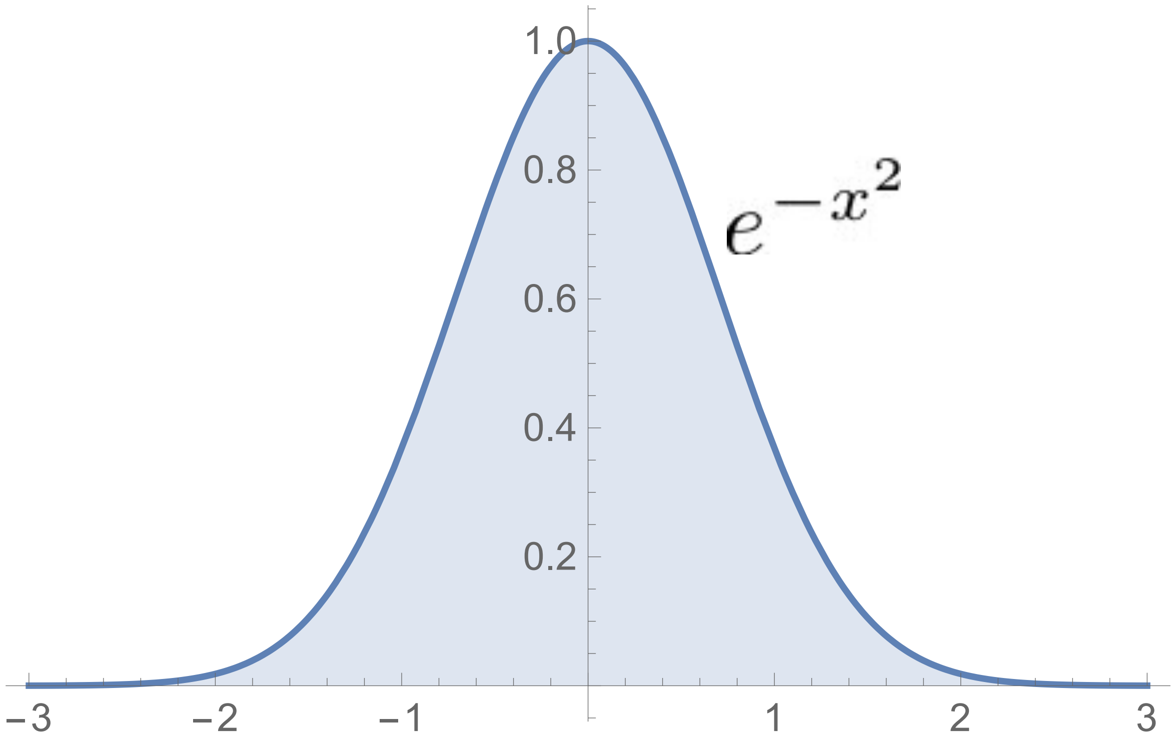
La variabile normale

L'integrale di Gauss

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$



Dimostrazione

$$\int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

$$I := \int_{-\infty}^{+\infty} e^{-x^2}\,dx$$

Calcoliamo in 2 modi l'integrale doppio $\int_{\mathbb{R}^2} e^{-x^2-y^2}\,dx\,dy$

Modo 1: formula di riduzione

$$\int_{\mathbb{R}^2} e^{-x^2-y^2}\,dx\,dy = \int_{-\infty}^{+\infty} e^{-x^2}\,dx \int_{-\infty}^{+\infty} e^{-y^2}\,dy = I^2$$

Modo 2: cambio di variabile

$$\int_{\mathbb{R}^2} e^{-x^2-y^2}\,dx\,dy = 2\pi \int_{0}^{+\infty} \rho e^{-\rho^2}\,d\rho = 2\pi \left[-\frac{1}{2}e^{-\rho^2}\right]_0^{+\infty} = \pi$$

Quindi $I^2 = \pi$, da cui $I = \sqrt{\pi}$

Conseguenza

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = 1$$

Verifica

$$t = x/\sqrt{2}$$

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2} \sqrt{2}\, dt$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-t^2}\, dt = 1$$

DEFINIZIONE

Una variabile Z si dice normale standard se

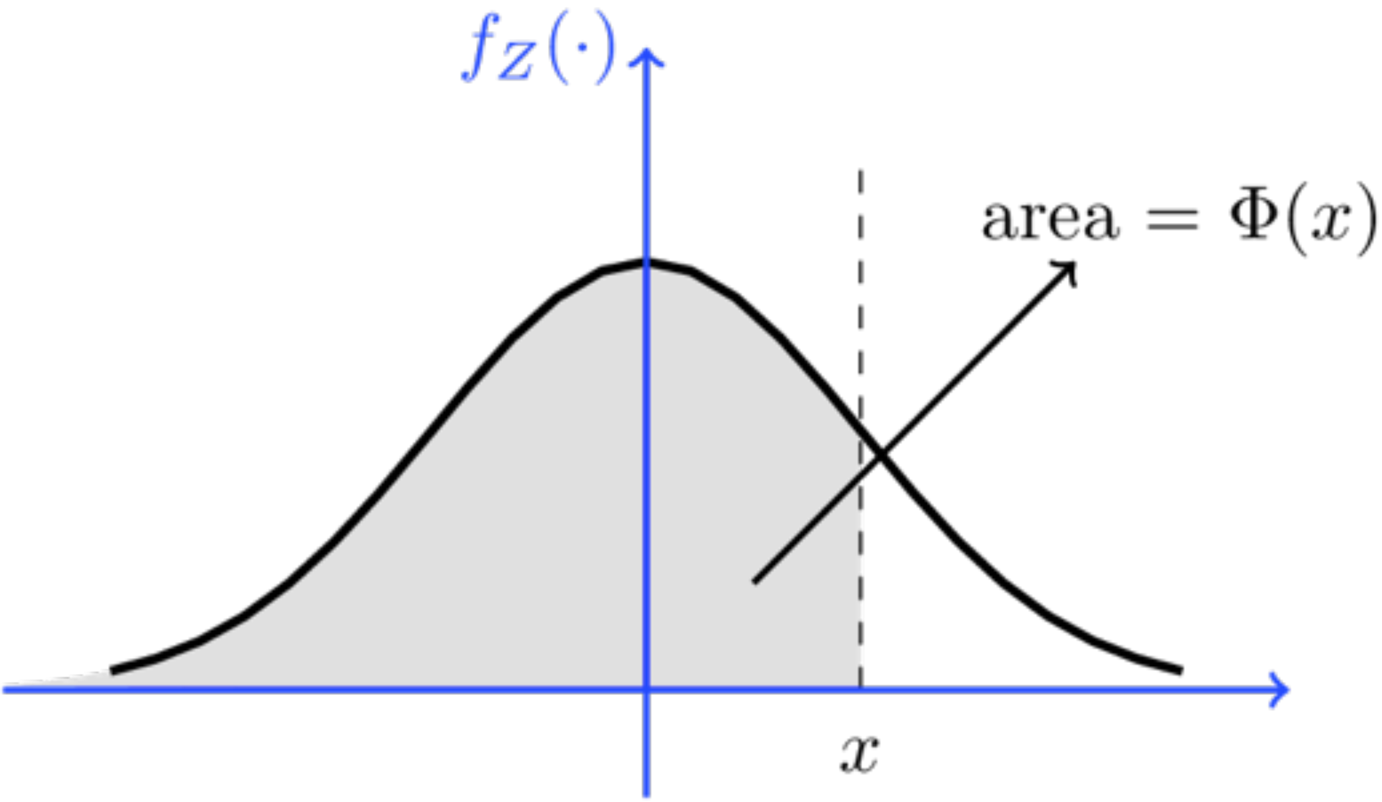
- Z è continua;
- la densità di Z è $f_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

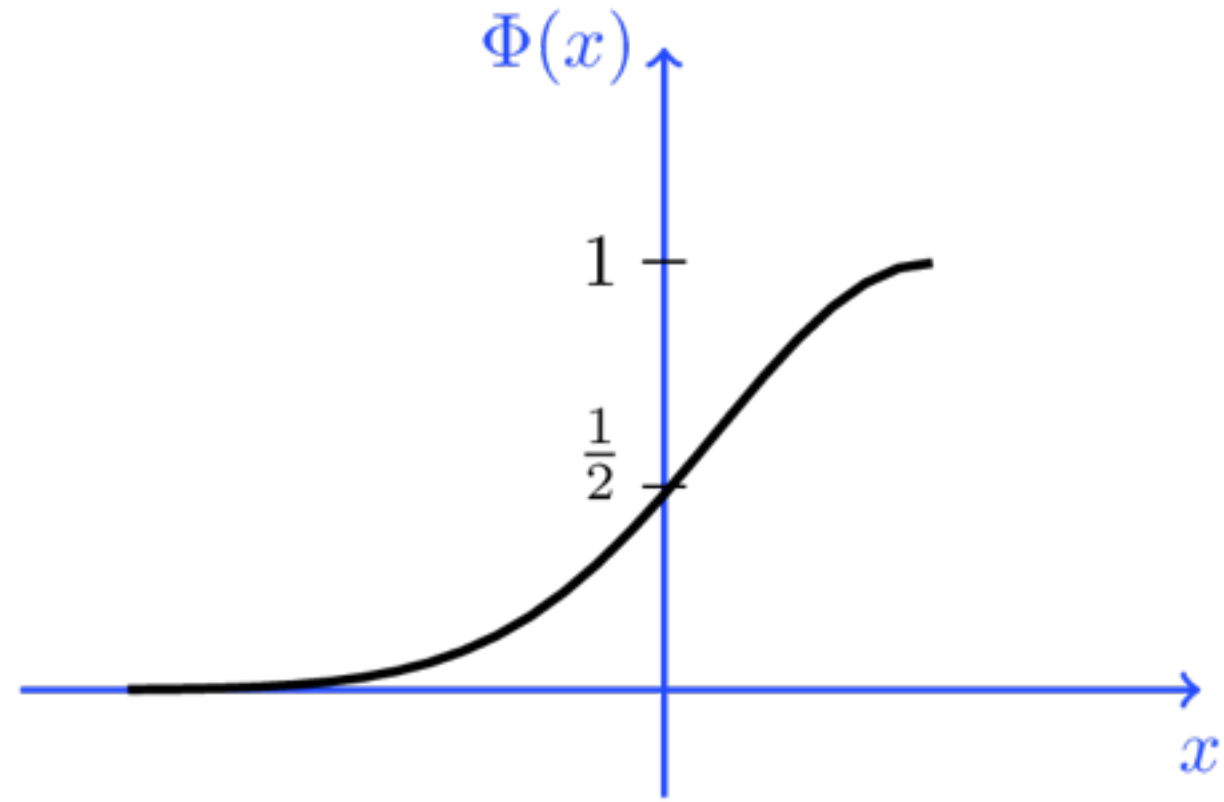
Si scrive $Z \sim N(0,1)$

La funzione di distribuzione di una variabile normale standard si indica $\Phi(x)$:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dx$$

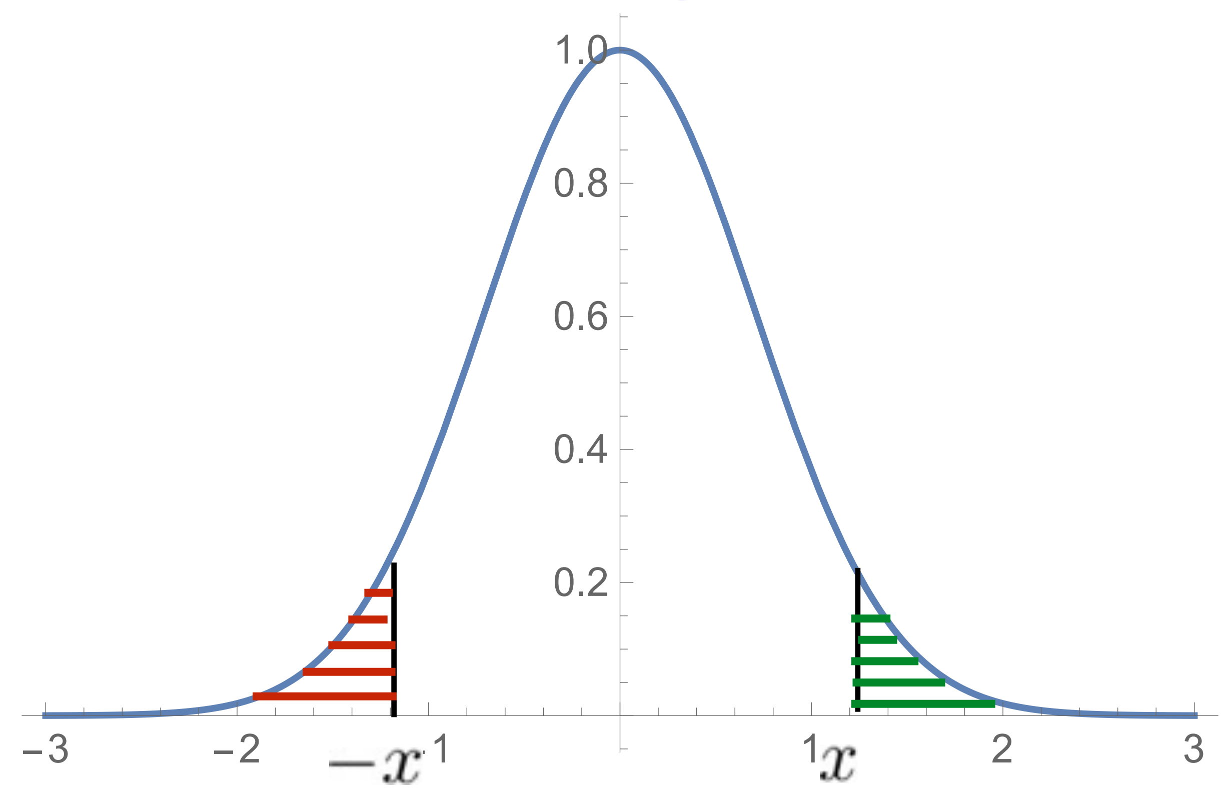
Proprietà di Φ



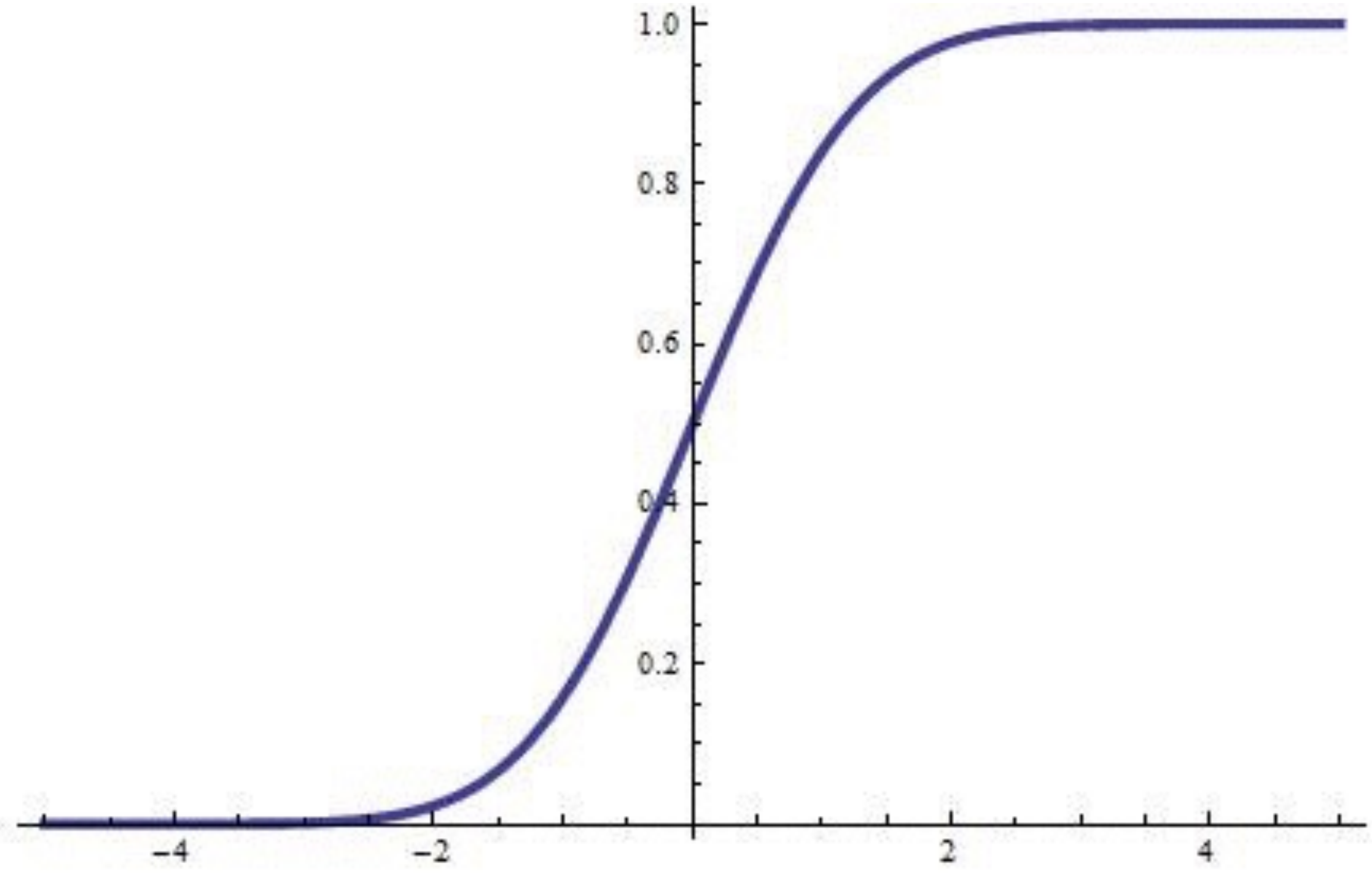


- $\Phi(0) = \dfrac{1}{2}$
- $\Phi(-x) = 1 - \Phi(x)$

Basta conoscere Φ sui reali positivi...



la funzione di distribuzione della normale standard



la tabella della normale

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

ESEMPIO

Calcolo di $\Phi(0.65) \approx 0.7422$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
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0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279

ESEMPIO Calcolo di $\Phi(-0.53)$

$$\Phi(-0.53) = 1 - \Phi(0.53) \approx 1 - 0.7019$$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
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ESEMPIO $Z \sim N(0,1)$

$$P(|Z| < 1) = P(-1 < Z < 1)$$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
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1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
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1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
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2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

ESEMPIO $Z \sim N(0,1)$

$$P(|Z| < 1) = P(-1 < Z < 1)$$

$$= \Phi(1) - \Phi(-1)$$

$$= \Phi(1) - (1 - \Phi(1))$$

$$= 2\Phi(1) - 1$$

$$\approx 2 \times 0.8413 - 1 = 0.6826$$

z	0.00	0.01
0.0	0.5000	0.5040
0.1	0.5398	0.5438
0.2	0.5793	0.5832
0.3	0.6179	0.6217
0.4	0.6554	0.6591
0.5	0.6915	0.6950
0.6	0.7257	0.7291
0.7	0.7580	0.7611
0.8	0.7881	0.7910
0.9	0.8159	0.8186
1.0	0.8413	0.8438
1.1	0.8643	0.8665
1.2	0.8849	0.8869
1.3	0.9032	0.9049
1.4	0.9192	0.9207

ESEMPIO $Z \sim N(0,1)$

$$P(|Z| < 2) = P(-2 < Z < 2)$$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
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0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
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1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
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1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
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1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
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2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
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2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

ESEMPIO $Z \sim N(0,1)$

$$P(|Z| < 2) = P(-2 < Z < 2)$$
$$= \Phi(2) - \Phi(-2)$$
$$= \Phi(2) - (1 - \Phi(2))$$
$$= 2\Phi(2) - 1$$
$$= 0.9544$$

Z	0.00	0.01
0.0	0.5000	0.5040
0.1	0.5398	0.5438
0.2	0.5793	0.5832
0.3	0.6179	0.6217
0.4	0.6554	0.6591
0.5	0.6915	0.6950
0.6	0.7257	0.7291
0.7	0.7580	0.7611
0.8	0.7881	0.7910
0.9	0.8159	0.8186
1.0	0.8413	0.8438
1.1	0.8643	0.8665
1.2	0.8849	0.8869
1.3	0.9032	0.9049
1.4	0.9192	0.9207
1.5	0.9332	0.9345
1.6	0.9452	0.9463
1.7	0.9554	0.9564
1.8	0.9641	0.9649
1.9	0.9713	0.9719
2.0	0.9772	0.9778
2.1	0.9821	0.9826

Valore atteso e varianza della normale standard

PROPOSIZIONE Sia Z normale standard.

- $E(Z) = 0$
- $Var(Z) = 1$

$Z \sim N(0, 1)$

Dim 1) valore atteso

$$|x| f_Z(x) = \frac{1}{\sqrt{2\pi}} |x| e^{-x^2/2} \quad \text{è integrabile in senso generalizzato}$$

$$\frac{|x| e^{-x^2/2}}{1/x^2} = |x^3| e^{-x^2/2} \to 0 \text{ per } x \to \pm\infty$$

$$E(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x e^{-x^2/2}\, dx = \frac{1}{\sqrt{2\pi}} \left[-e^{-x^2/2} \right]_{-\infty}^{+\infty} = 0.$$

2) varianza

$$t = x/\sqrt{2}$$

$$E(X^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x^2 e^{-x^2/2}\, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} 2t^2 e^{-t^2} \sqrt{2}\, dt$$

$$= \frac{2}{\sqrt{\pi}} \int_{-\infty}^{+\infty} t^2 e^{-t^2}\, dt$$

$$t^2 e^{-t^2} = t\, t e^{-t^2} = t \left(-\frac{1}{2} e^{-t^2} \right)'$$

$$\int_{-\infty}^{+\infty} t \left(-\frac{1}{2} e^{-t^2} \right)' dt = \left[-\frac{t}{2} e^{-t^2} \right]_{-\infty}^{+\infty} + \frac{1}{2} \int_{-\infty}^{+\infty} e^{-t^2}\, dt$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} e^{-t^2}\, dt = \frac{\sqrt{\pi}}{2}$$

$$E(X^2) = \frac{2}{\sqrt{\pi}} \frac{\sqrt{\pi}}{2} = 1. \qquad Var(X) = E(X^2) - (E(X))^2 = 1.$$

La variabile normale di parametri (μ, σ^2)

DEFINIZIONE $\mu \in \mathbb{R}$, $\sigma > 0$.

X è variabile normale di parametri (μ, σ^2) se

$$X = \mu + \sigma Z, \text{ con } Z \sim N(0,1)$$

Si scrive $X \sim N(\mu, \sigma^2)$

OSSERVAZIONE

$X \sim N(\mu, \sigma^2) \Rightarrow$

- $E(X) = \mu$;
- $Var(X) = \sigma^2$

Esercizio $X \sim N(10, 36) = N(10, 6^2)$

- calcolare $P(X \leq 8)$

$$X = 10 + 6Z, Z \sim N(0,1)$$

$$X \leq 8 \Leftrightarrow 10 + 6Z \leq 8 \Leftrightarrow Z \leq \frac{8-10}{6} = -\frac{1}{3}$$

$$P(X \leq 8) = \Phi\left(-\frac{1}{3}\right) = 1 - \Phi\left(\frac{1}{3}\right) \approx 1 - 0.6293 = 0.3707.$$

Z	0.00	0.01	0.02	0.03
0.0	0.5000	0.5040	0.5080	0.5120
0.1	0.5398	0.5438	0.5478	0.5517
0.2	0.5793	0.5832	0.5871	0.5910
0.3	0.6179	0.6217	0.6255	0.6293
0.4	0.6554	0.6591	0.6628	0.6664
0.5	0.6915	0.6950	0.6985	0.7019
0.6	0.7257	0.7291	0.7324	0.7357
0.7	0.7580	0.7611	0.7642	0.7673
0.8	0.7881	0.7910	0.7939	0.7967
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

$$X \sim N(10, 36) = N(10, 6^2)$$

- calcolare $P(4 \leq X < 16)$

$$X = 10 + 6Z, Z \sim N(0, 1)$$

$$4 \leq X < 16 \Leftrightarrow 4 \leq 10 + 6Z < 16 \Leftrightarrow -1 \leq Z < 1$$

$$P(4 \leq X < 16) = P(-1 \leq Z < 1) = \Phi(1) - \Phi(-1)$$

$$= 2\Phi(1) - 1 \approx 2 \times 0.8413 - 1 = 0.6826$$

ESEMPIO $X \sim N(\mu, \sigma^2)$ Calcolare $P(|X-\mu| \leq \sigma)$

$$X = \mu + \sigma Z,\ Z \sim N(0,1)$$

$$|X-\mu| \leq \sigma \Leftrightarrow |Z| \leq 1$$

$$P(|X-\mu| \leq \sigma) = P(|Z| \leq 1) = \Phi(1) - \Phi(-1)$$

$$= 2\Phi(1) - 1 \approx 0.6826$$

Funzione di distribuzione di una variabile normale

$X \sim N(\mu, \sigma^2) \Rightarrow$

- X è continua;
- La densità è $f_X(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Dim. Determiniamo $F_X(x)$. Sia $Z \sim N(0,1)$ con $X = \mu + \sigma Z$

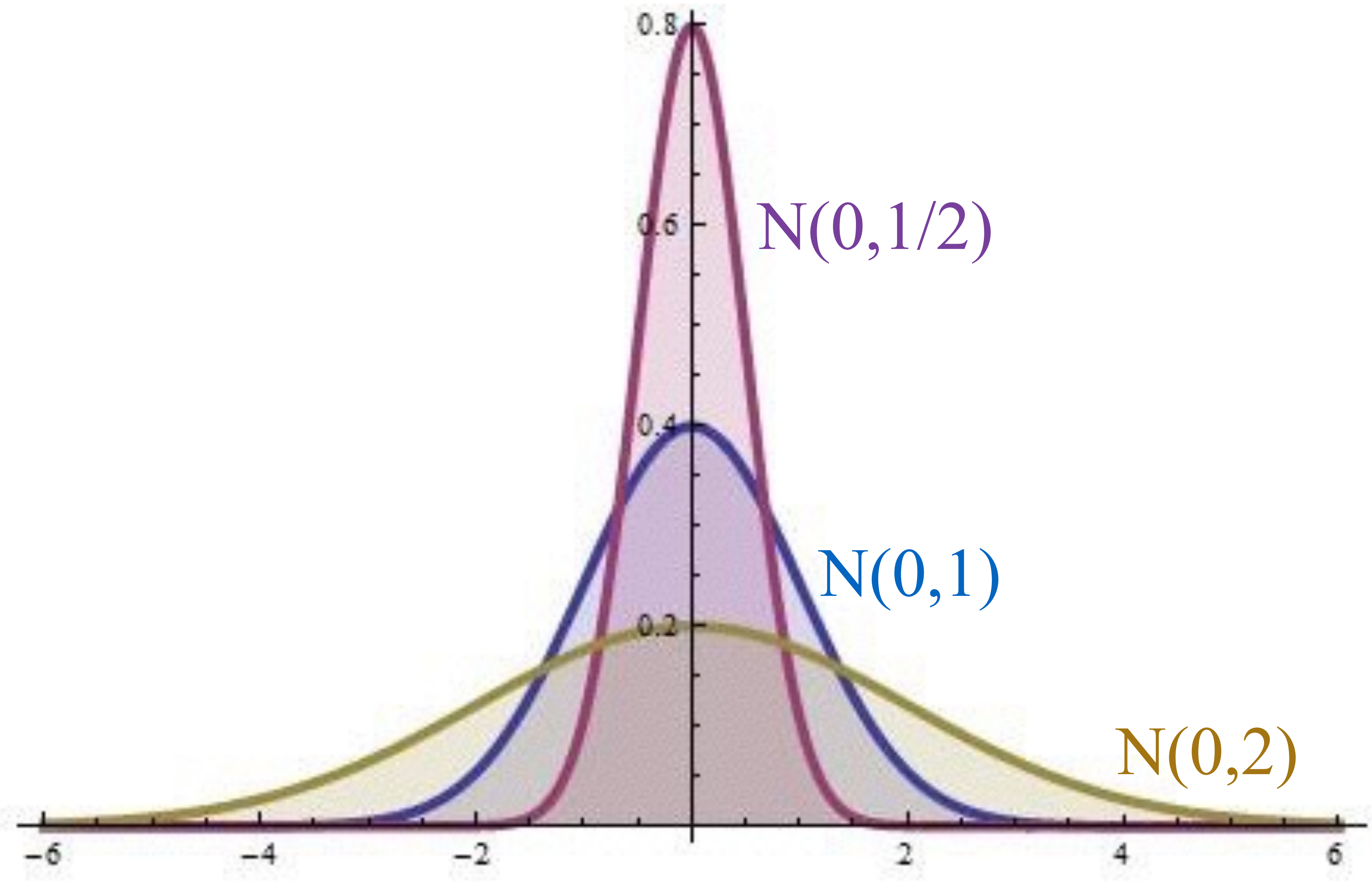
Se $x \in \mathbb{R}$ è $X \le x$ se e solo se $\quad \mu + \sigma Z \le x \Leftrightarrow Z \le \dfrac{x-\mu}{\sigma}$

$$F_X(x) = P(X \le x) = P\left(Z \le \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

$$f_X(x) = \frac{d}{dx}\Phi\left(\frac{x-\mu}{\sigma}\right) = \Phi'\left(\frac{x-\mu}{\sigma}\right)\frac{1}{\sigma}$$

$$= f_Z\left(\frac{x-\mu}{\sigma}\right)\frac{1}{\sigma} = \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2/2}$$

Alcune densità normali

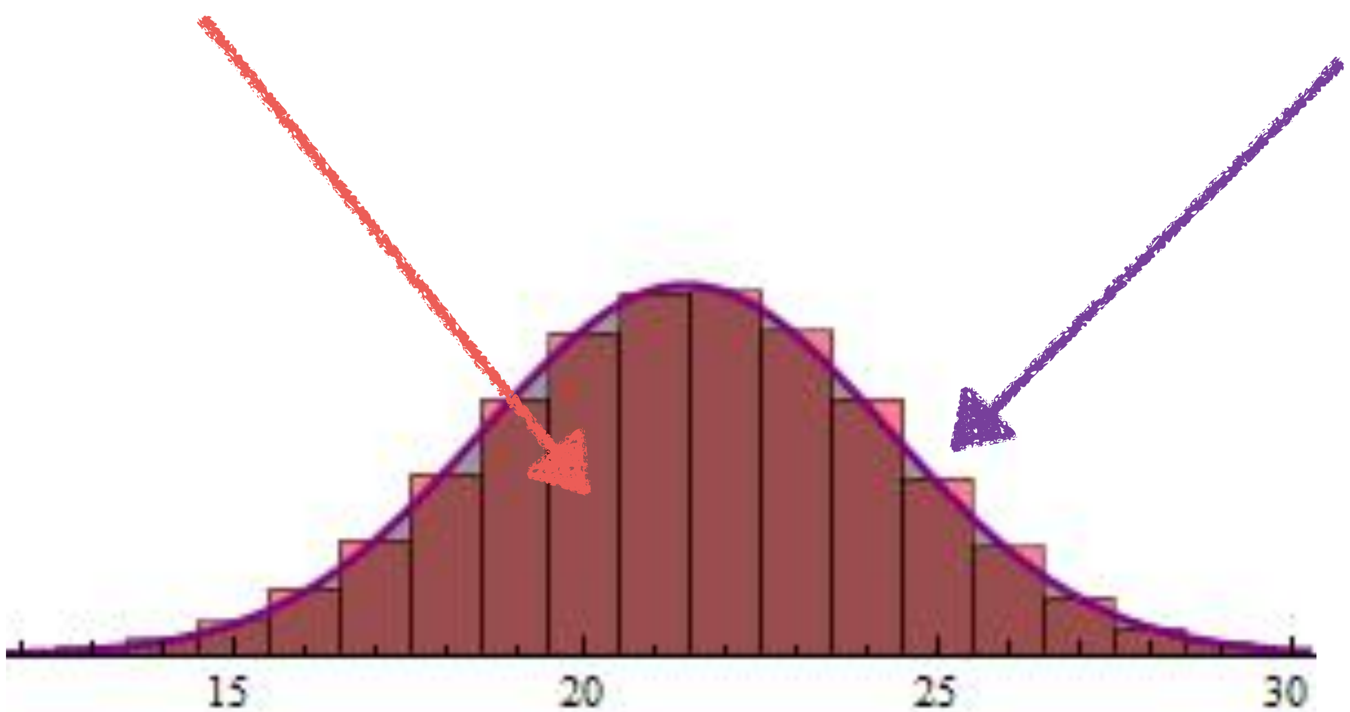


Teorema centrale del limite

Qualche osservazione…

densità discreta della binomiale $B(30, 0.7)$

densità della normale $N(21, 6.3)$

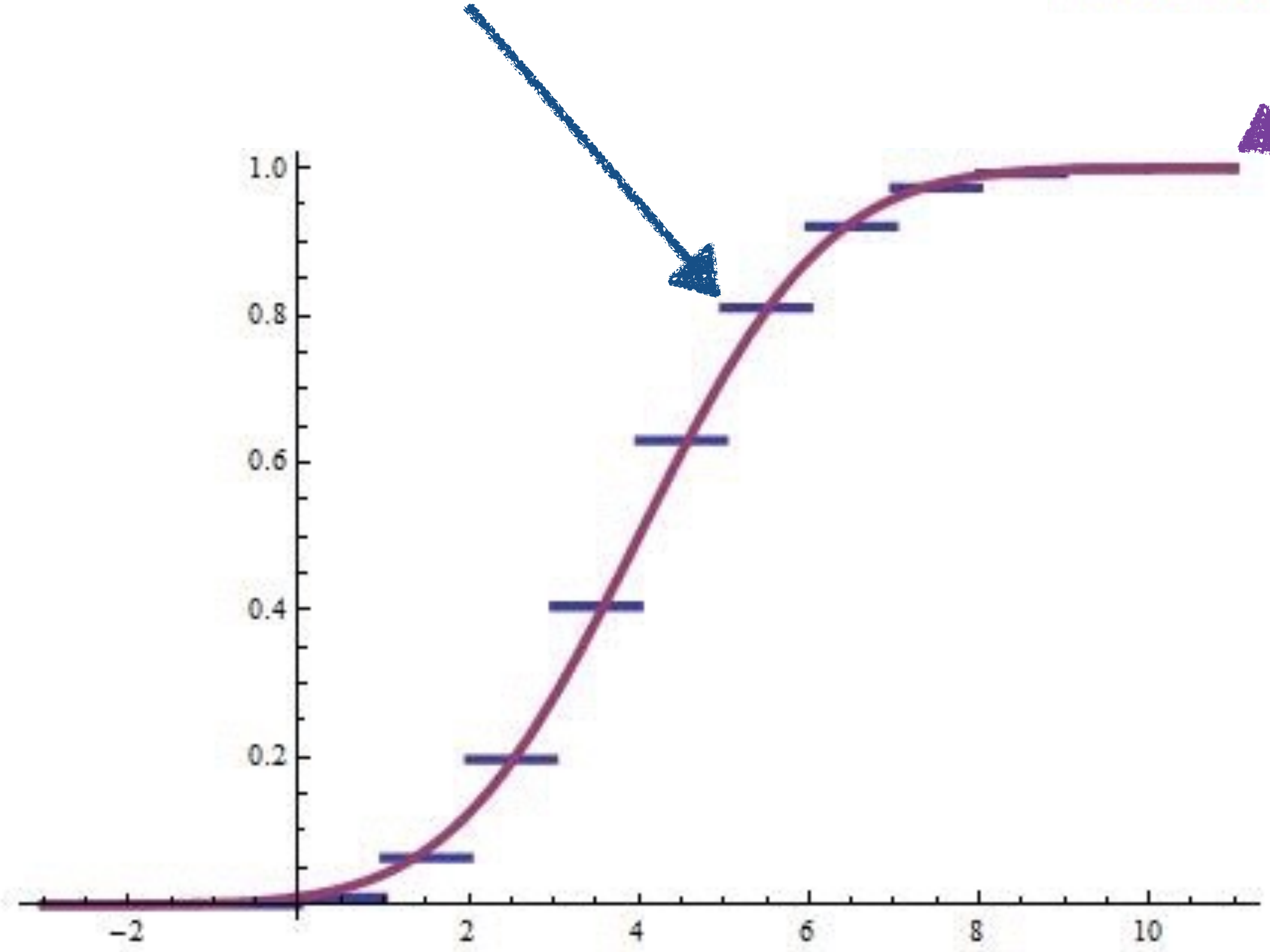


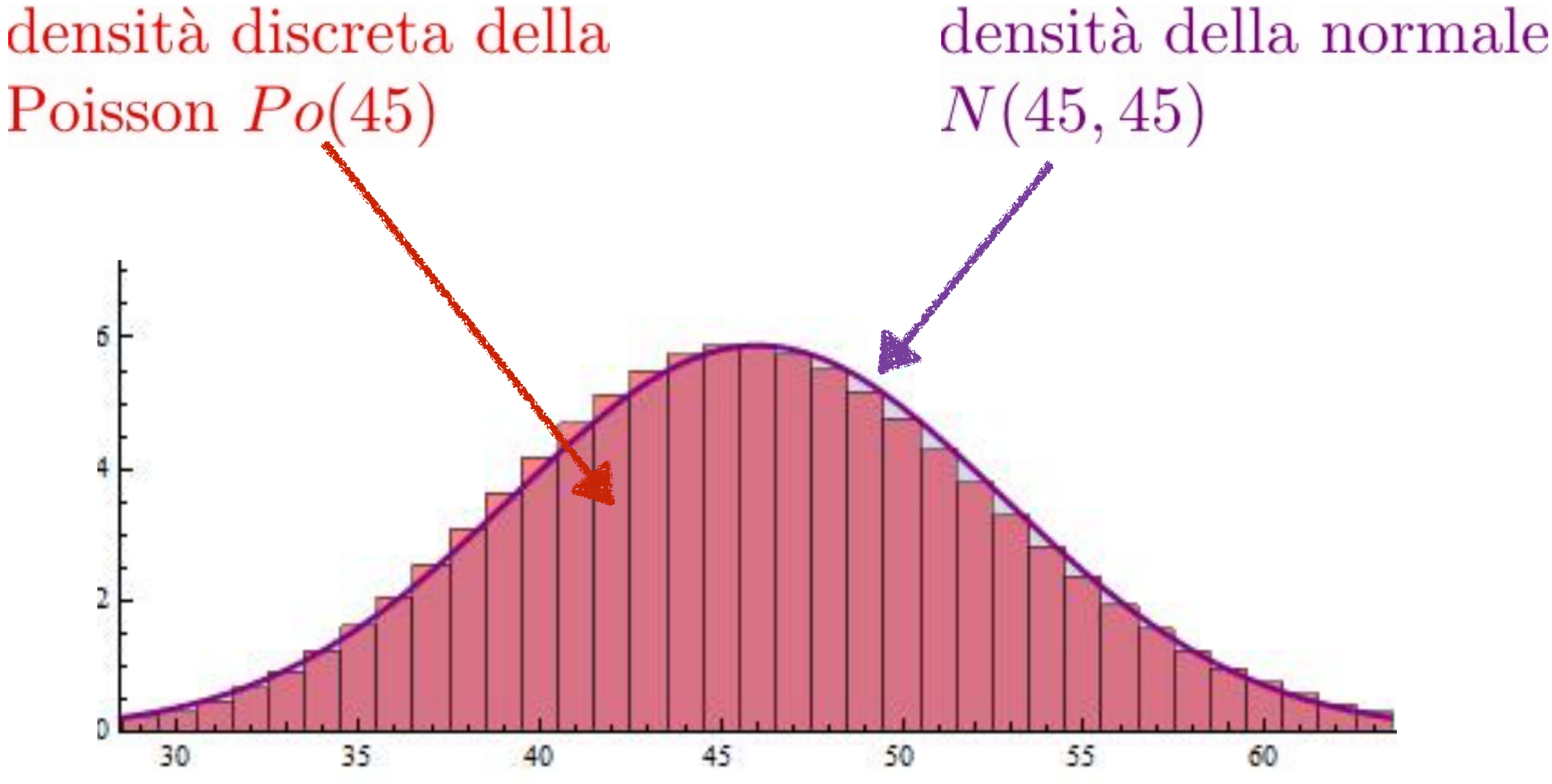
media: 21

varianza: $30 \cdot 0.7 \cdot 0.3 = 6.3$

distribuzione della
binomiale $B(16, \frac{1}{4})$

distribuzione della
normale $N(16 \cdot \frac{1}{4}, 16 \cdot \frac{1}{4} \cdot \frac{3}{4})$





Stessa media e varianza: 45

distribuzione della
Poisson di media 16

distribuzione della
Normale di parametri $(16, 16)$