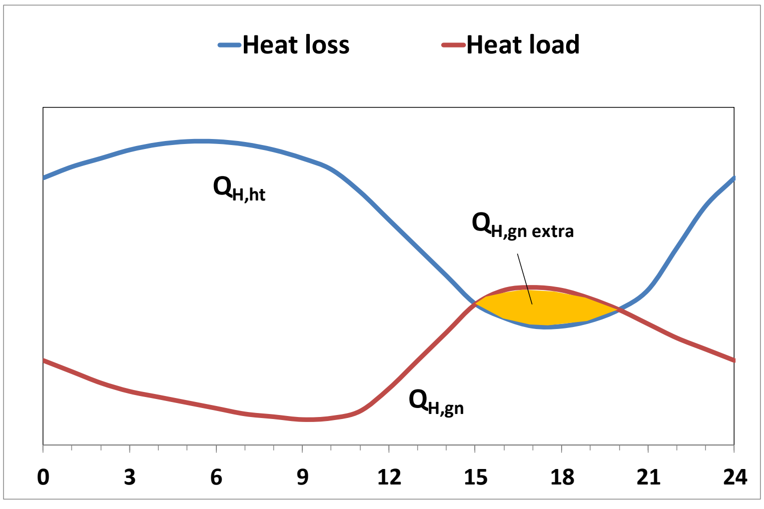


Figure 8: Heat losses and heat gains in an average day in winter period

3.2 Building energy demand for space cooling

The building energy need for space cooling in the reference period for a room ($Q_{C,nd}$) for conditions of continuous cooling, is calculated as given by the following equation:

$$Q_{C,nd} = Q_{C,gn} - \eta_{C,ht} \cdot Q_{C,ht} \qquad (23)$$

where $Q_{C,gn}$ gives the total heat gains (solar radiation and internal loads) for the cooling mode, $Q_{C,ht}$ is the total heat transfer for the cooling mode and $\eta_{C,ht}$ is the dimensionless utilization factor for heat losses.

The definition of the utilization factor for the heat losses can be understood by using Figure 9. In the cooling period the heat load has a peak during the day, mainly due to solar radiation. The shape of the heat load curve is smoothed, due to the effect of the structures and the time constant of the room. The heat loss shape is similar to the one in winter time, but the average value is lower due to the reduced temperature difference between indoor and outdoor in summer time and, in the afternoon, it becomes negative, since the outdoor temperature is higher than the indoor temperature. The negative heat loss is called $Q^{-}_{C,ht}$ and it becomes an extra load to be removed by the cooling system, which has to face the amount of energy $Q_{C,gn,extra}$ + $|Q^{-}_{C,ht}|$ (pink and red areas in Figure 9). During night time the heat loss might exceed the heat load, thus leading to a free cooling. In any case the surplus of heat loss ($Q_{C,ht,extra}$) cannot be used, hence the useful part of the heat loss for free cooling is $Q_{C,ht}$ - $Q_{C,ht,extra}$ - $|Q^{-}_{C,ht}|$.

Therefore, the utilization factor for heat losses can be calculated as the useful part of heat loss and the overall amount of heat loss ($Q_{C,gn}$ - $|Q^{-}_{C,ht}|$):

$$\eta_{C,ht} = \frac{Q_{C,ht} - Q_{C,ht,extra} - |Q^{-}_{C,ht}|}{Q_{C,ht} - |Q^{-}_{C,ht}|} \qquad (24)$$

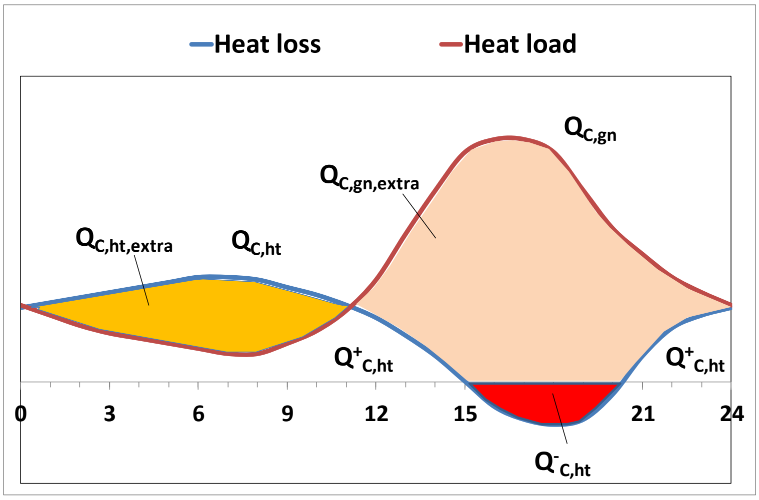


Figure 9: Heat losses and heat gains in an average day in summertime

4 Detailed model of the thermal balance of a room

4.1 Equations for the thermal balance

Let us consider a room with a uniform air temperature t_a. For a generic internal surface the following balance equation can be written (Figure 10):

$$q_{d,i} + q_{c,i} + q_{r,i} + q_{s,i} + q_{l,i} = 0 \qquad i = 1, \ldots, n \qquad (25)$$

where $q_{d,i}$ is the conduction heat flow, $q_{c,i}$ is the convective heat flow, $q_{r,i}$ is the infrared radiation heat flow with other surfaces, $q_{s,i}$ is the high frequency radiant heat flow due to solar radiation and $q_{l,i}$ is the radiant heat flow due to lighting or other internal gains.

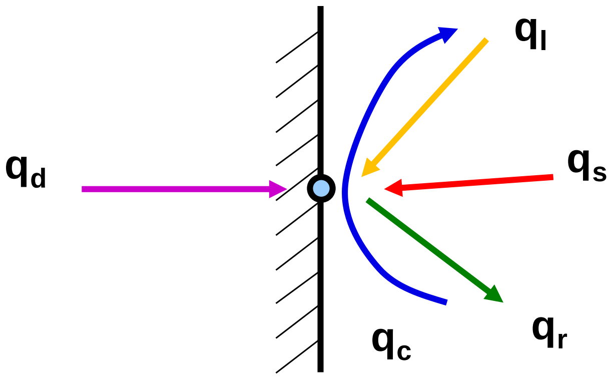


Figure 10: Heat flows to be considered on a generic surface of the room

As far as the conduction is concerned, the heat flow can be calculated under steady state or under unsteady state conditions. The conduction heat flux under steady state conditions can be calculated as:

$$q_{d,i} = \frac{S_i (t_{s,i} - t_{s,o})}{R_i} \qquad (26)$$

where S_i is the area of the considered surface, R_i is the thermal resistance of the wall between internal surface at temperature ($t_{s,i}$) and the outer surface temperature ($t_{s,o}$), which could be external or another indoor environment at the same temperature or at different temperature. The conduction heat flow under dynamic conditions can be evaluated by means of different techniques (usually commercial software calculate the thermal balance of a room by means of the response factors technique).
The convective heat flow from a generic surface S_i (with uniform temperature t_i) can be expressed by means of:

$$q_{c,i} = S_i h_{ci} (t_{s,i} - t_a) \qquad (27)$$

where h_{ci} is the convective heat flow between surface and air. In the most simple thermal balance model, the air is assumed to be at constant temperature. In some cases, especially with large environments and with ventilation techniques which are based on stratification (e.g. displacement ventilation), the different temperatures of the air in the room should be taken into account, as shown in [4]. Further details on the convective heat transfer coefficients are discussed in paragraph 3.2.

As for the radiant thermal flow between two internal surfaces (with absolute temperatures $T_{s,i}$ e $T_{s,j}$, and mean value $T_{s,m}$), the hypothesis of radiantly grey surfaces with emissivity close to one can be assumed, which leads to:

$$q_{r,i} = 4 \cdot \sigma \cdot S_i \sum_{j=1}^{n} \left[F_{i-j} \cdot \varepsilon_i \cdot T_{s,m}^3 \left(T_{s,i} - T_{s,j} \right) \right] \tag{28}$$

where σ is the Stefan-Boltzman constant equal to 5,67x10^{-8} W/(m^2 K^4), F_{i-j} is the view factor between the i-th and the j-th surface, ε_i the emissivity of the i-th surface. Details on the infrared radiant heat exchange between surfaces are reported in paragraph 3.3.
As for the heat flow due to solar radiation, there are several possible solutions, as described in paragraph 3.5, depending on the detail used for the calculation. Usually the most simple approach (leading to accurate results) is the assumption of uniform distribution of the solar radiation in the room. Under these conditions, there are different ways to calculate the solar load from a window, as described in paragraph 3.4. In all cases, once determined the solar energy passing through each window (named $q_{s,k}$), the heat flow impinging the i-th surface due to solar radiation can be calculated as:

$$q_{s,i} = \frac{S_i}{S_t - \sum_{k=1}^{f} S_k} \sum_{k=1}^{f} \left(q_{s,k} \right) \tag{29}$$

where q_{sk} is the energy due to solar radiation entering the room through each window with surface S_k and S_t is the total internal surface of the room. As can be seen, in this case the solar energy is considered only for opaque walls, since for glazed elements the thermal balance with respect to solar radiation has already been considered, as described in 3.4.
The heat load due to internal radiant sources $q_{l,i}$ can be written in a similar way:

$$q_{l,i} = \frac{S_i \left(\sum_{j=1}^{m} q_{l,j} \right)}{S_t} \tag{30}$$

As can be seen, in this case the internal radiant loads are impinging both opaque and glazed surfaces.
The thermal balance of the air (Figure 11) can be written as:

$$\sum_{i=1}^{n} q_{c,i} + q_{c,\text{int}} + q_g + q_p = \frac{M_a c_v \left(t_a - \left(t_a \right)_{-\Delta\tau} \right)}{\Delta\tau} \tag{31}$$

where $q_{c,i}$ are the convective heat flows exchanged with internal room surfaces, equation (27) changed in sign, $q_{C,int}$ is the heat flow due to convective internal gains, q_g is the sensible heat flow due to the inlet air flow rates and q_p is the convective heat power of the internal plants. The term on the right side of the equation represents the internal energy variation in time: t_a and $(t_a)_{-\Delta\tau}$ are the temperatures of the whole air mass M_a respectively at the current time step and at the previous time step, c_v is the specific heat capacity of air at constant volume.

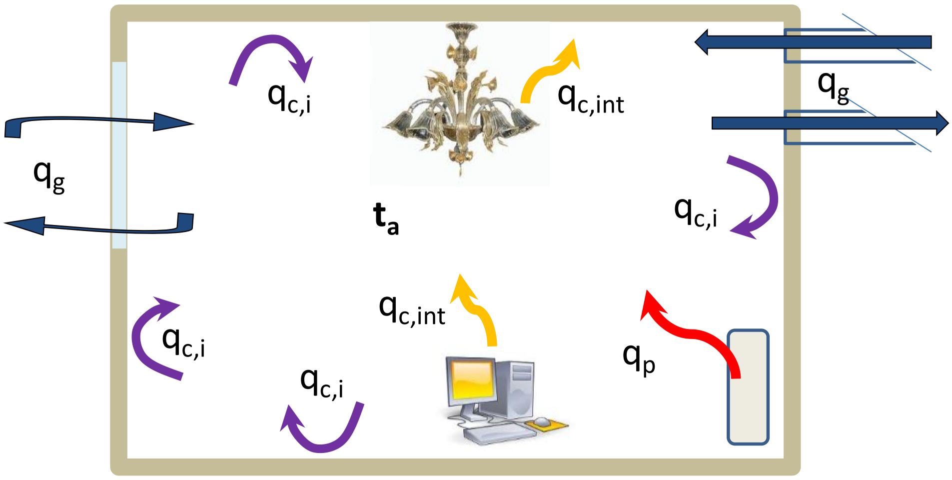


Figure 11: Thermal balance of the air in a room

The heat load due to all internal convective sources $q_{c,u}$ can be written as the sum of all convective sources:

$$q_{c,\mathrm{int}} = \sum_{u=1}^{s} q_{c,u} \tag{32}$$

The convective heat flow q_g due to the incoming and outgoing air rates in the room can be written as follows:

$$q_g = \sum_{z=1}^{r} \left[G_{a,z} c_p (t_{a,z} - t_a) \right] \tag{33}$$

where $G_{a,z}$ are the air flow rates at temperature $t_{a,z}$, which can be due to infiltration (external temperature) or due to mechanical ventilation devices handling in case the air.
It is possible, in this way, to write n equations (25) and one equation (31), thus obtaining a system of n+1 equations in the n+2 variables, represented by the n internal surface temperatures $t_{s,i}$, by q_p, and by t_a. Therefore, there are two cases:

- if the air temperature t_a is fixed, a convective heating or cooling system, (e.g. fan-coil units) can be simulated and the result is the heat flow required by the plant (q_p) to maintain the set-point temperature;
- if the heat flow required by the plant (q_p) is fixed (usually it is null, in case the plant is switched off), the resulting air temperature t_a of the room can be calculated.

For simulating heating and cooling radiant systems, there are two ways. In the first method the system of equations allows to evaluate the surface temperature which gives a null value to q_p: the correspondent convective or radiant heat flow from the considered surface is the power required by the radiant system [5]. In this way it is possible to have some simplified methods which take into account the effective distribution of the heat flows. The second method is more complicated, since it needs additional equations for accounting the thermal balance and the mass balance of the fluid flowing in the pipes, as described in paragraph 4.
The vapour mass balance can be calculated as well in the following way:

$$G_{v,in} + G_{v,p} + \sum_{z=1}^{r}\left[G_{a,z}(\xi_{a,z} - \xi_a)\right] = 0 \quad (34)$$

where $G_{v,in}$ is the rate of internal vapour production due to all sources, $G_{v,p}$ is the vapour generation/extraction due to the plant, $G_{a,z}$ are the air flow rates entering the room at specific humidity $\xi_{a,z}$, which can be due to infiltration (external temperature) or due to mechanical ventilation devices, handling in case the air. In this case it is supposed that the humidity is not absorbed by materials and internal surfaces of the walls. This assumption is quite common. There are applications where sorption of vapour in structures have to be considered, like old historical buildings, churches etc. In this case hygrothermal simulation models have to be considered which require some basic knowledge about material properties. Most building materials are hygroscopic which means that they absorb water vapour from the environment until equilibrium conditions are achieved. This behaviour can be described by sorption curves over a relative humidity range between 0 and 95%. More details on these models can be found in [6].

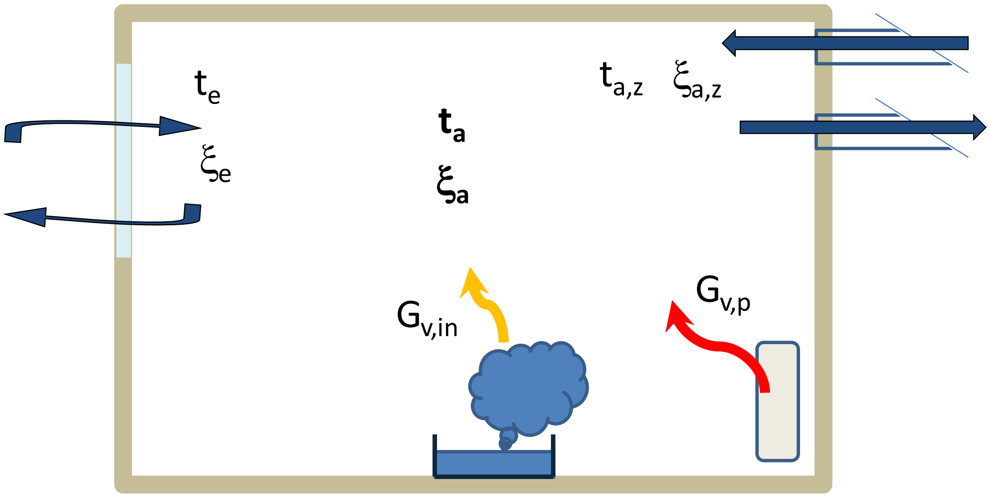


Figure 12: Vapour balance in a room

4.2 Convective heat exchange

The convective heat flows for the *i*-th general surface element facing air can be expressed by means of equation (27). In natural convection the air flow may be laminar or turbulent, depending on the position of the surface and the temperature difference between the surface and the fluid. Since transition occurs in the range $10^7 < Ra < 10^9$, in building applications the problem of fluid motion is generally turbulent (Rayleigh numbers are circa 1×10^{10}).
Three types of correlations to estimate internal heat transfer coefficient for natural convection can be found in literature: correlations for isolated horizontal plates, for enclosed spaces and from studies in three-dimensional rooms. Based on a literature review [7], the most reliable analyses are those based on measurements in full scale test rooms, as they reproduce the most realistic conditions of a heated/cooled environment. In the past a lot of studies were carried out on heat transfer in test rooms, aimed at deriving correlations describing natural convection phenomena.
Awbi [8] used experimental results to validate the numerical predictions of CFD codes. Convective heat transfer coefficients, valid for heated floors, ceilings and walls, were

calculated from measurements at various temperature differences. Named D_e the equivalent diameter of the room, the following equations were proposed for heated floors, ceilings and walls respectively:

$$h_C = 2.175 \cdot \frac{(t_s - t_a)^{0.308}}{D_e^{0.076}} \tag{35}$$

$$h_C = 0.704 \cdot \frac{(t_s - t_a)^{0.133}}{D_e^{0.601}} \tag{36}$$

$$h_C = 1.823 \cdot \frac{(t_s - t_a)^{0.293}}{D_e^{0.121}} \tag{37}$$

In Olesen et al. [9] heat exchange coefficients for floor cooling are calculated with different reference measured temperatures by means of four different approaches. In the first approach, radiant heat exchange is calculated using 5.5 W m^{-2} K^{-1} as radiant heat transfer coefficient and the reference temperature for the convection is the air temperature at two different heights (0.6 m and 1.1 m), for 9 test conditions. Results of this work have been further elaborated [7] to obtain the following expression:

$$h_c = 1.26 \cdot \frac{(t_s - t_a)^{0.25}}{D_e^{0.25}} \tag{38}$$

Karadağ [10] determines radiant and convective ceiling heat transfer coefficients using a commercial computational fluid dynamics tool. Convective heat transfer is simulated numerically for the cooled ceiling case, avoiding radiant heat transfer (ε =0); the resulting equation is:

$$h_C = 3.1 \cdot (t_s - t_a)^{0.22} \tag{39}$$

A recent analysis has shown that, if the purpose is to maintain the comfort conditions in the room, the conditions should allow moderate values of the parameters involved in the thermal balance of the room, including heat exchange coefficients [11]. The difference in using variable or constant parameters for the convective heat exchange coefficients is negligible, therefore the calculation should be simplified, since there is no need to calculate convective coefficients with iterative solutions. The suggested values for the convective heat exchange coefficients are reported in Table 4.

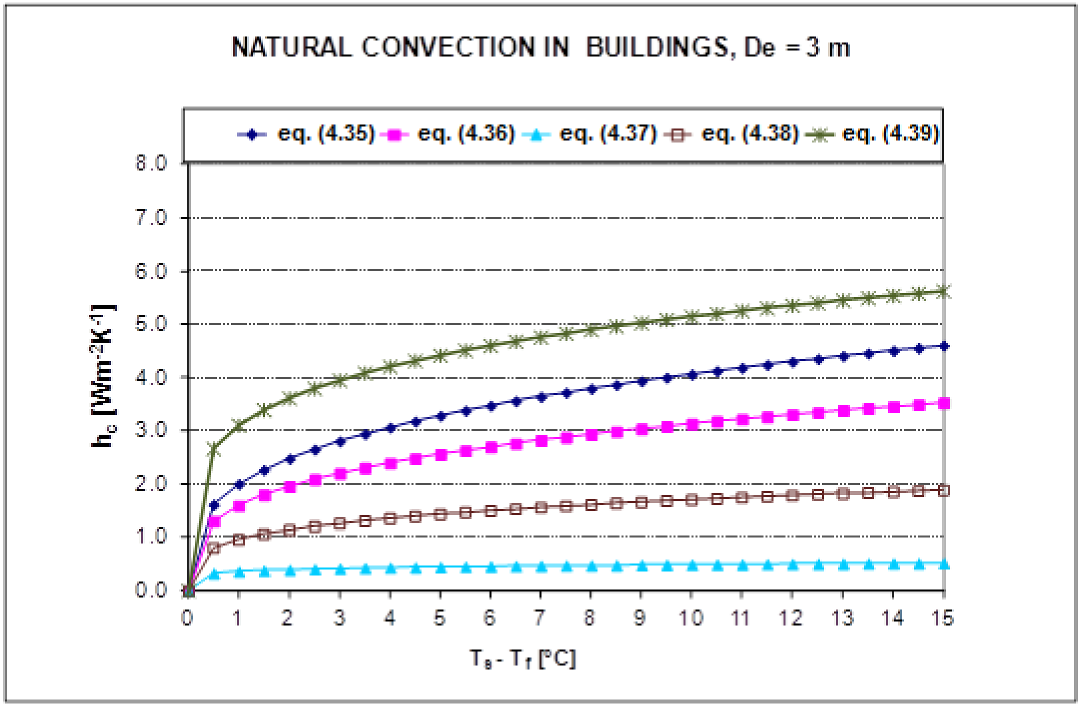


Figure 13: Convective heat transfer coefficients calculated with equation (35) (floor heating), euquation (36) (wall heating), equation (37) (ceiling heating), equation (38) (floor cooling) and equation (39) (ceiling cooling), by considering an equivalent diameter of the surface D_e of 3 m

Table 4: Constant convective heat exchange coefficients of surfaces [W m^{-2} K^{-1}] which can be considered in a thermal balance of a room.

	Heating	Cooling
Floor	5.3	1.5
Wall	2.5	2.5
Ceiling	0.5	5.3

4.3 Radiant heat exchange

For mutual radiation, in the range of usual temperatures in the indoor environments, equation (28) can be simplified in the following equation:

$$q_{r,i} = h_r S_i \sum_{j=1}^{n} \left[F_{i-j} \left(T_{s,i} - T_{s,j} \right) \right] \qquad (40)$$

where the radiant heat exchange coefficient $h_r \approx 4\ \sigma\,\varepsilon\, T_m^3$ = 5.5 W/(m^2 K).
The view factors (Figure 14) should be calculated as:

$$F_{i-j} = \frac{1}{\pi \cdot S_i} \int_{Si} \int_{Sj} \cos(\phi_j) \cdot \cos(\phi_i) \frac{dS_j \cdot dS_i}{r^2} \qquad (41)$$

where, being r the distance between the centres of surfaces dS_i and dS_j, and ϕ_i and ϕ_j are the angles between the segment connecting the centres and the surface normal n_i and n_j.
It is interesting to remark that, if the surface discretisation is fine enough, the view factors may be approximated as follows:

$$F_{i-j} \cong \frac{\cos\phi_i \cos\phi_j \, S_i}{\pi \, r^2} \tag{42}$$

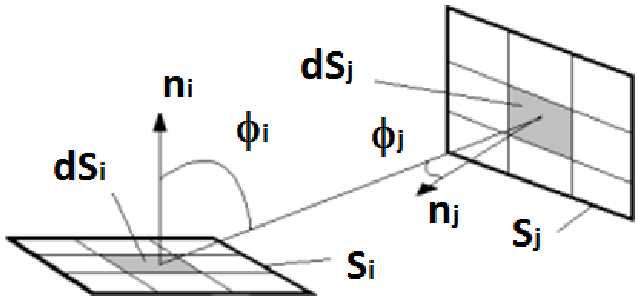
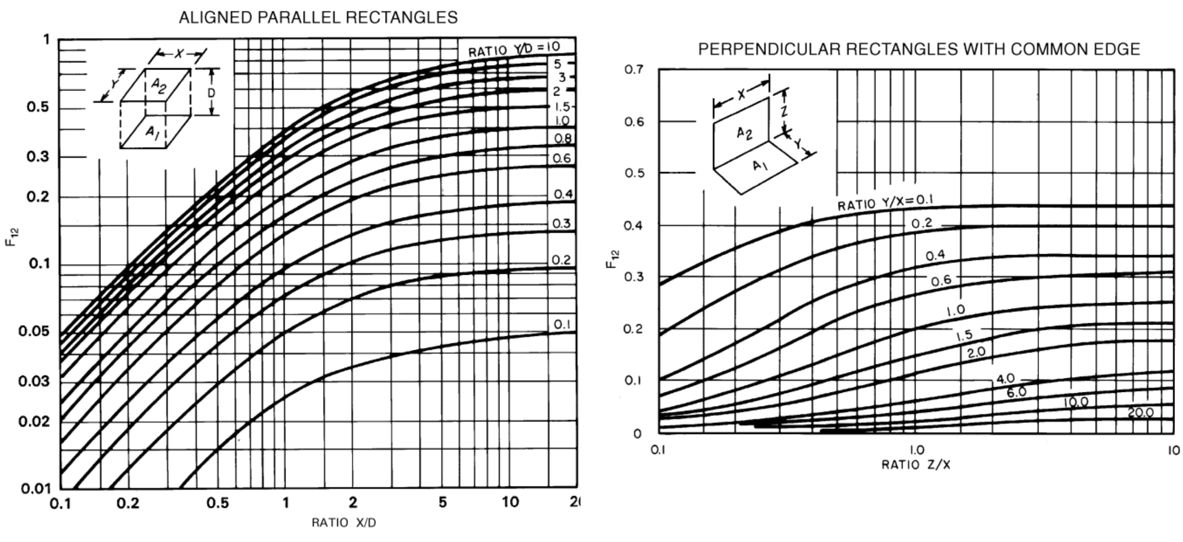


Figure 14: Determination of the view factors between building elements

A different way to estimate the mutual infrared heat exchange radiation is to resort to diagrams, as the ones reported in Figure 15.
An alternative way to calculate view factors is the use of equations [12].



a **b**

Figure 15: Determination of the view factors by means of diagrams
Source: [13]

4.4 Solar radiation modelling in the rooms

There are different models for dealing with solar radiation inside a room. Four possible models are here presented. Two of the models contain the standard criteria which are used in building simulation techniques while the other two are detailed. In the latest two models the surfaces of the room have to be discretised in sub-regions, i.e. in surface elements.

4.4.1 Standard simplified models

In the simplest model, solar radiation is distributed in a uniform way, without taking into account the absorption coefficients of the surface elements or the colour of the surfaces. This is the most common model used in room simulations, where the solar radiation is not assigned to the glazed elements. The room is a black body that absorbs all of the solar radiation with no radiation exiting through the windows. The general equation to be used in the room balance is the equation (29).
An alternative model has been proposed in CEN TC 89WG 6 [14], in order to calculate the design performance of the radiant systems under cooling conditions. The overall radiation entering the room is distributed depending on the type of surface (floor, ceiling, wall or window) and the colour of the floor (tables 5 and 6). Usually, with this model the incoming solar radiation is supposed to remain in the room, i.e. it does not exit through windows. Internal radiant gains are usually also shared between surfaces with the same criterion.

Table 5: The solar energy distribution coefficients for the dark-coloured floor

Floor	Ceiling	Wall	Window
$\frac{2 \cdot S_f}{S_f + S_{tot}}$	$\frac{S_c}{S_f + S_{tot}}$	$\frac{S_{op}}{S_f + S_{tot}}$	0

Table 6: The solar energy distribution coefficients for the bright-coloured floor

Floor	Ceiling	Wall	Window
$\frac{S_f}{S_{tot}}$	$\frac{S_c}{S_{tot}}$	$\frac{S_{op}}{S_{tot}}$	0

4.4.2 Detailed models

In each time step of the first detailed model, depending on the reciprocal position of the surface element and the Sun, the surface elements that are struck by direct solar radiation through the window are determined (Figure 16). Such surface elements have a non-null reflection coefficient; thus, the direct solar radiation is redistributed to the surrounding elements. The multiple reflections between the surface elements can be expressed by (Figure 17):

$$I_j = I_{bj} + \sum_{i=1}^{N} F_{j-k} \cdot (1-\alpha_k) \cdot I_k \tag{43}$$

which leads to the following linear system:

$$\begin{pmatrix} 1 & \dots & -(1-\alpha_1)F_{N1} \\ . & . & . \\ . & . & . \\ . & . & . \\ -(1-\alpha_1)F_{N1} & \dots & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ . \\ . \\ . \\ I_N \end{pmatrix} = \begin{pmatrix} I_{b1} \\ . \\ . \\ . \\ I_{bN} \end{pmatrix} \tag{44}$$

The direct solar radiation is calculated at each time step under the hypothesis of a clear sky, as proposed by Wen and Smith [15]. Windows are assumed to have an influence on energy transmission but not on solar direction. In this way, it is not possible to couple windows with internal shading devices because the shading device itself acts as a diffuse radiation source.
The internal distribution of the solar radiation is affected by the discretisation of the room. In this case, if a surface element is only partially subject to direct solar radiation, it is assumed that no solar radiation impinges on the surface element and that the related part of the direct solar radiation is added to the diffuse radiation.
After the distribution of the direct solar radiation in the room has been determined, the diffuse radiation and the albedo solar radiation are distributed as follows:

$$I_d = \frac{(\Phi_d + \Phi_a) \cdot S_w}{S_{tot}} \tag{45}$$

The sum of I and I_D is the overall incident solar radiation on the single-surface element. To determine the dynamic thermal behaviour of the room, the calculation of the radiation that is absorbed by each surface element is important because it is a boundary condition for the calculation of the thermal balance of the room. Therefore, for the generic *i-th* element, the following equation can be written:

$$(I_a)_i = \alpha_i \cdot I_i + I_d \tag{46}$$

If the internal radiant gains are present, the load is distributed and added to the absorbed solar radiation that is expressed by equation (46).

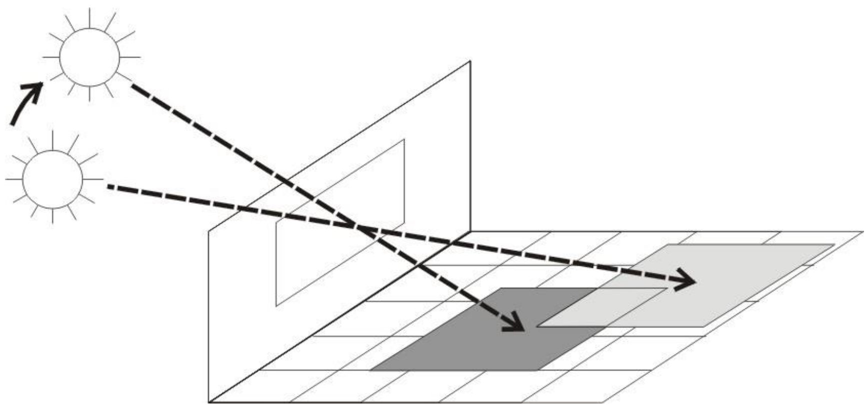


Figure 16: Identification of building elements that are subject to direct solar radiation

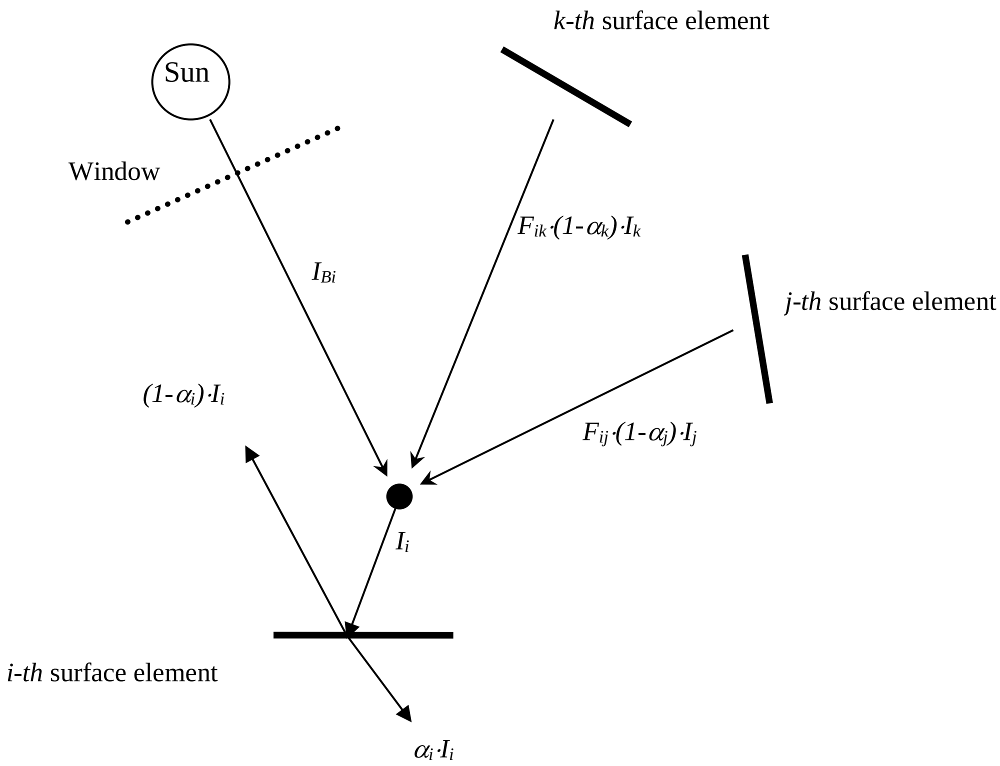


Figure 17: Schematic of the distribution of solar radiation inside of the room in the detailed model

There is an alternative way to build up a detailed model. In this case surface elements that are subject to direct solar radiation are determined as in the previous model but, in this case, the direct radiation that is absorbed by the surface also depends on the incidental angle of the solar radiation on the surface, i.e. on the angle between the impinging radiation and the normal to the surface. Such a model allows the assignment of direct solar radiation considering the mutual orientation of the surfaces based on the position of the centre of the elements. Therefore, the discretisation may affect the calculations in this model and errors may occur in two cases:

- the area of the whole illuminated surface is null (no centre of the elements is lighted) but the power entering from the window is not zero;
- the surface of the window that is projected onto the plane perpendicular to the incident angle of solar radiation differs from the sum of the projections of the single surface elements on the same plane.

The correction procedure changes depending on the case. In the first case, the whole direct solar radiation entering the room is added to the diffuse radiation and is distributed as described hereafter. In the second case, the remaining radiation is added to the impinged surface elements via the equation:

$$I_b^* = I_b \cdot \frac{S_w \cdot \cos \vartheta_v}{\sum_{i=1}^{P} S_i \cdot \cos \vartheta_i} \qquad (47)$$

From equation (4.70), if the area of the window is bigger than the sum of the area impinged by the surface elements, the incident solar radiation is greater than the real one and vice versa. Nevertheless, this is a minor error and, hence, can be considered negligible.
The diffuse radiation that passes through the window is assigned to the surface elements of the window, which acts as a radiant source for the other internal surface elements, i.e. the window is a perfect diffuse radiation surface source [16]. This model can be summarised using the following equation:

$$I_j = I_{bj}^* \cdot \cos \vartheta_j + \sum_{i=1}^{N} F_{jk} \cdot \left[(1-\alpha_k) \cdot I_k + I_{dk} + \Phi_{\text{int}} \right] \qquad (48)$$

where I_{dk} is not null for surfaces that are not windows.

4.4.3 Remarks on the models of solar radiation in a room

In a recent work [17] it has been shown that if one aims to accurately predict localised thermal behaviour of a room, the detailed method, which takes into account the re-irradiation, must be utilised. If the evaluation is based on average values (temperatures, heat flows and comfort), then the rough models are also valid. Furthermore, when solar radiation is considered to be uniformly distributed comfort conditions at a single, central point of the room can be calculated. Whereas, in the case of the accurate models, comfort conditions at specific points in the room can be predicted, thus aiding space distribution in the design phase.
Once a model with only one surface and no element discretisation is considered, it is unnecessary to split the radiation into different methods in order to take into account the colour of the surfaces or their position. More detailed models do not achieve results which are significantly different from those of simplified models which assume solar radiation entering a room to be diffuse.

4.5 Example of a balance in steady state conditions

In this paragraph a simplified method for a first sizing of a cooling system is presented for a better understanding of the thermal balance of a room. In this case (Figure 18) one external surface (the glazed roof) and one internal surface (the floor) are considered as surface elements. The solar radiation is partly absorbed by the glazed roof (I_a) and the remaining transmitted part is absorbed by the floor (I_t). Internal gains are partly convective ($q_{c,int}$) and partly radiant (q_l). Handled air enters the room at temperature $T_{a,in}$.
As for the absorbed solar radiation, a simple approach is to fix the thermal characteristics of the glazed surface (thermal resistance R_{ceil}) and to split the absorbed solar energy into the two inner and outer surfaces.
Four nodes are considered for determining the system of equations defining the thermal balance of the model (Fig. 19).

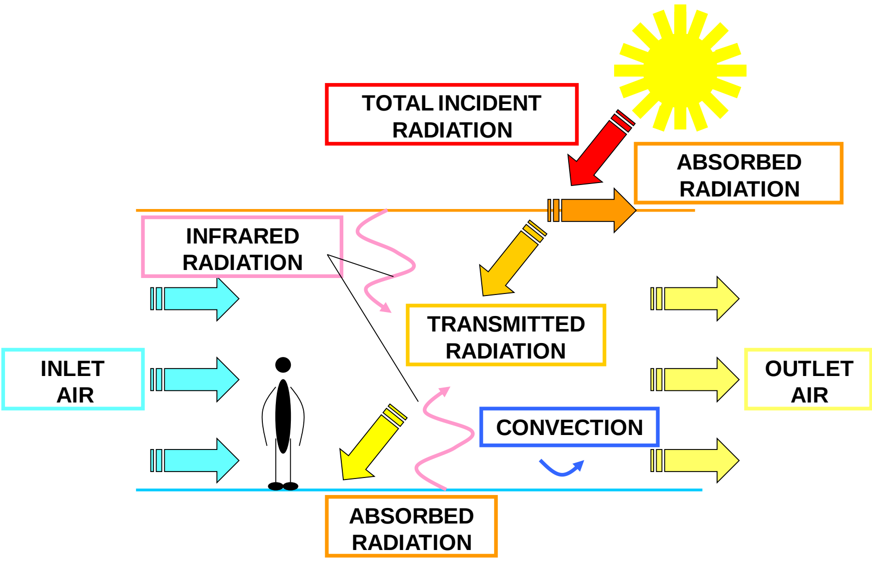


Figure 18: Schematic of the thermal balance for a simplified model

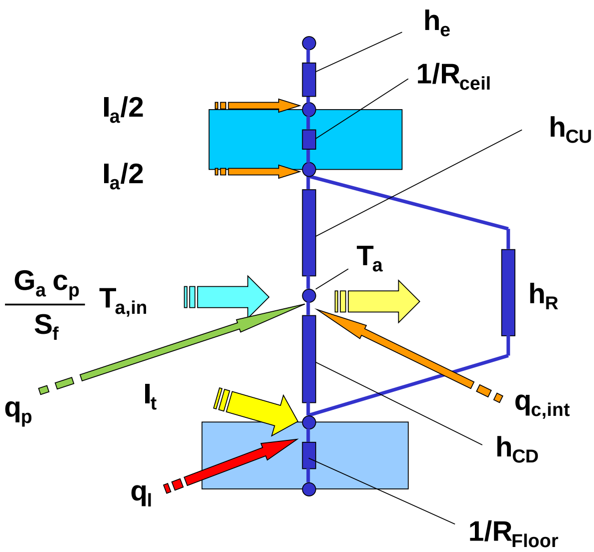


Figure 19: Schematic of the distribution of solar radiation inside of the room in the detailed model

For the floor surface node, with temperature t_f:

$$\frac{(t_f - t_w)}{R_{Floor}} + h_R(t_f - t_{CeD}) + h_{CD}(t_f - t_a) = q_l + I_t \tag{49}$$

where t_w is the temperature of the boundary conditions below the floor and R_{floor} the thermal resistance of the floor.

For the air node balance the following equation can be written:

$$h_{CD}(t_a - t_f) + \frac{G_a c_p}{S_f} \cdot (t_a - t_{a,in}) + h_{CU}(t_a - t_{CeD}) = q_p + q_{c,\mathrm{int}} \tag{50}$$

For the inner surface temperature node (t_{ceD}):

$$h_R(t_{ceD} - t_f) + h_{CU}(t_{CeD} - t_a) + \frac{(t_{ceD} - t_{ceU})}{R_{ceil}} = \frac{I_a}{2} \tag{51}$$

For the outer surface temperature node (t_{ceU}):

$$\frac{(t_{ceU} - t_{ceD})}{R_{ceil}} + h_e(t_{ceU} - t_{amb}) = \frac{I_a}{2} \tag{52}$$

Once decided to fix the power of the convective plant q_p, the linear system of equations is:

$$\begin{pmatrix} \frac{1}{R_{floor}} + h_{CD} + h_R & -h_{CD} & -h_R & 0 \\ -h_{CD} & \frac{G_a c_p}{S_f} + h_{CD} + h_{CU} & -h_{CU} & 0 \\ -h_R & -h_{CU} & h_R + h_{CU} + \frac{1}{R_{ceil}} & -\frac{1}{R_{ceil}} \\ 0 & 0 & -\frac{1}{R_{ceil}} & \frac{1}{R_{ceil}} + h_{ext} \end{pmatrix} \cdot \left\{ \begin{matrix} t_f \\ t_a \\ t_{ceD} \\ t_{ceU} \end{matrix} \right\} = \left\{ \begin{matrix} q_l + I_t + \frac{t_w}{R_{floor}} \\ q_p + q_{c,\text{int}} + \frac{G_a c_p}{S_f} t_{a,in} \\ \frac{I_a}{2} \\ \frac{I_a}{2} + h_{ext} t_{amb} \end{matrix} \right\} \tag{53}$$

5 Boundary conditions on external surfaces

5.1 Sol-air temperature

When dealing with boundary conditions, the outdoor climatic conditions are defined through the sol-air temperature, which includes the combined effect of the ambient temperature (t_{amb}) and the solar radiation. This is usually applied as contact temperature on external surface:

$$t_{sa} = t_{amb} + \frac{\alpha \cdot f_{sh} \cdot I}{h_e} \tag{54}$$

where α is the solar absorbance of the external wall, f_{sh} is the shading factor of the considered wall, I is the solar incidence radiation and h_e is the convective heat exchange coefficient of the outer wall; a suitable value for h_e is equal to 15 W/(m^2 K). The effect of considering the outdoor temperature or the sol-air temperature is shown in Figures 20 and 21 for a period in winter and summer respectively in Venice (Italy).

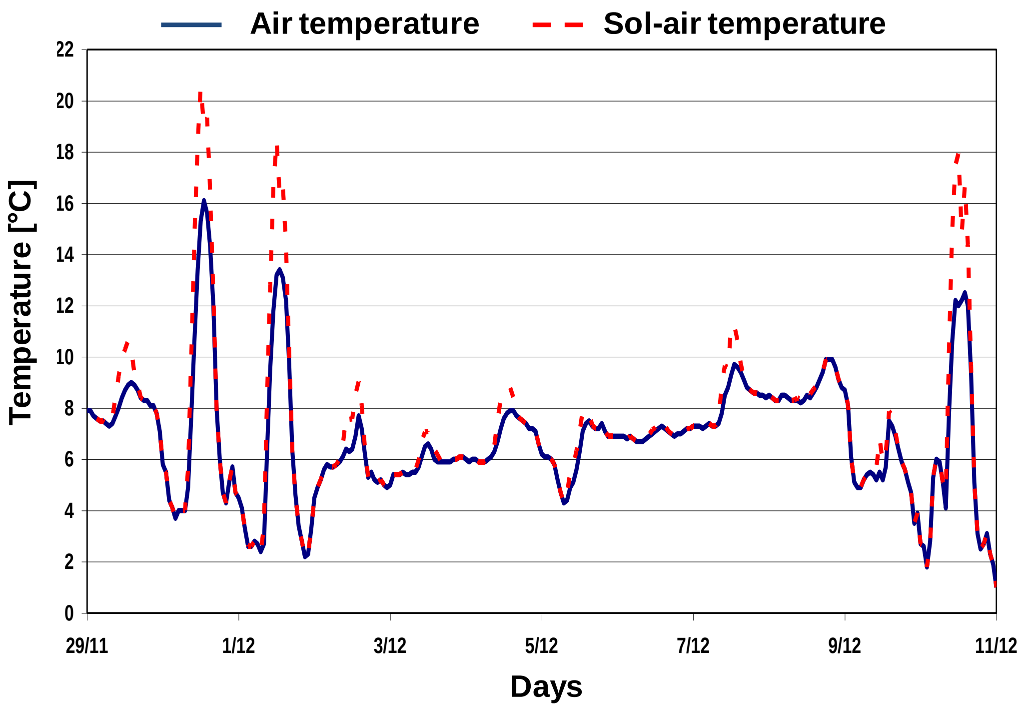


Figure 20: Effect of solar radiation in winter time for a cold week in Venice

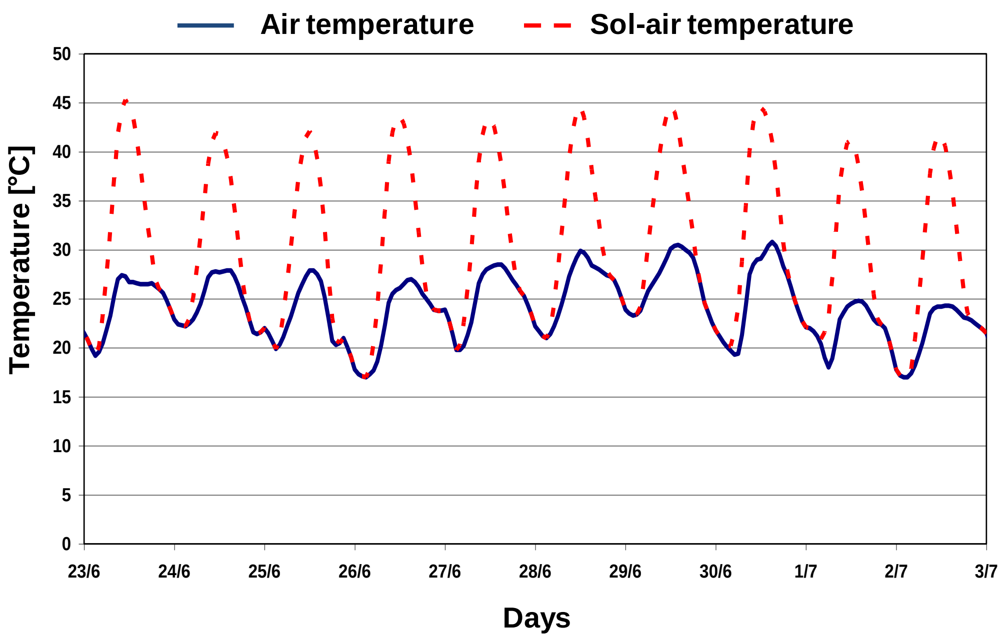


Figure 21: Effect of solar radiation in summer time for a warm week in Venice

5.2 Heat exchange with the sky

The long-wave radiation heat exchange from a surface (an external surface of the building or the ground) to the atmosphere can be calculated by means of a fictive sky temperature. In the calculation the sky is assumed to be an ideal black surface. The actual emittance of the clear and clouded sky must be known. Thus, the fictive sky temperature is a function of ambient temperature, air humidity, cloudiness factor of the sky, local air pressure. If the weather data do not include the cloudiness factor of the sky, the cloudiness factor K_{cluod} can be determined a s a function of the ratio of the diffuse radiation on horizontal $I_{d,0}$ and the overall radiation on horizontal $I_{T,0}$, according to the following equation [18]:

$$K_{cloud} = \sqrt{1.4286\frac{I_{d,0}}{I_{T,0}} - 0.3} \tag{55}$$

The atmospheric pressure is determined according to the barometric height (h) formula for the considered location:

$$p_{atm} = p_0 \cdot e^{\frac{g\rho_0 h}{p_0}} \tag{56}$$

The emittance of the clear sky can be derived by the saturation temperature of the air ($T_{amb,sat}$), which depends on the ambient conditions (temperature and air humidity) [19]:

$$\begin{aligned}\varepsilon_0 &= 0.711 + 0.005 \cdot T_{sat} + 7.3 \cdot 10^{-5} \cdot T_{sat}{}^2 + 0.013 \cdot \cos\left[2\pi\frac{\tau_h}{24}\right] + \\ &+ 12 \cdot 10^{-5}\left(p_{atm} - p_0\right)\end{aligned} \tag{57}$$

where the variable time τ_h corresponds to the hour of the day.

The fictive sky temperature can then be determined by [19]:

$$T_{sky} = T_{amb}\left[\varepsilon_0 + 0.8 \cdot (1 - \varepsilon_0)K_{cloud}\right]^{0.25} \tag{58}$$

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