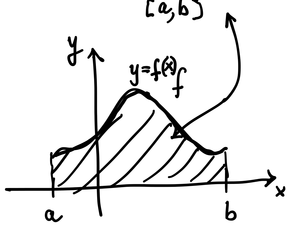
Multiple Integration

Goal: define and compute

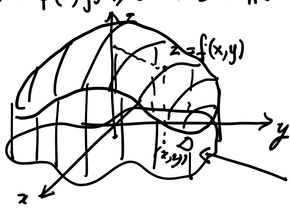
$$\int_D f(x_1, \dots x_n)\, dx_1 \dots dx_n$$

Recall that $\int_{[a,b]} f(x)\,dx$, $f \geq 0$



Integral for funct of 1 var → 2 dim → areas of plane figures.

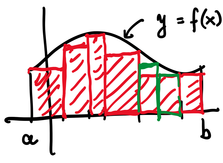
If $f = f(x,y) \geq 0 : D \subset \mathbb{R}^2 \longrightarrow [0, +\infty[$



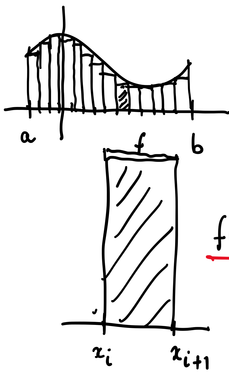
$\int_D f(x,y)\,dx\,dy$ ⇝ 3 dim volumes of solid figures.

$\int_D f(x,y,z)\,dx\,dy\,dz$ ⟶ 4 dim vol

How the int is defd?



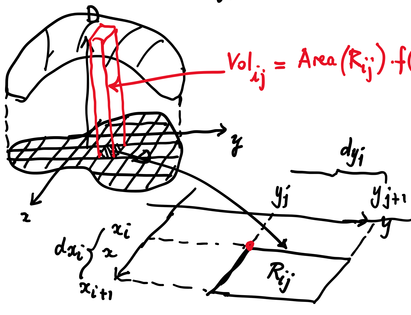
$$\int_a^b f(x)\,dx \approx \sum_i f(x_i)\,dx_i$$



$f(x_i) \cdot (x_{i+1} - x_i)$

Now if we wish to define

$$\int f(x,y)\,dx\,dy, \qquad f \geq 0$$



$$\int_D f(x,y)\,dx\,dy \approx \sum_{i,j} f(x_i, y_j)\,dx_i\,dy_j$$

So in this way, we can define the integral of $f = f(\vec{x})$ $\vec{x} \in \mathbb{R}^n$, $f \geq 0$

$$\int_D f(\vec{x})\,d\vec{x}$$

$$= \text{measure}\left\{(\vec{x}, z) \in \mathbb{R}^{n+1} : \vec{x} \in D,\ 0 \leq z \leq f(\vec{x})\right\}$$