Università degli Studi di Padova

Dipartimento di Ingegneria Industriale

Energy and buildings

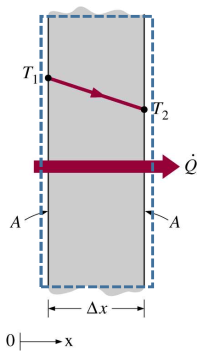
Energy balance of a room

Building energy balance

The **building energy needs** depend on several overlapping physical phenomena:

- Heat conduction through the walls
- Heat convection and radiant heat exchange on internal and external surfaces
- Internal generation of heat due to people and appliances
- Solar radiation absorbed by opaque building components
- Solar radiation transmitted through glazed building components

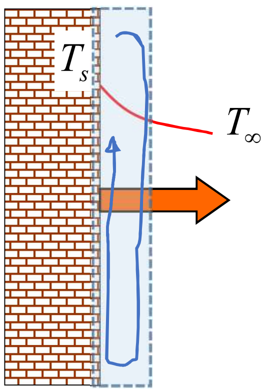
Heat conduction through the envelope



Heat conduction is a heat transfer mechanism between two systems (either **solids, liquids or gases**) at different temperatures **with no significant mass transfer** and responds to the Fourier Law

$$\vec{q} = -k \; \nabla T$$

Heat transfer by convection



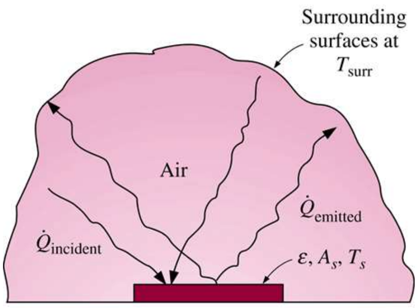
Thermal convection is a heat exchange mechanism that takes place between two systems placed **in contact with each other** and at different temperatures, where at least one of the two systems is **fluid**.

A necessary condition for this energy exchange to take place is that there is **relative motion** between the two systems.

$$\mathrm{q} = h_c A(T_s - T_\infty)$$

Heat transfer by radiation

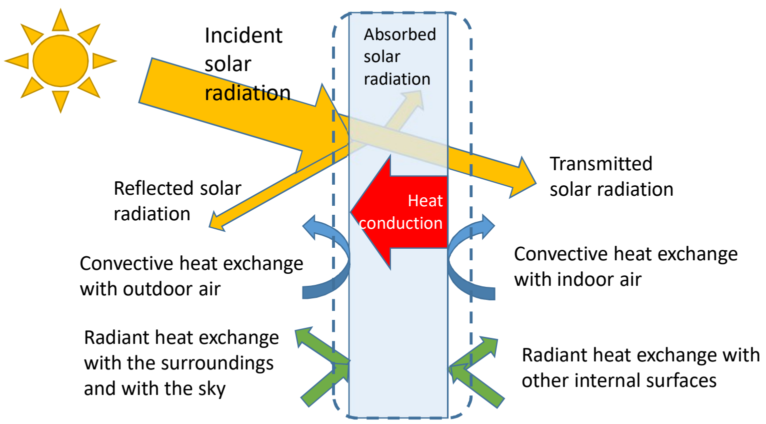
The **emissivity ε** of a material expresses the global emission E of that material relative to the global emission of a black body at the same temperature E_b.



$$q_{rad} = \varepsilon\,\sigma A_s (T_s^4 - T_{surr}^4)$$

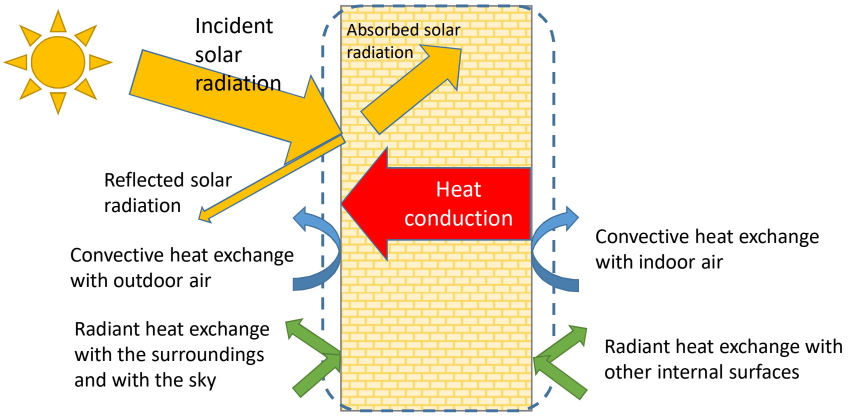
Heat balance of the building envelope

Glazed building components



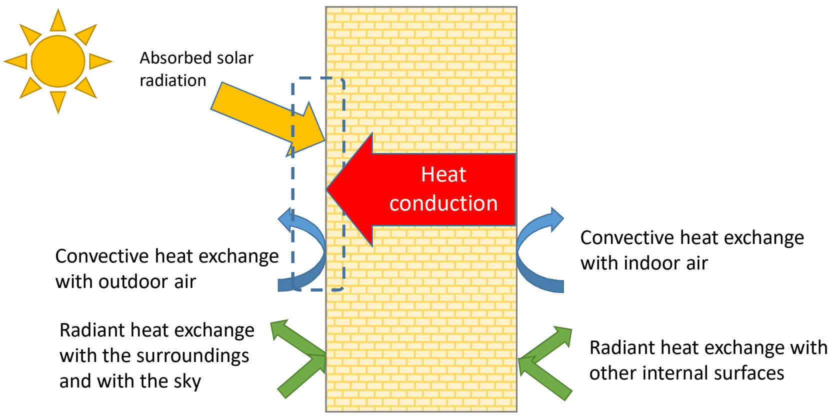
Heat balance of the building envelope

Opaque building components



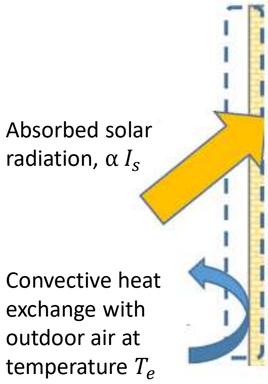
Heat balance of the building envelope

Opaque building components



Heat balance of the building envelope

Opaque building components: sol-air temperature



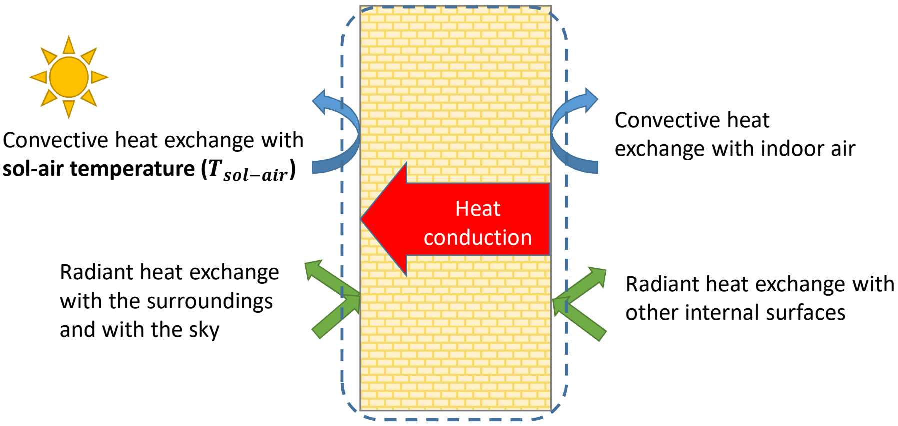
$$h_{c,e}(T_s - T_e) - \alpha\, f_{sh} I_s = h_{c,e}(T_s - T_{sol-air})$$

$$h_{c,e}(T_{sol-air} - T_e) = \alpha\, f_{sh} I_s$$

$$\boldsymbol{T_{sol-air} = T_e + \frac{\alpha\, f_{sh} I_s}{h_{c,e}}}$$

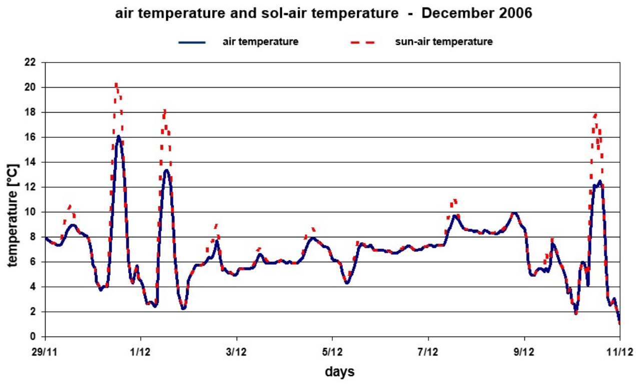
Heat balance of the building envelope

Opaque building components: sol-air temperature



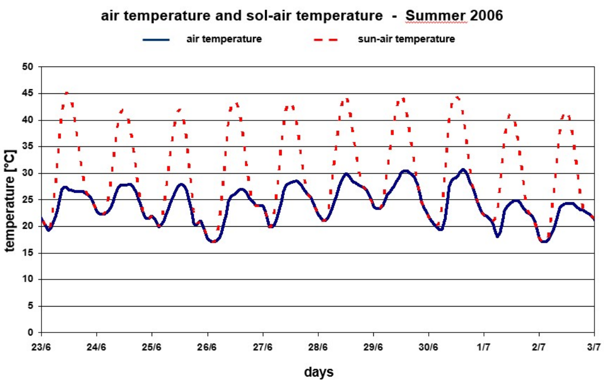
Heat balance of the building envelope

Opaque building components: sol-air temperature



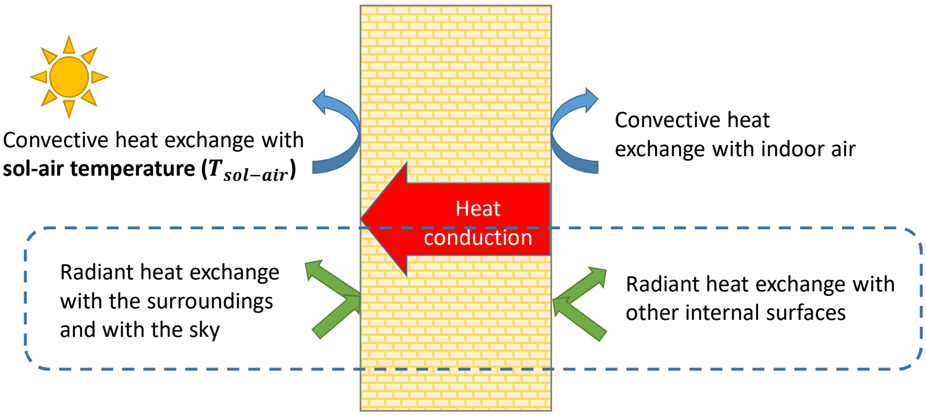
Heat balance of the building envelope

Opaque building components: sol-air temperature



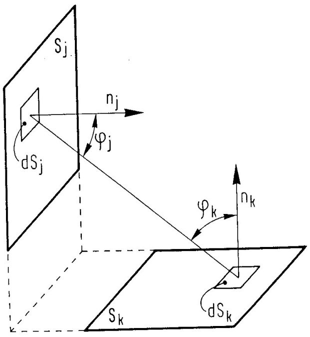
Heat balance of the building envelope

Radiant heat exchange between surfaces



Heat balance of the building envelope

Radiant heat exchange between surfaces



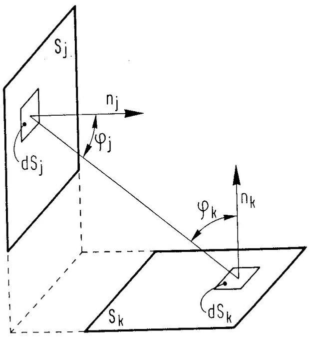
View factor F_{k-j}: fraction of the radiation that reaches directly the k-th surface from the j-th surface

$$F_{k-j} = \frac{1}{2\pi S_j} \iint_{S_k S_j} \left(\frac{\cos\varphi_k \cos\varphi_j}{d^2} \right) dS_k dS_j$$

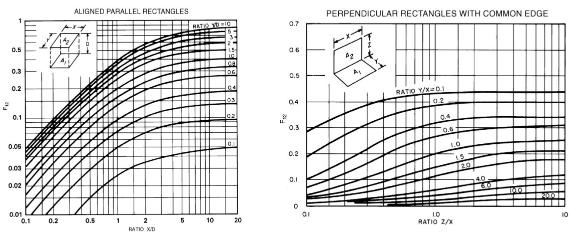
$$q_{k-j} = A_k F_{k-j} \sigma \left(T_k^4 - T_j^4 \right) = A_j F_{j-k} \sigma \left(T_k^4 - T_j^4 \right)$$

Heat balance of the building envelope

Radiant heat exchange between surfaces

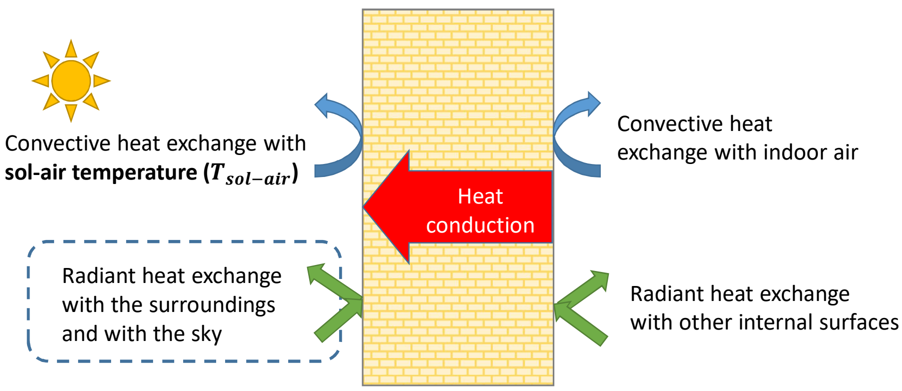


Charts with pre-calculated view factors $\boldsymbol{F_{k-j}}$ for given geometries of the mutual connections



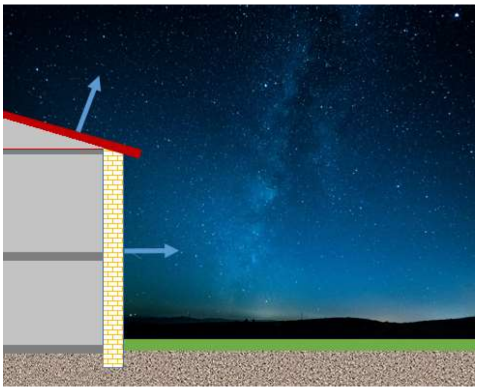
Heat balance of the building envelope

Radiant heat exchange towards the sky



Heat balance of the building envelope

Thermal radiation to the sky



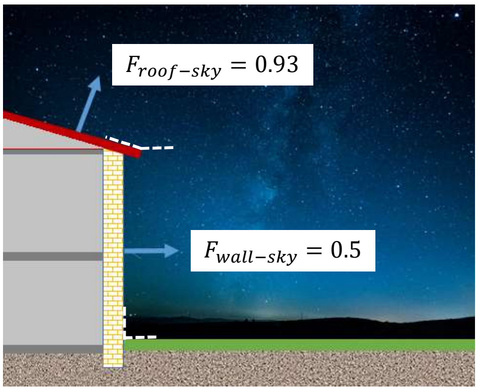
The radiant heat emission of the j-th outer surface of the building towards the sky can be expressed by:

$$q_{sky,j} = \varepsilon_j \sigma A_j F_{j-sky} (T_j^4 - T_{sky}^4)$$

The effective sky temperature T_{sky} can be calculated based on the local air temperature and relative humidity (Bliss, 1961)

Heat balance of the building envelope

Thermal radiation to the sky



$$q_{gnd,j} = \varepsilon_j \sigma A_j F_{j-gnd} (T_j^4 - T_{gnd}^4)$$

$$F_{j-sky} = \frac{1-cos\theta}{2}\,; F_{j-gnd} = 1 - F_{j-sky}$$

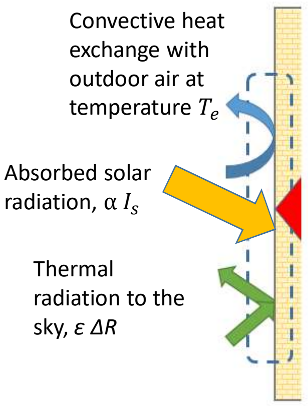
$$q_{gnd,j} + q_{sky,j} = \varepsilon_j \sigma\, A_j (..)$$

$$\frac{q_{gnd,j}+q_{sky,j}}{A_j} = \varepsilon_j\, \Delta R$$

ASHRAE Handbook suggests ΔR = 63 W/m²

Heat balance of the building envelope

Opaque building components: sol-air temperature with thermal radiation to the sky

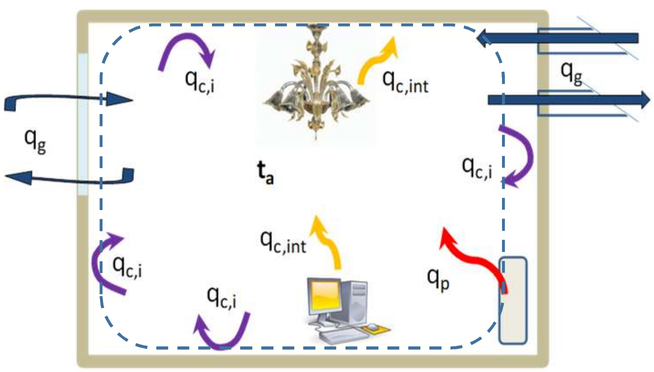


$$h_{c,e}(T_s - T_e) - \alpha\, f_{sh} I_s + \varepsilon\, \Delta R = h_{c,e}(T_s - T_{sol-a}\quad)$$

$$h_{c,e}(T_{sol-air} - T_e) = \alpha\, f_{sh} I_s - \varepsilon\, \Delta R$$

$$T_{sol-air} = T_e + \frac{\alpha\, f_{sh} I_s}{h_{c,e}} - \frac{\varepsilon\, \Delta R}{h_{c,e}}$$

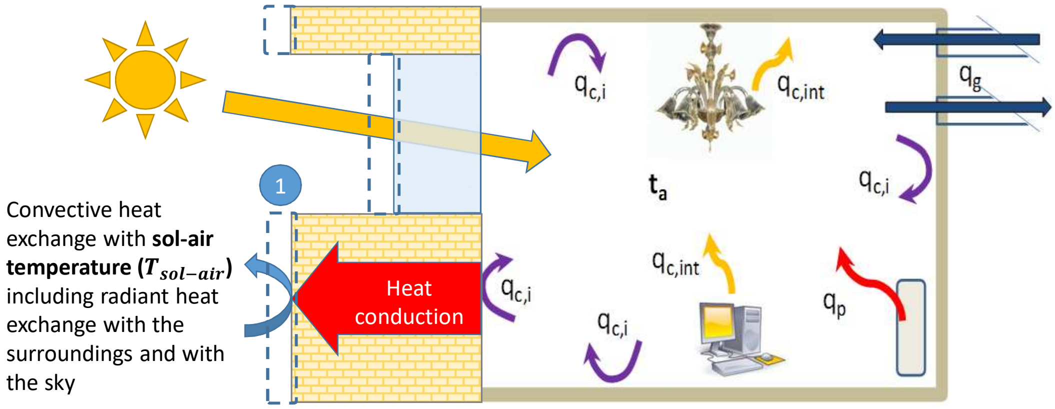
Heat balance of the indoor air



- Convective heat exchange with internal surfaces of the building envelope $q_{c,i}$
- Convective heat exchange with internal heat sources such as lights and appliances $q_{c,int}$
- Convective heat exchange with HVAC system $q_{c,p}$
- Convective heat exchange due to ventilation and infiltration q_g

Overall heat balance of the room

Heat exchange at the external surface (1)



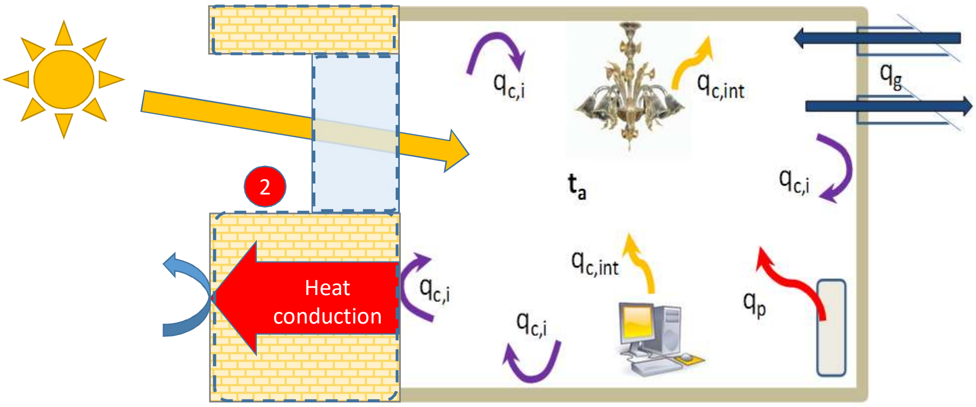
Overall heat balance of the room

Heat exchange at the external surface (1)

1 $$-h_{ce,k}S_k\left(T_{se,k}-T_{sol,air}\right)+\ q_{cond}=0$$

Overall heat balance of the room

Heat conduction through the building components (2)



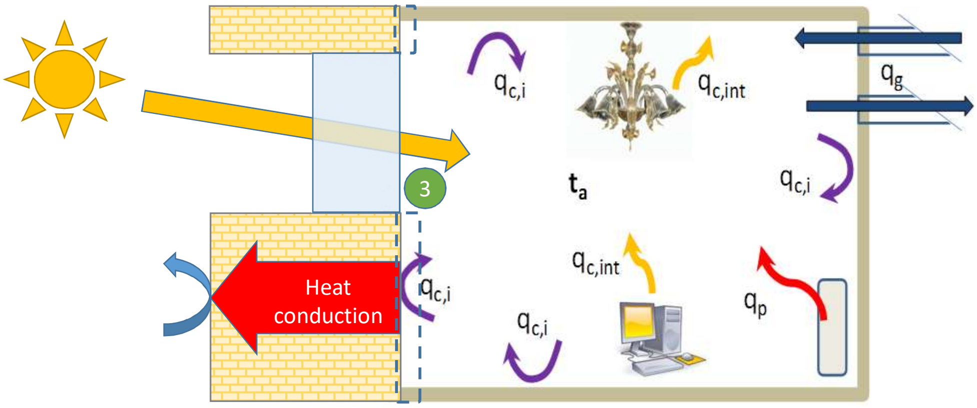
Overall heat balance of the room

1 $$-h_{ce,k}S_k\left(T_{se,k}-T_{sol,air}\right)+\ q_{cond}\ =0$$

2 $$q_{cond,k}=S_k\ \cdot f\left(T_{si,k}-T_{se,k}\right)$$

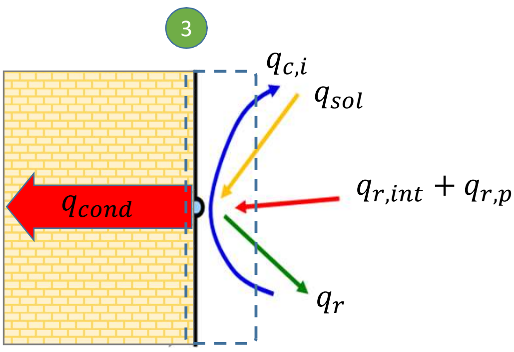
Overall heat balance of the room

Heat exchange at the internal surface (3)



Overall heat balance of the room

Heat exchange at the internal surface (3)

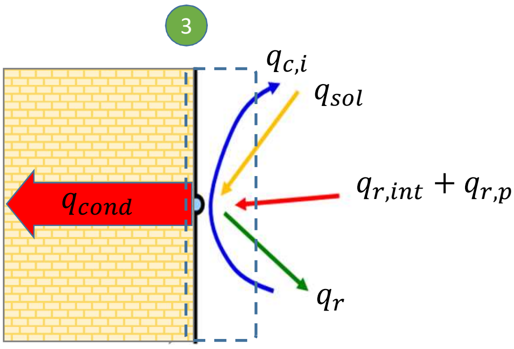


$$q_{c,i,k} = h_{c,i,k} S_k (T_a - T_{si,k})$$

$$q_{sol,k} = \frac{S_k}{S_{tot} - S_{windows}} \sum_{w=1}^{m} q_{sol,w}$$

Overall heat balance of the room

Heat exchange at the internal surface (3)



$$q_{r,int,k} = \frac{S_k}{S_{tot} - S_{windows}} q_{r,int}$$

$$q_{r,p,k} = \frac{S_k}{S_{tot} - S_{windows}} q_{r,p}$$

$$q_{r,k} = h_{r,k} \, S_k \sum_{j=1}^{n} F_{k-j} \left(T_{si,k} - T_{si,j}\right)$$

Overall heat balance of the room

1 $$-h_{ce,k} S_k \left(T_{se,k} - T_{sol,air}\right) + q_{cond,k} = 0$$

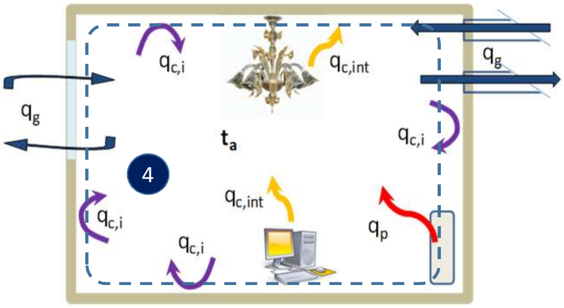
2 $$q_{cond,k} = S_k \cdot f\left(T_{si,k} - T_{se,k}\right)$$

3 $$-q_{cond,k} + q_{c,i,k} + q_{sol,k} + q_{r,int,k} + q_{r,p,k} - q_{r,k} = 0$$

Overall heat balance in the room

Heat exchange with the internal air volume (4)

$$-q_{c,i,k} + q_{c,int,k} + q_{c,p,k} + \dot{m}_a c_{p,a}(T_e - T_a) = \rho_a V_a c_{p,a}\frac{dT_a}{dt}$$



Overall heat balance of the room

1 $$-h_{ce,k}S_k\left(T_{se,k} - T_{sol,air}\right) + \; q_{cond,k} = 0$$

2 $$q_{cond,k} = S_k \cdot f\left(T_{si,k} - T_{se,k}\right)$$

3 $$-q_{cond,k} + q_{c,i,k} + q_{sol,k} + q_{r,int,k} + q_{r,p,k} - q_{r,k} = 0$$

Repeated for each surface

4 $$-q_{c,i,k} + q_{c,int,k} + q_{c,p,k} + \dot{m}_a c_{p,a}(T_e - T_a) = \rho_a V_a c_{p,a}\frac{dT_a}{dt}$$

Overall heat balance of the room

1 $$-h_{ce,k}S_k\left(T_{se,k}-T_{sol,air}\right)+S_k\cdot f\left(T_{si,k}-T_{se,k}\right)=0$$

3 $$-S_k\ \cdot f\left(T_{si,k}-T_{se,k}\right)+q_{c,i,k}+q_{sol,k}+q_{r,int,k}+q_{r,p,k}-q_{r,k}=0$$

Repeated for each surface

4 $$-q_{c,i,k}+q_{c,int,k}+q_{c,p,k}+\dot{m}_a c_{p,a}(T_e-T_a)=\rho_a V_a c_{p,a}\frac{dT_a}{dt}$$

Overall heat balance of the room

Main assumptions

- Indoor air stratification neglected (single air node)
- Steady-state heat conduction through the walls
- Uniform distribution of the radiation in the room (solar, mutual and from internal sources)

Overall heat balance of the room

Main assumptions

- Indoor air stratification neglected (single air node)
- Uniform distribution of the radiation in the room (solar, mutual and from internal sources)
- $f_{int,c}$ is the convective fraction of internal heat gains
- $f_{p,c}$ is the convective fraction of heat emitted by the HVAC system

Overall heat balance of the room

1 $$-h_{ce,k} S_k \left(\boldsymbol{T_{se,k}} - T_{sol,air}\right) + Sk \cdot f\left(\boldsymbol{T_{si,k}} - \boldsymbol{T_{se,k}}\right) = 0$$

3 $$-S_k \cdot f(\boldsymbol{T_{si,k}} - \boldsymbol{T_{se,k}}) + q_{c,i,k} + q_{sol,k} + (1 - f_{int,c}) q_{int,k} + (1 - f_{p,c})\, q_{p,k} - q_{r,k} = 0$$

4 $$-q_{c,i,k} + f_{int,c}\, q_{in\ ,k} + f_{p,c}\, q_{p,k} + \dot{m}_a c_{p,a} (T_e - \boldsymbol{T_a}) = \rho_a V_a c_{p,a} \frac{d\boldsymbol{T_a}}{dt}$$

Overall heat balance of the room

1

3

Repeated for each surface

N surfaces → 2*N equations,
2*N variables
$(T_{si,1}..T_{si,N}, T_{se,1}..T_{se,N})$

4

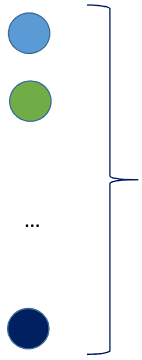
1 air node → 1 equation
2 variables (indoor air temperature T_a and heat exchange with the HVAC system q_p)

Overall heat balance of the room

…

Linear system
of
(2*N+1) equations, (2*N+2) variables

Overall heat balance of the room



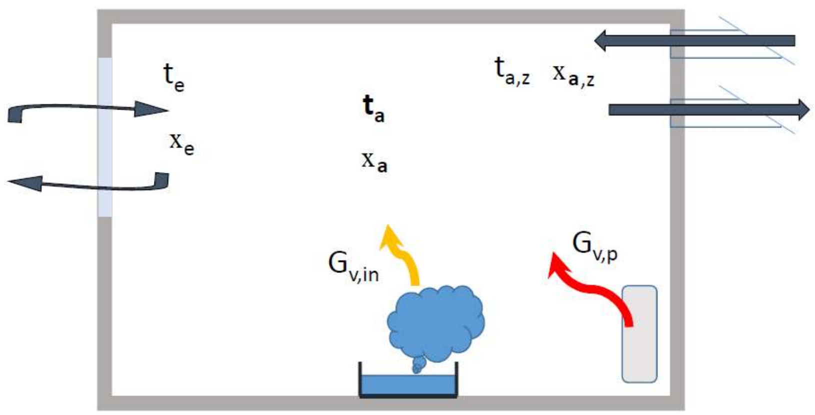
Two possible solutions:

1) Fixed temperature → Calculate energy needs for sensible heating/cooling

2) Fixed heating/cooling load → Calculate indoor air temperature

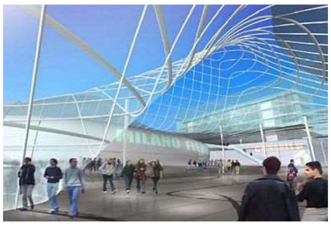
Overall heat balance of the room

Vapour mass balance

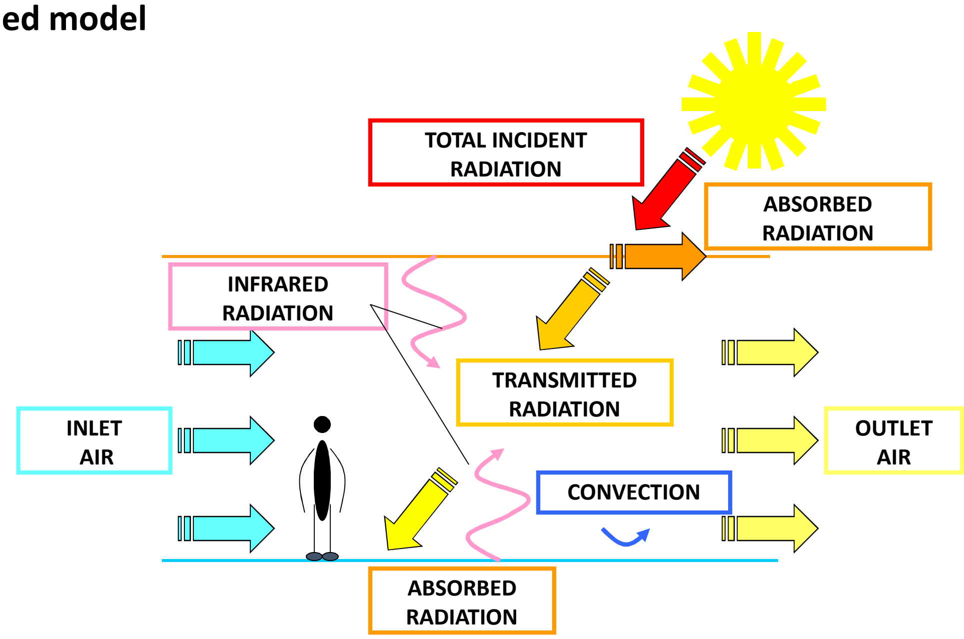


Example

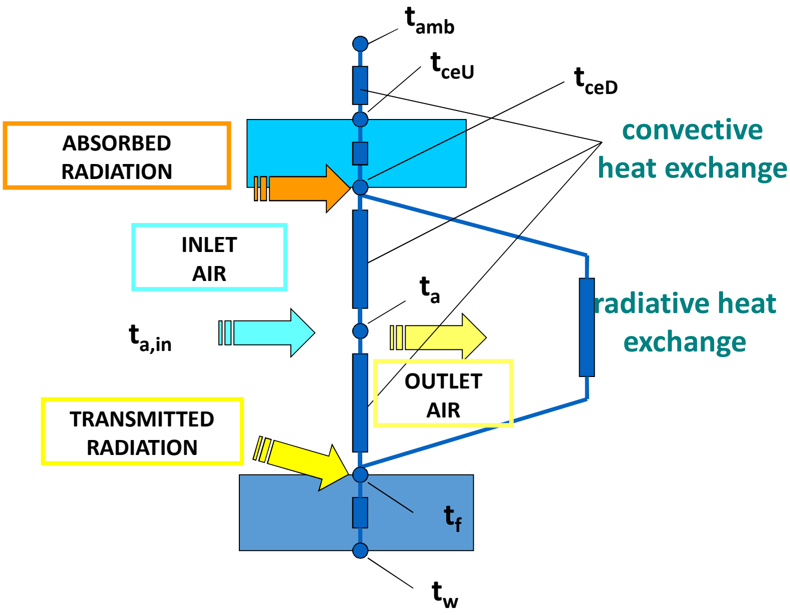
Analyze and semplify the problem
An example: the hall of a congress palace

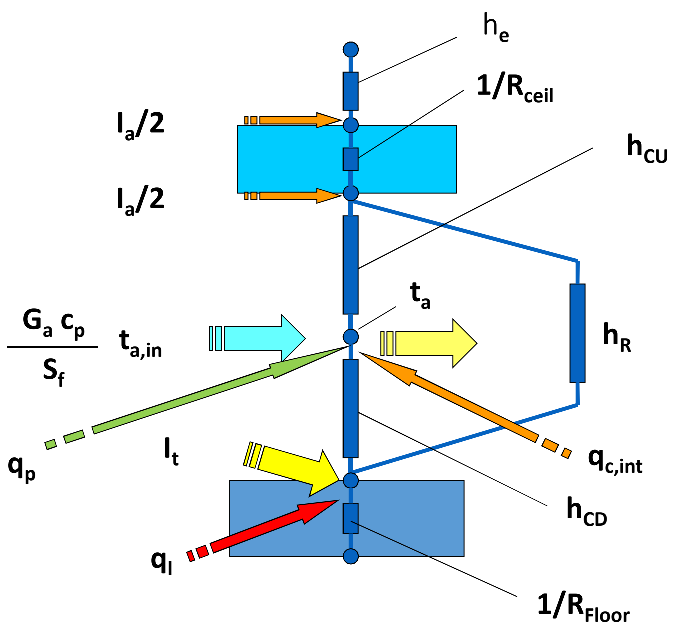


Simplified model



Example of a balance





$$① \quad \frac{(t_f - t_w)}{R_{Floor}} + h_R(t_f - t_{CeD}) + h_{CD}(t_f - t_a) = q_l + I_t$$

$$② \quad h_{CD}(t_a - t_f) + \frac{G_a c_p}{S_f} \cdot (t_a - t_{a,in}) + h_{CU}(t_a - t_{CeD}) = q_p + q_{c,\mathrm{int}}$$

$$③ \quad h_R(t_{ceD} - t_f) + h_{CU}(t_{CeD} - t_a) + \frac{(t_{ceD} - t_{ceU})}{R_{ceil}} = \frac{I_a}{2}$$

$$④ \quad \frac{(t_{ceU} - t_{ceD})}{R_{ceil}} + h_e(t_{ceU} - t_{amb}) = \frac{I_a}{2}$$

$$\begin{pmatrix} \frac{1}{R_{floor}} + h_{CD} + h_R & -h_{CD} & -h_R & 0 \\ -h_{CD} & \frac{G_a c_p}{S_f} + h_{CD} + h_{CU} & -h_{CU} & 0 \\ -h_R & -h_{CU} & h_R + h_{CU} + \frac{1}{R_{ceil}} & -\frac{1}{R_{ceil}} \\ 0 & 0 & -\frac{1}{R_{ceil}} & \frac{1}{R_{ceil}} + h_{ext} \end{pmatrix} \cdot$$

$$\cdot \begin{Bmatrix} t_f \\ t_a \\ t_{ceD} \\ t_{ceU} \end{Bmatrix} = \begin{Bmatrix} q_l + I_t + \frac{t_w}{R_{floor}} \\ q_p + q_{c,\mathrm{int}} + \frac{G_a c_p}{S_f} t_{a,in} \\ \frac{I_a}{2} \\ \frac{I_a}{2} + h_{ext} t_{amb} \end{Bmatrix}$$

References

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