

Geophysics for Natural Risks and Resources
ENVIRONMENTAL and ENGINEERING GEOPHYSICS

Introduction to
**GEOPHYSICAL PROSPECTING for
ENGINEERING**

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Outline

Geophysical methods for engineering

- Electrical methods (ERT Electrical resistivity methods, IP methods)
- Electro-magnetic methods (EM, RADAR)
- Seismic methods (Refraction Seismic, Reflection seismic, surface waves seismic)
 - definition of inversion problem
 - resolution and depth penetration
 - sketch of signal processing

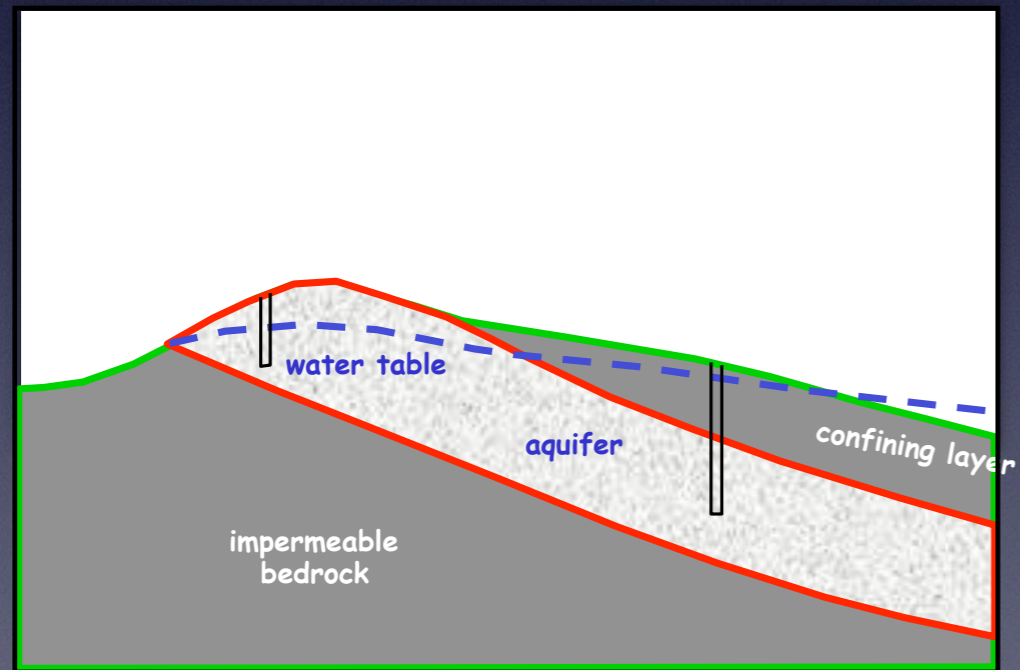


Geophysical methods for the subsoil exploration

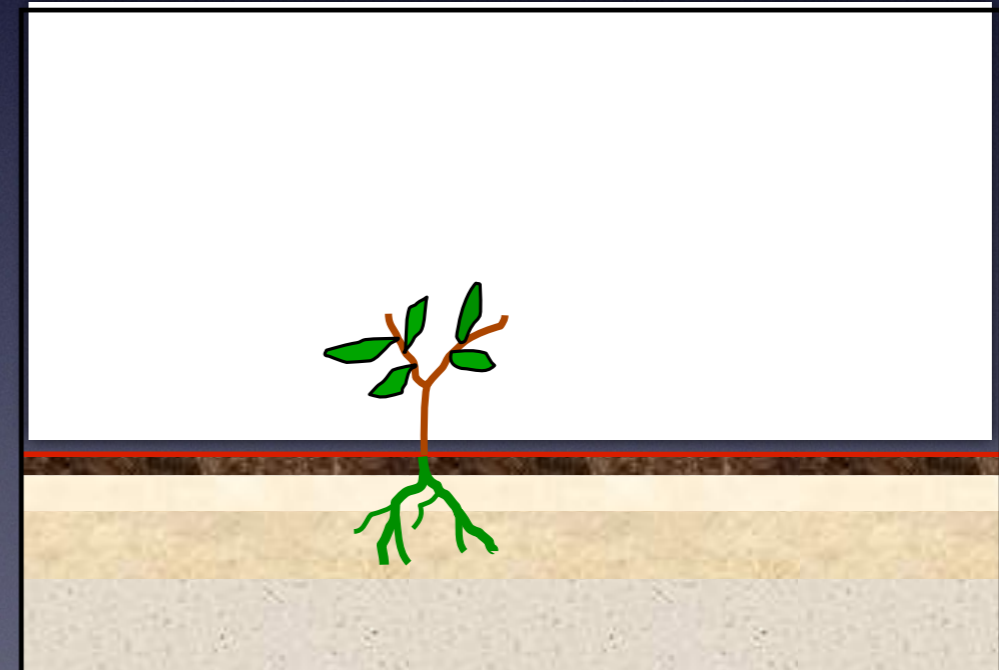
Method	Structure	Dynamic
Seismic	++	
Electro-Magnetic	+	++
DC resistivity methods	++	++
Ground Penetration Radar	++	+
Distributed Temp. Sensing		++
Magnetics	+	
Gravimetry	+	+
Spectral Induced Polarization	+	
Self Potential		+
Borehole logs	++	+

What geophysical methods can help define

- structure / texture (Seismic methods, EM methods, Electrical methods, Gravity methods, Radar etc)



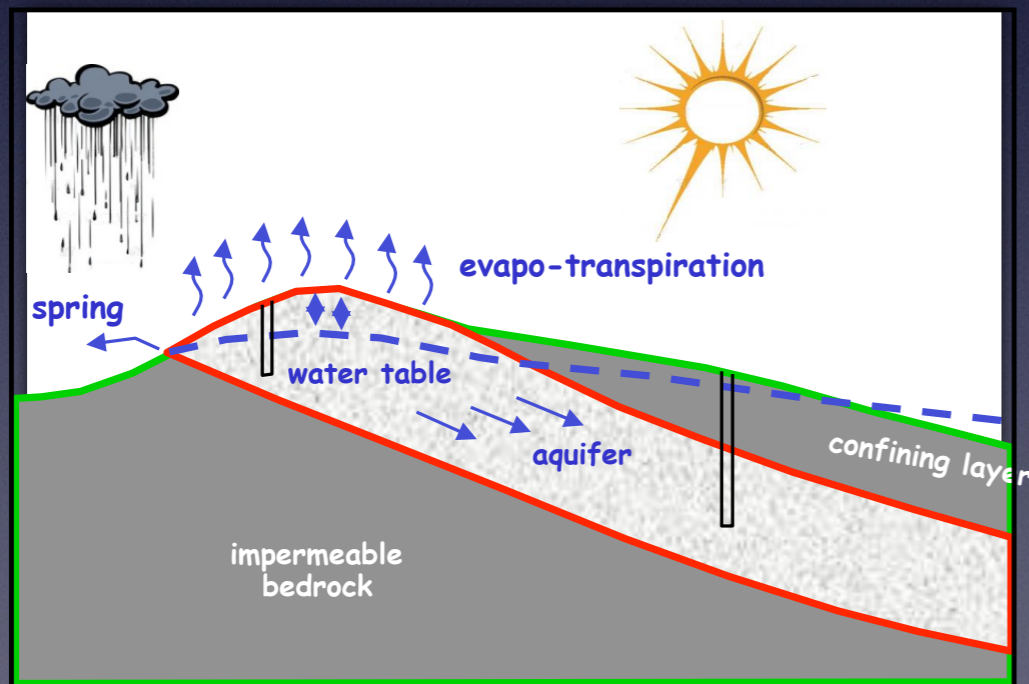
large scale



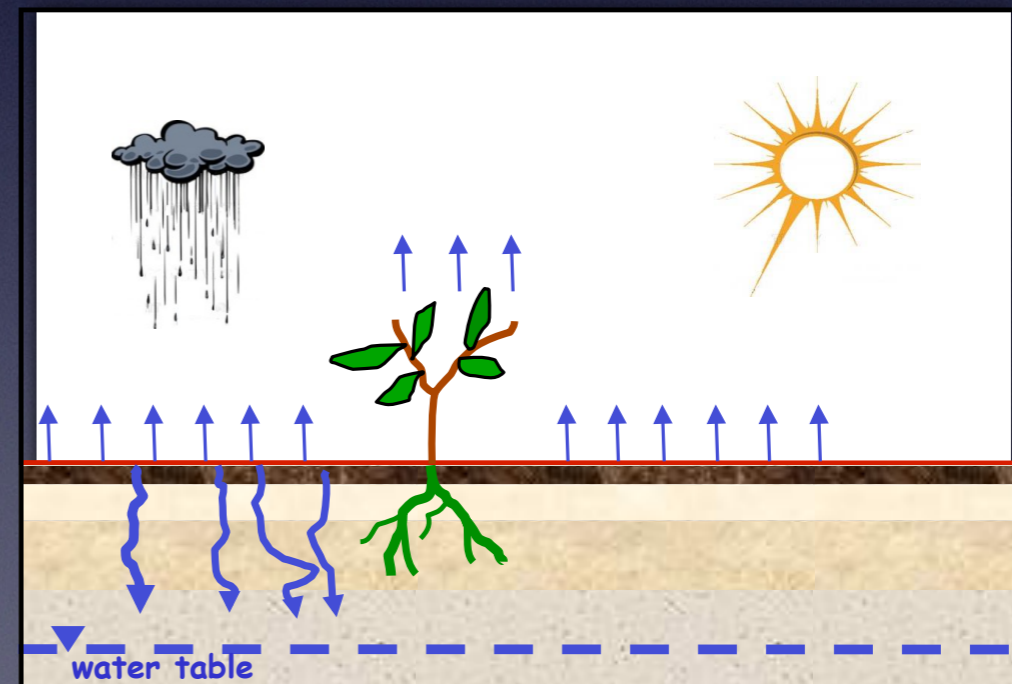
small scale

What geophysical methods can help define

- structure / texture (Seismic methods, EM methods, Electrical methods, Gravity methods, Radar etc)
- fluid-dynamics: e.g. time-lapse evolution of moisture content (DC resistivity methods, EM methods, GPR etc)



large scale

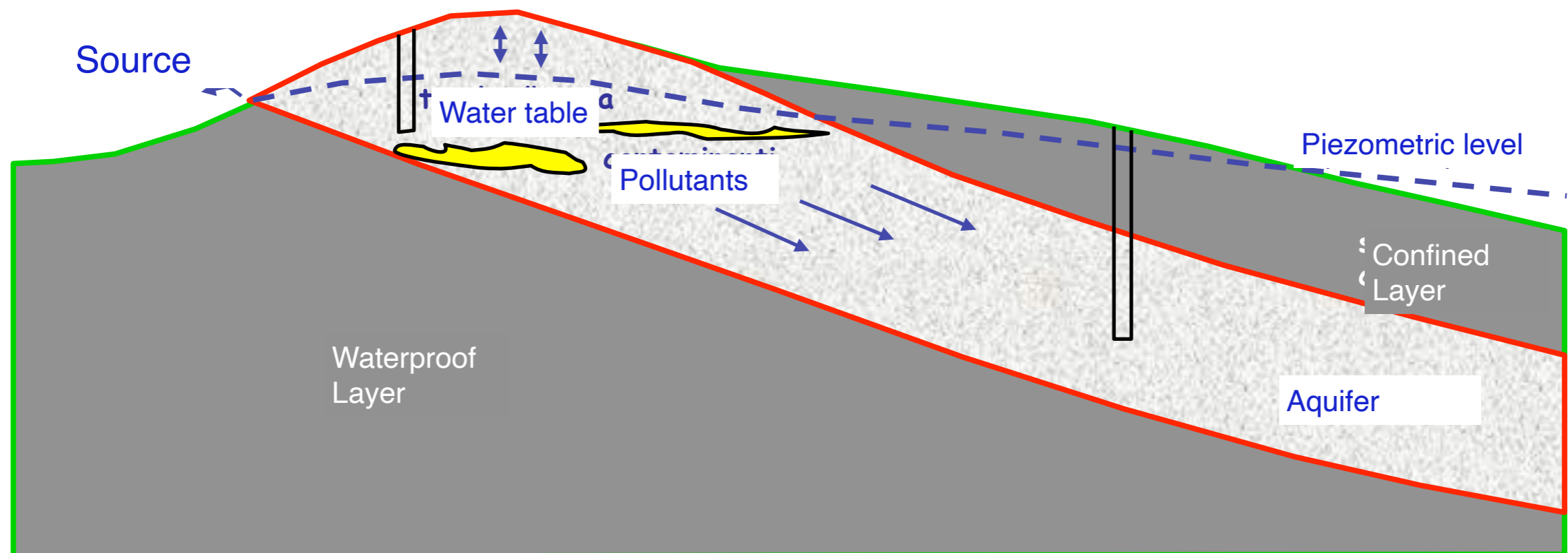


small scale



Aspects of environmental problem:

- Subsoil Structure
- Fluids dynamics.
- Presence of pollutants.



The geophysical measurement

An indirect measurement

Instrument



$D =$ geophysical quantity measured (DATA)

$P =$ geophysical parameter who rule D

Investigated domain

$D = D (P, F = \text{forcing conditions})$

PHYSICAL PROPERTIES (P)

- ❑ **Seismic** : elastic moduli and density
- ❑ **Gravimetry** : density
- ❑ **Magnetic Methods** : magnetic susceptibility
- ❑ **GeoElectrics** : electric conductivity
- ❑ **Electro-Magnetic methods** : electric conductivity
- ❑ **Induced polarization** : electric complex conductivity
- ❑ **Spontaneous potential** : electric conductivity sources
- ❑ **Ground penetrating radar** : dielectric constant

PHYSICAL PROPERTIES (P)

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Do I need this course ?

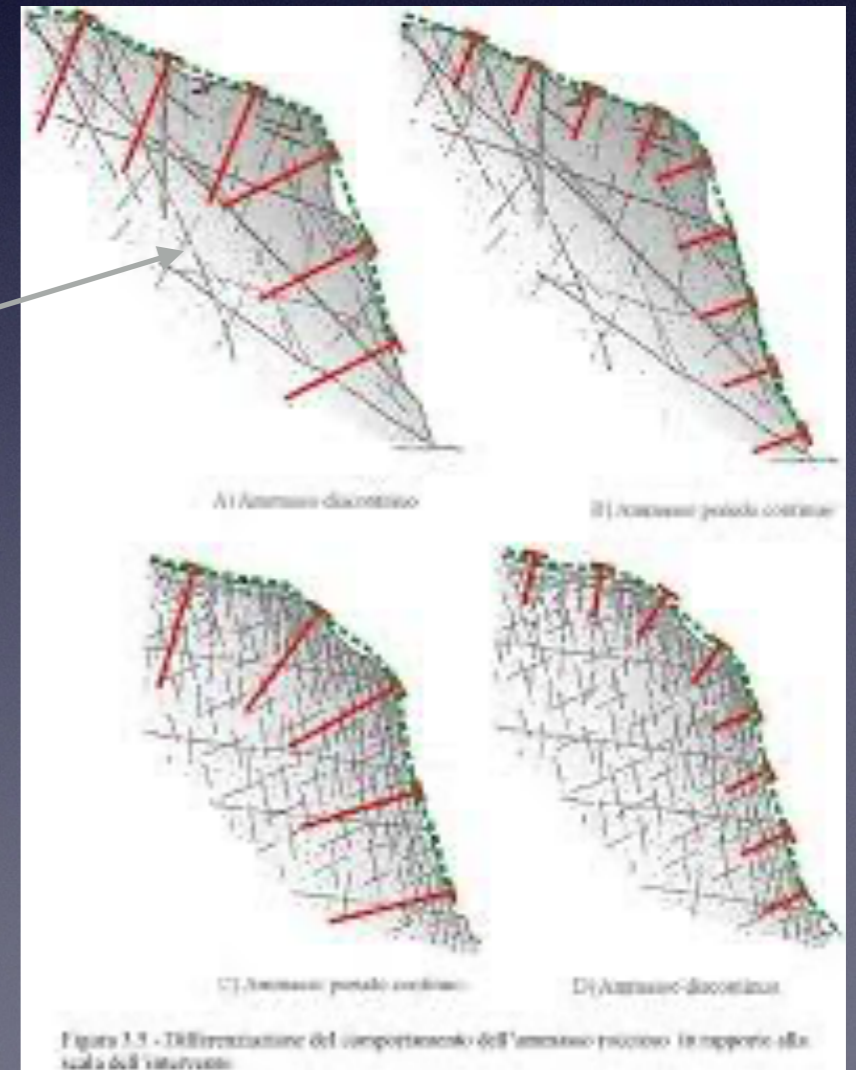


Practical problem:
Design a classical net
defence for rock falls



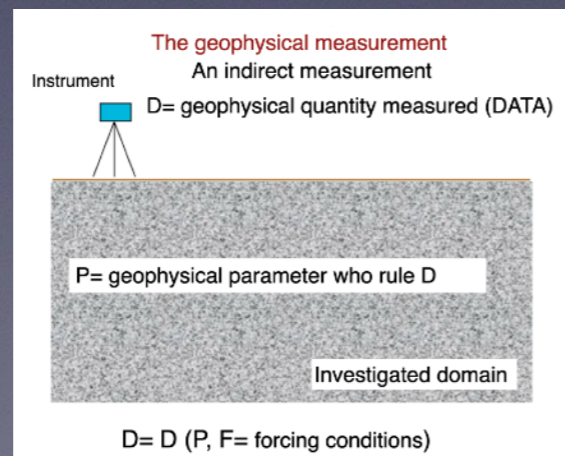
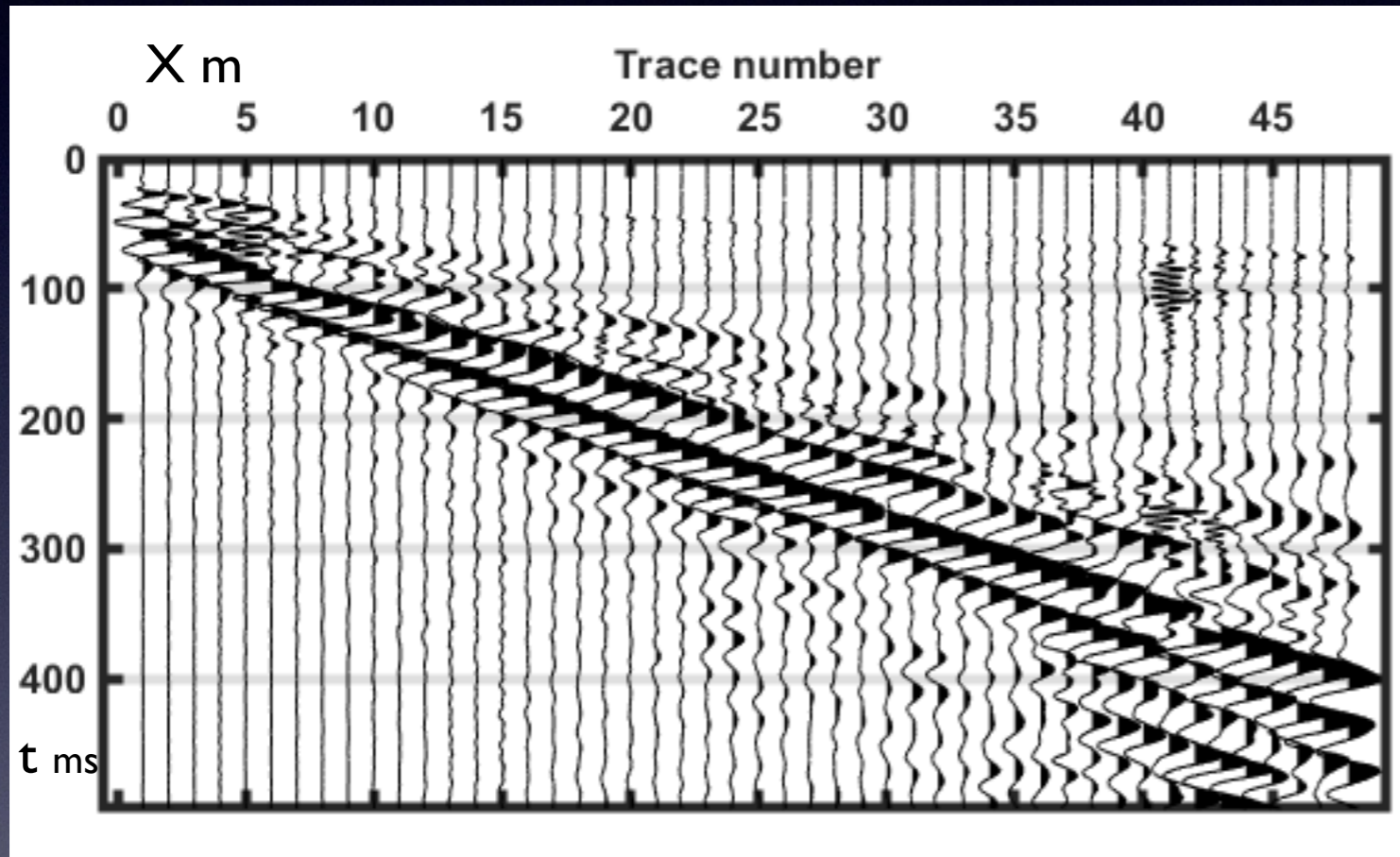
Nails must be anchored
in the solid rock
underneath debris...

How to do?



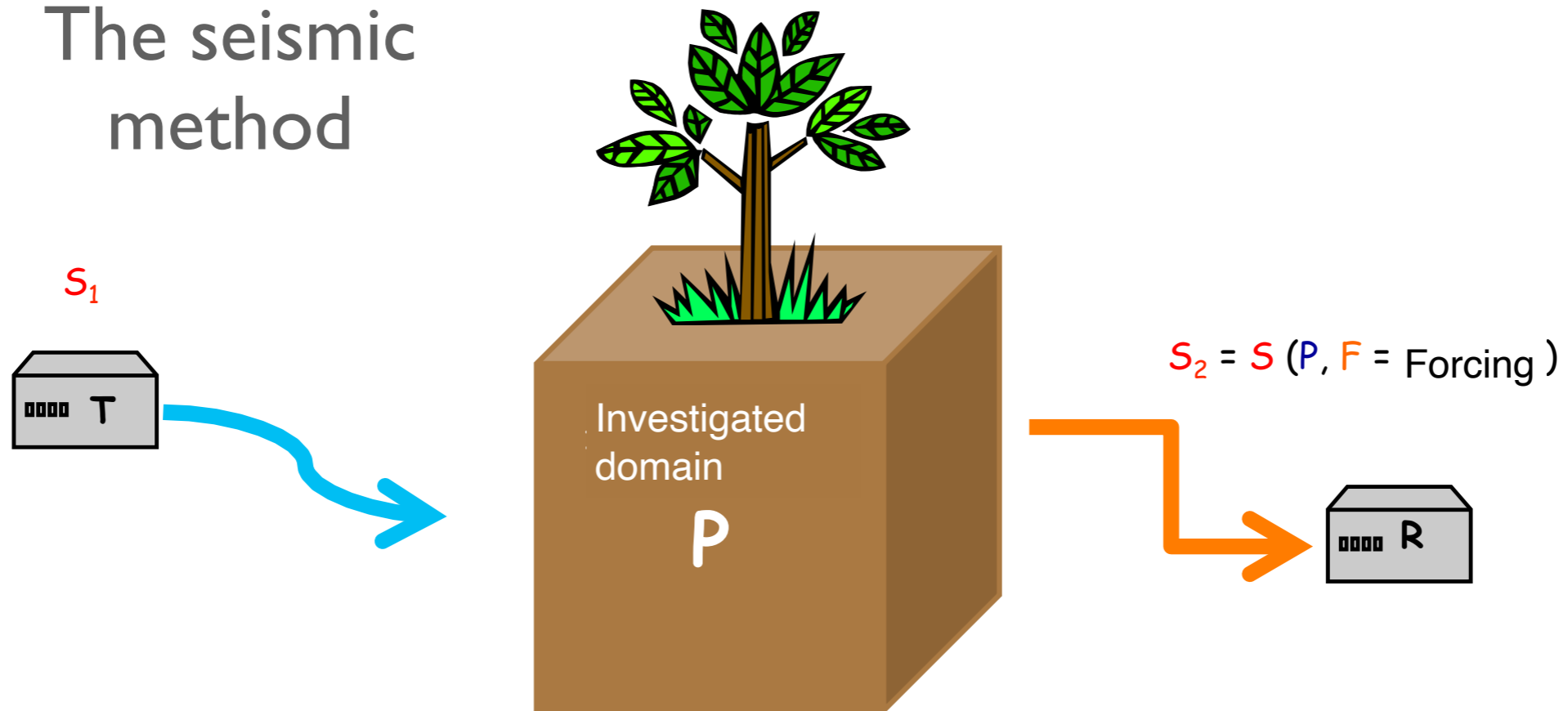
Seismic Methods

Study the wave elastic propagation in the subsoil and furnish the mechanical parameters



Physics parameter P
=
Elastic properties

The seismic method



S_1 = Signal =

Elastic wave source

S_2 = Signal =

Induced vibration

P = Physical parameter

Elastic moduli/ density



Borehole seismic

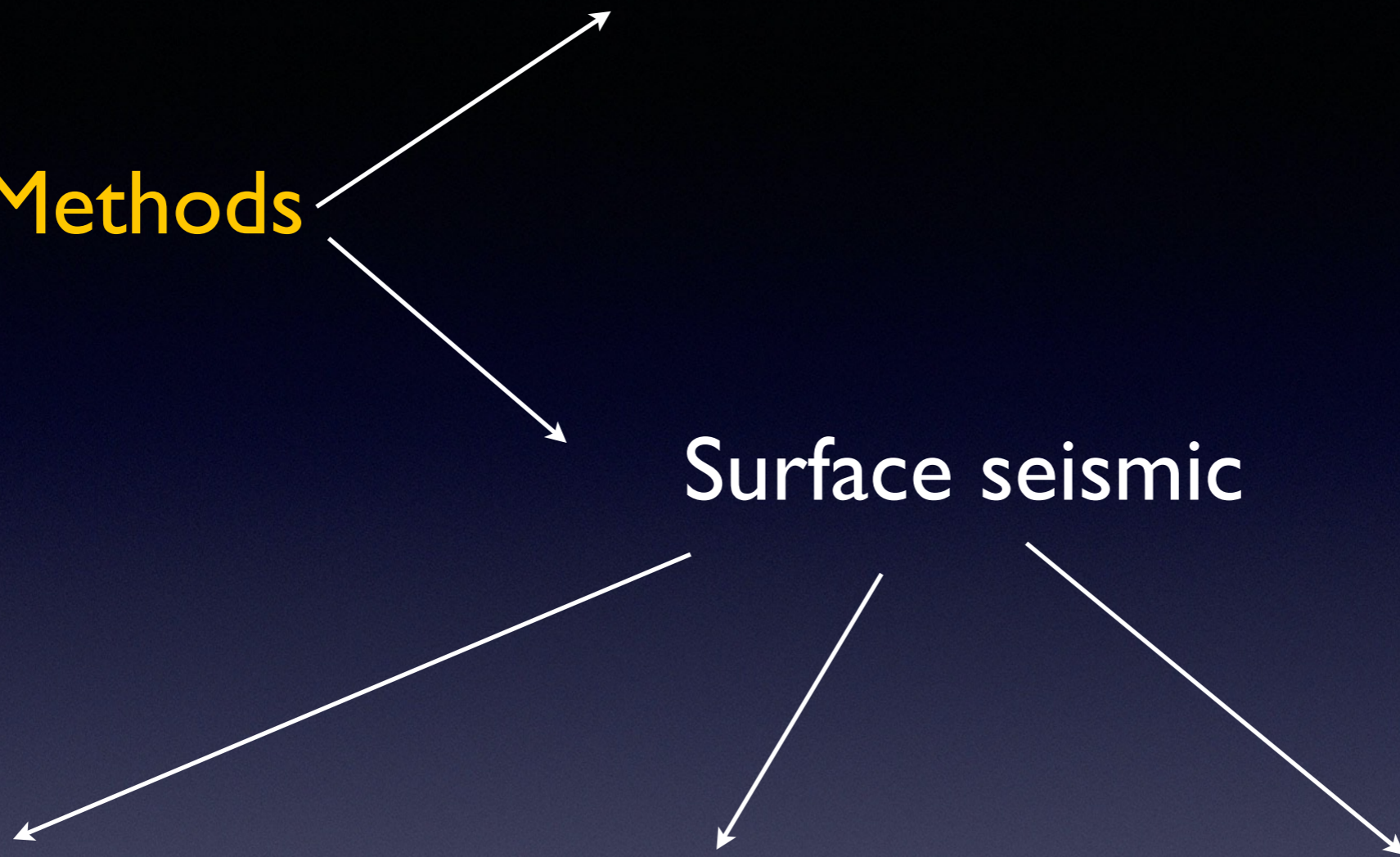
Seismic Methods

Surface seismic

Reflection
seismic

Refraction
seismic

Surface wave
seismic



Seismic Methods

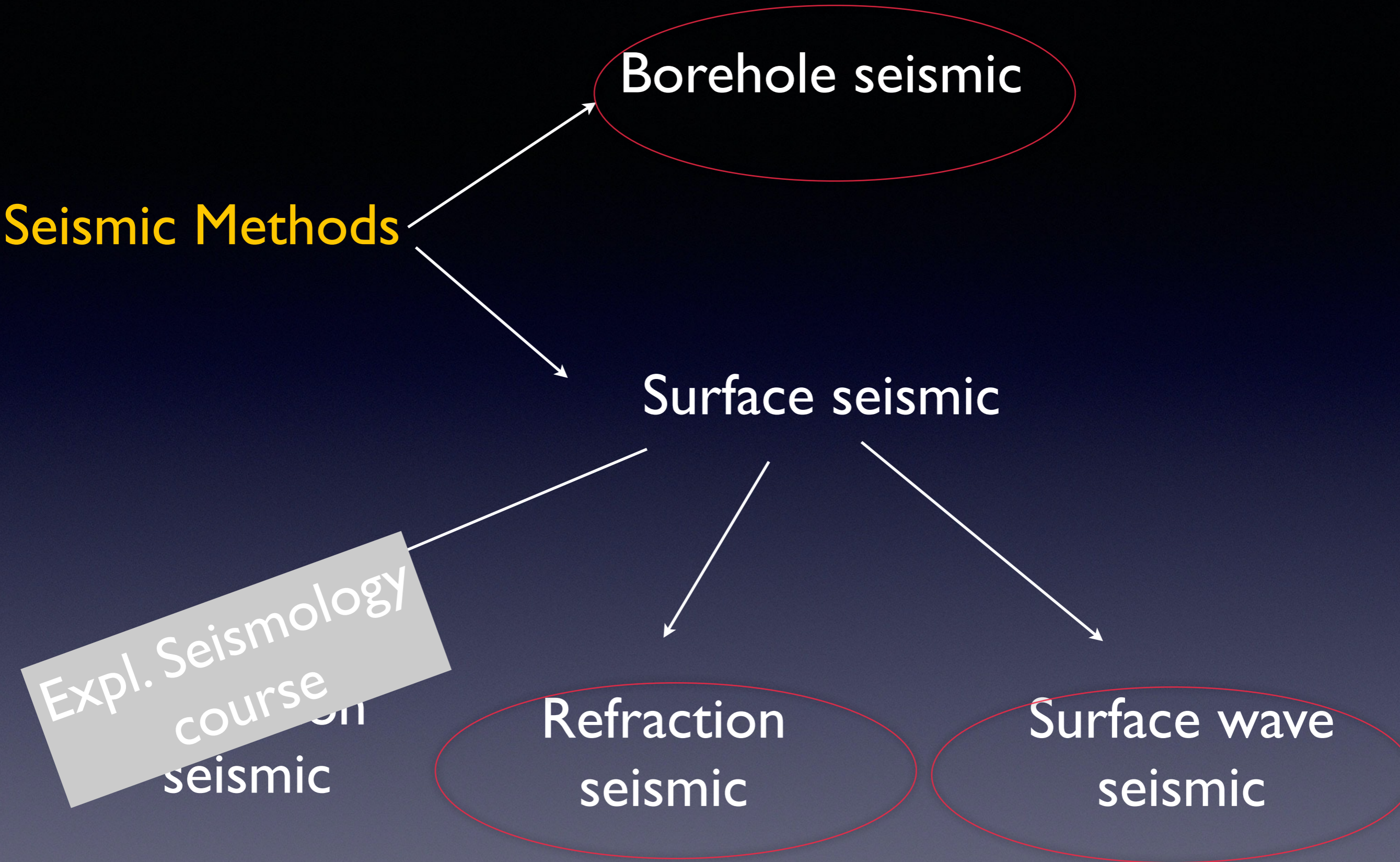
Borehole seismic

Surface seismic

Refraction seismic

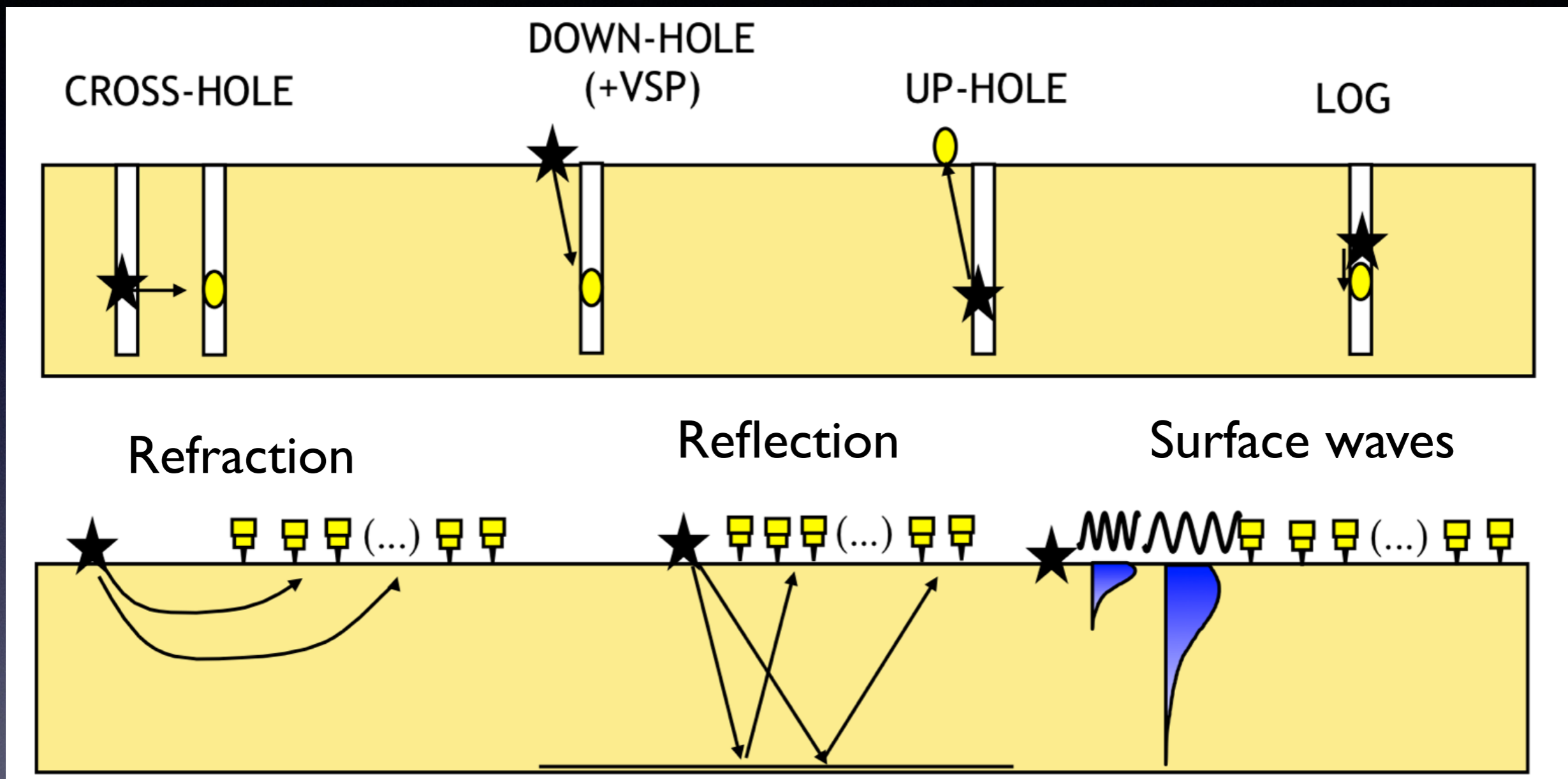
Surface wave seismic

Expl. Seismology course on seismic



Seismic Methods

Borehole Seismic



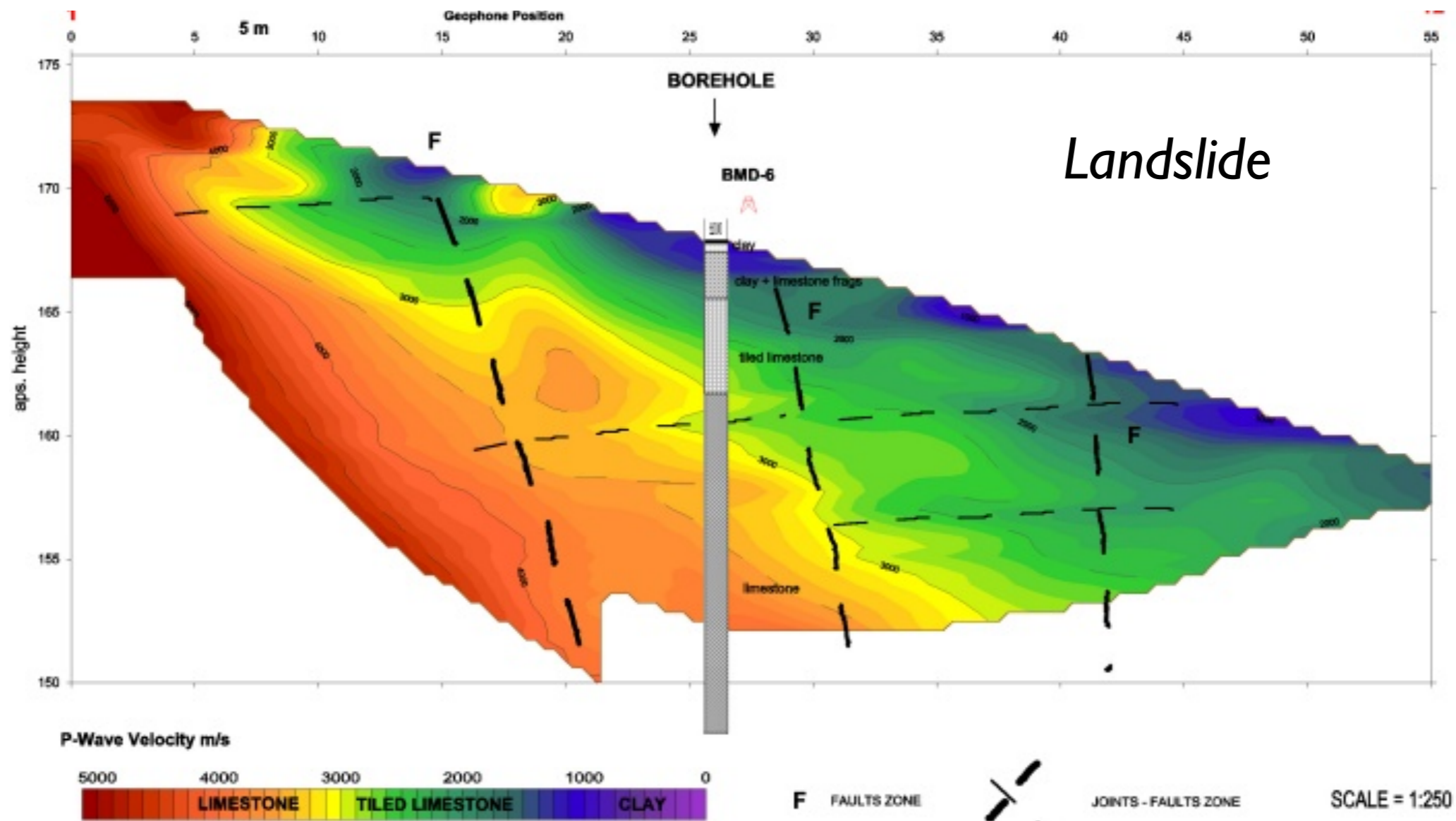
Surface seismic

Seismic Methods

Surface Seismic

Refraction seismic

2D Tomographic reconstruction of the seismic velocities

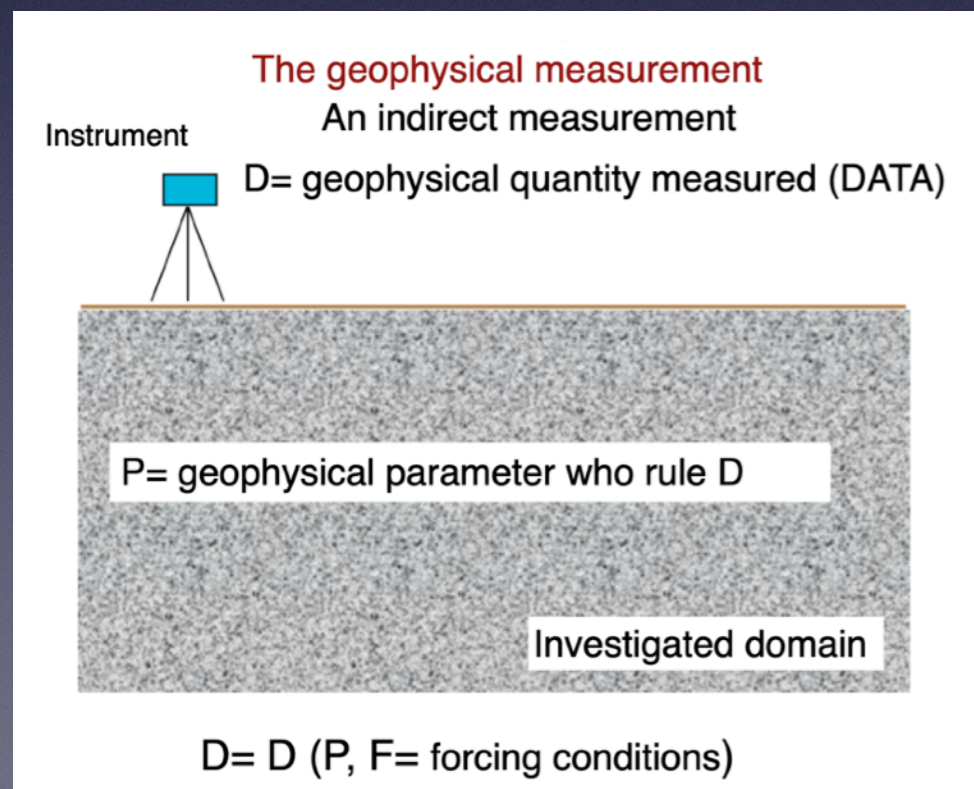


V_p

II) Electric and ElectroMagnetic methods

Environmental and geological purposes

- e.g. HYDROgeophysics -



Physics parameter P
=
Electric properties

Electric Methods

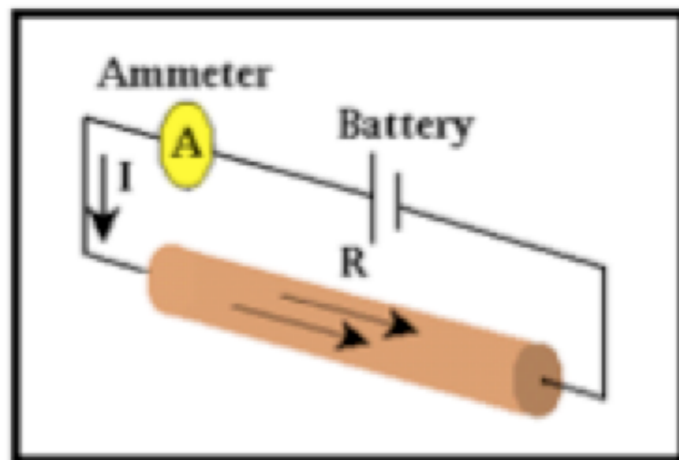
ELECTRIC RESISTIVITY TOMOGRAPHY

ERT

electrical-resistivity-tomography

OHM LAW

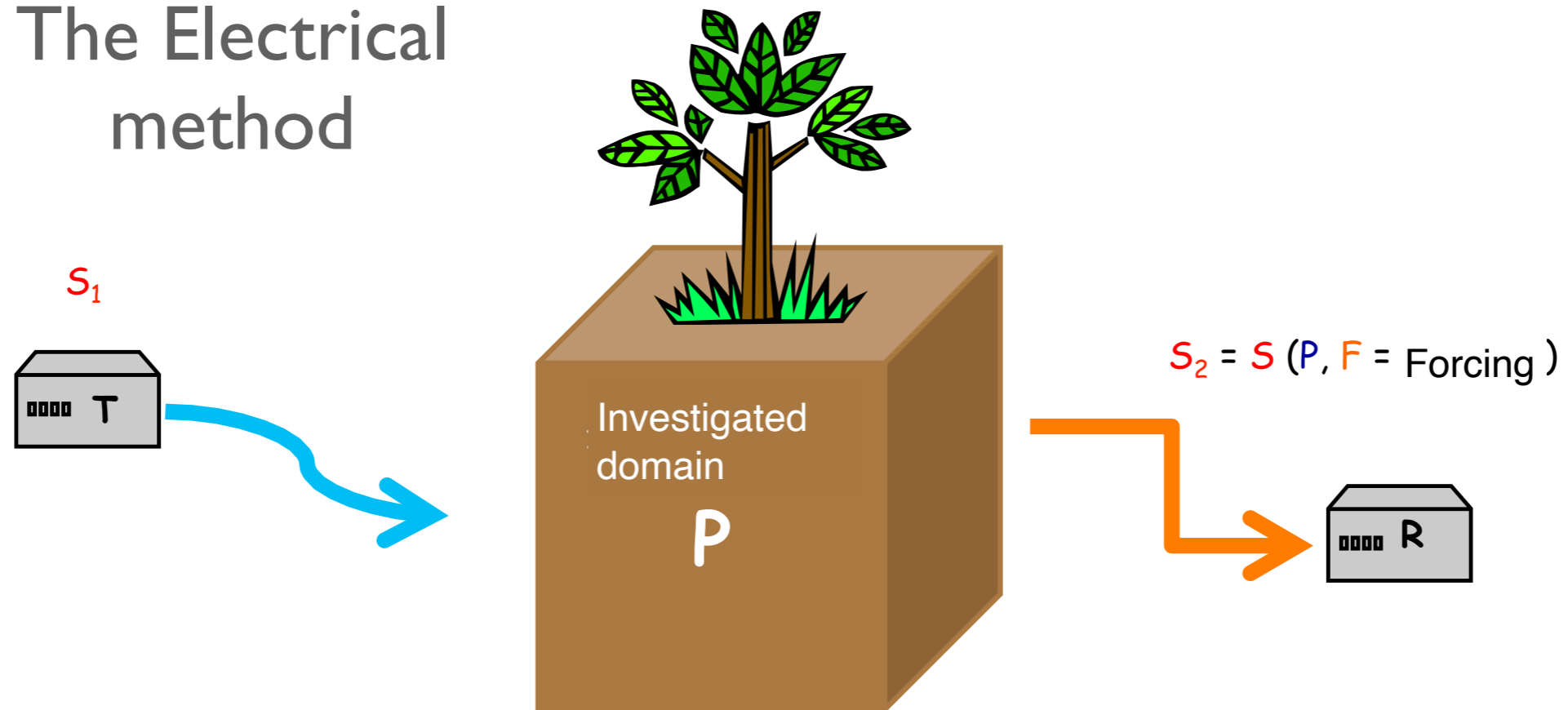
$$\Delta V = I \cdot R$$



Electric conductivity: $\sigma = \frac{1}{\rho}$

Electric Resistivity \uparrow

The Electrical method



S_1 = Signal = Injected current

S_2 = Signal = Potential voltage measured

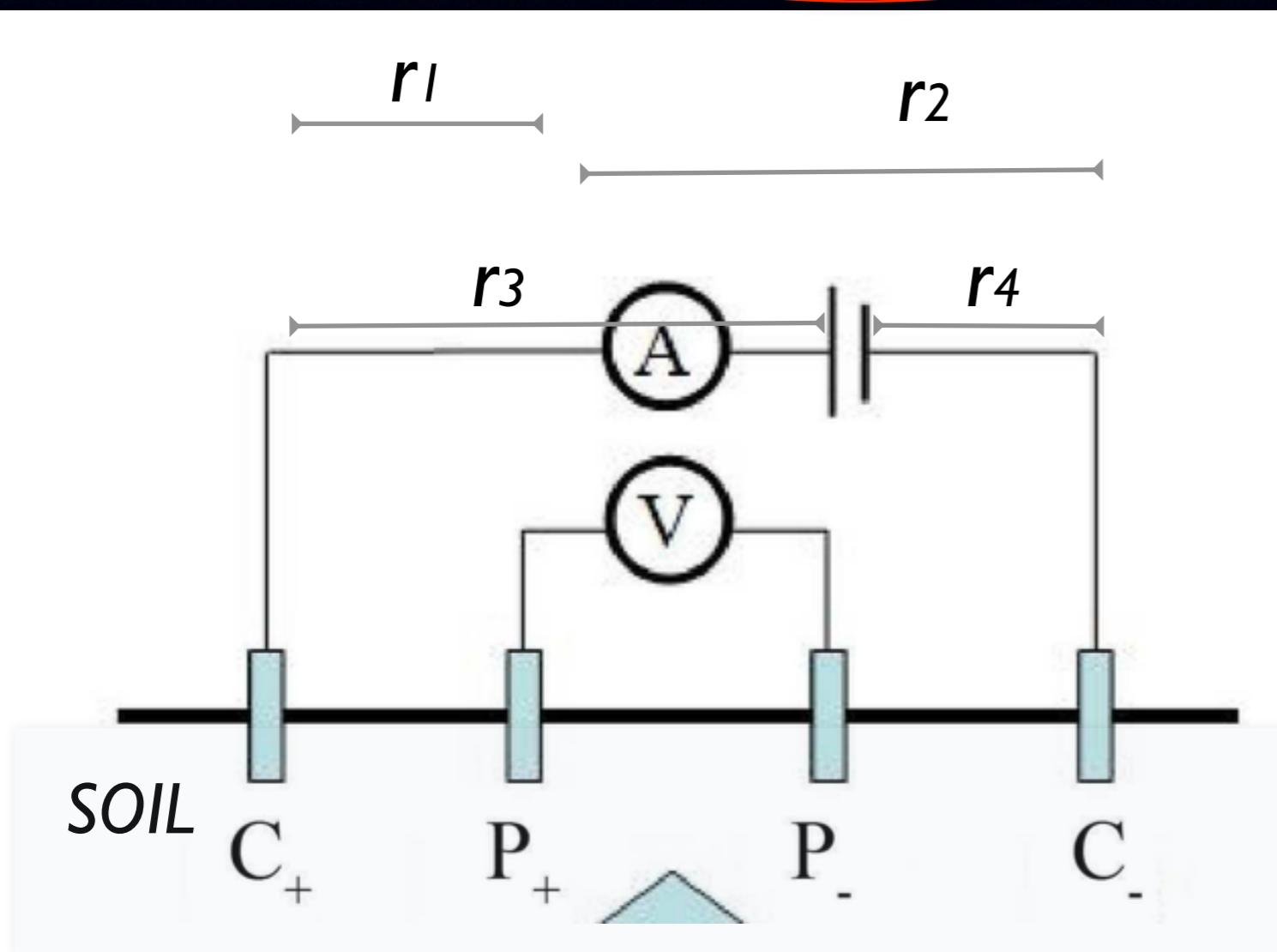
P = Physical parameter = Resistivity

ELECTRIC QUADRIPOLE

1)

$$\Delta V = \frac{I\rho}{2\pi} \left[\left(\frac{1}{r_1} - \frac{1}{r_3} \right) - \left(\frac{1}{r_2} - \frac{1}{r_4} \right) \right]$$

K
Geometric Factor



Apparent Resistivity

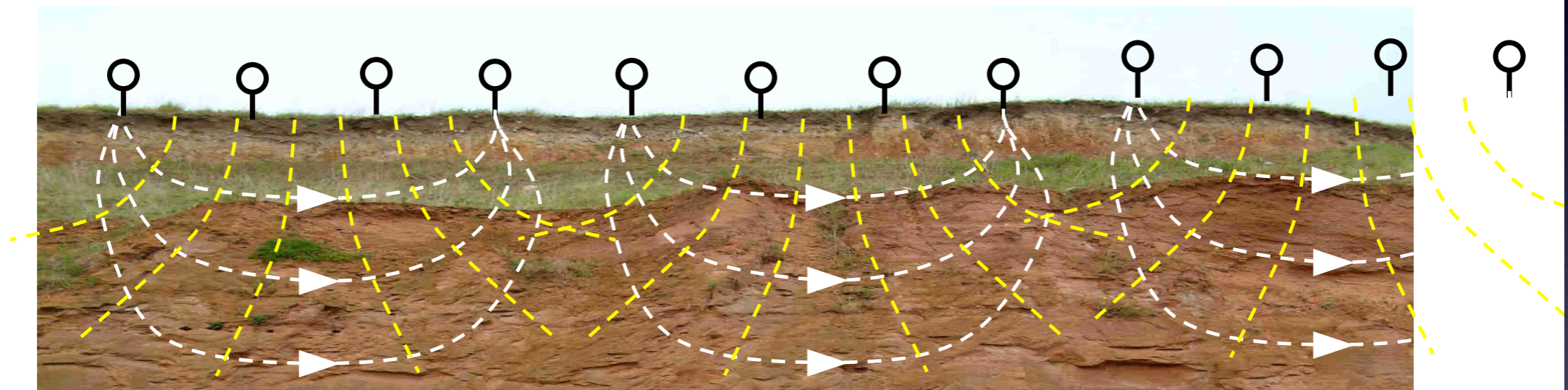
$$\rho_a = \frac{k\Delta V}{I}$$

ERT (Electrical Resistivity tomography)

Resistivity profiling

We can profile the subsurface by moving our array

C+ P+ P- C- C+ P+ P- C- C+ P+ P- C-



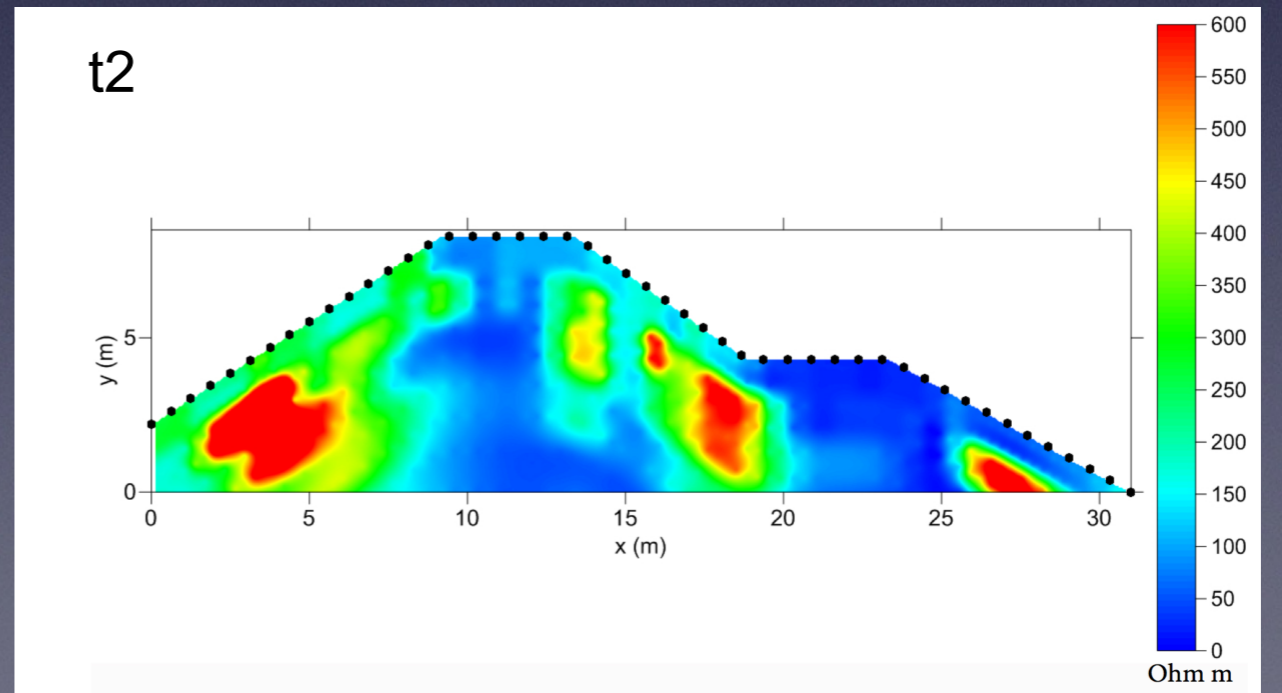
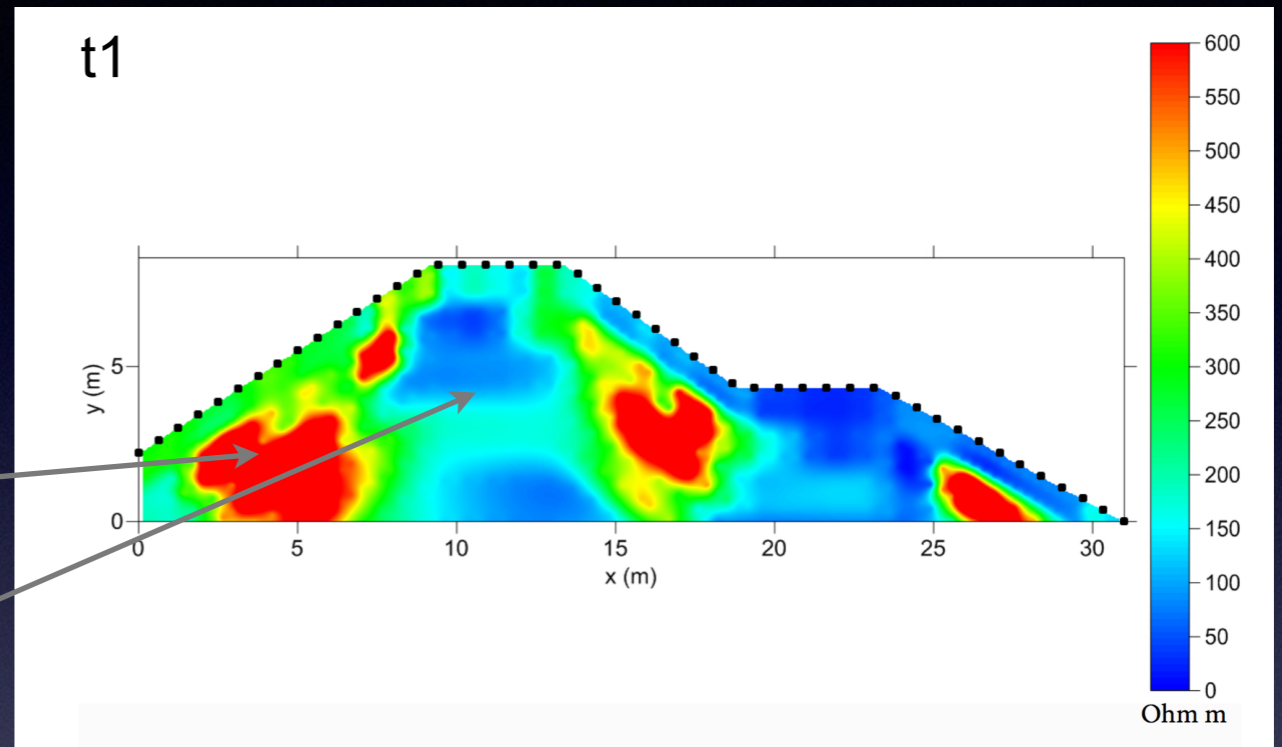
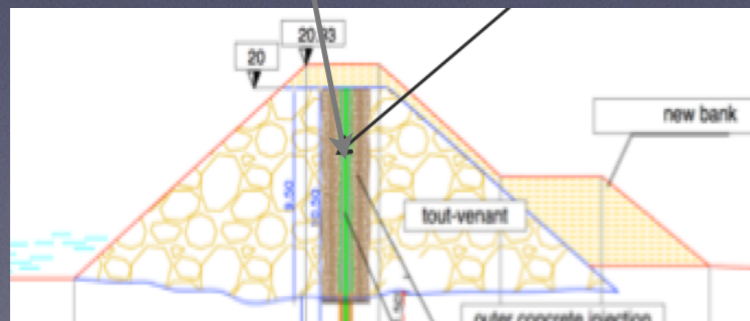
The depth we are sensitive to will depend on the array configuration and the subsurface properties. For the array above we may assume that the apparent resistivity is at about half the electrode spacing.

ERT Examples

ERT on EMBANKMENT - Top Resolution

Tout Venant

Jet Grouting Septum

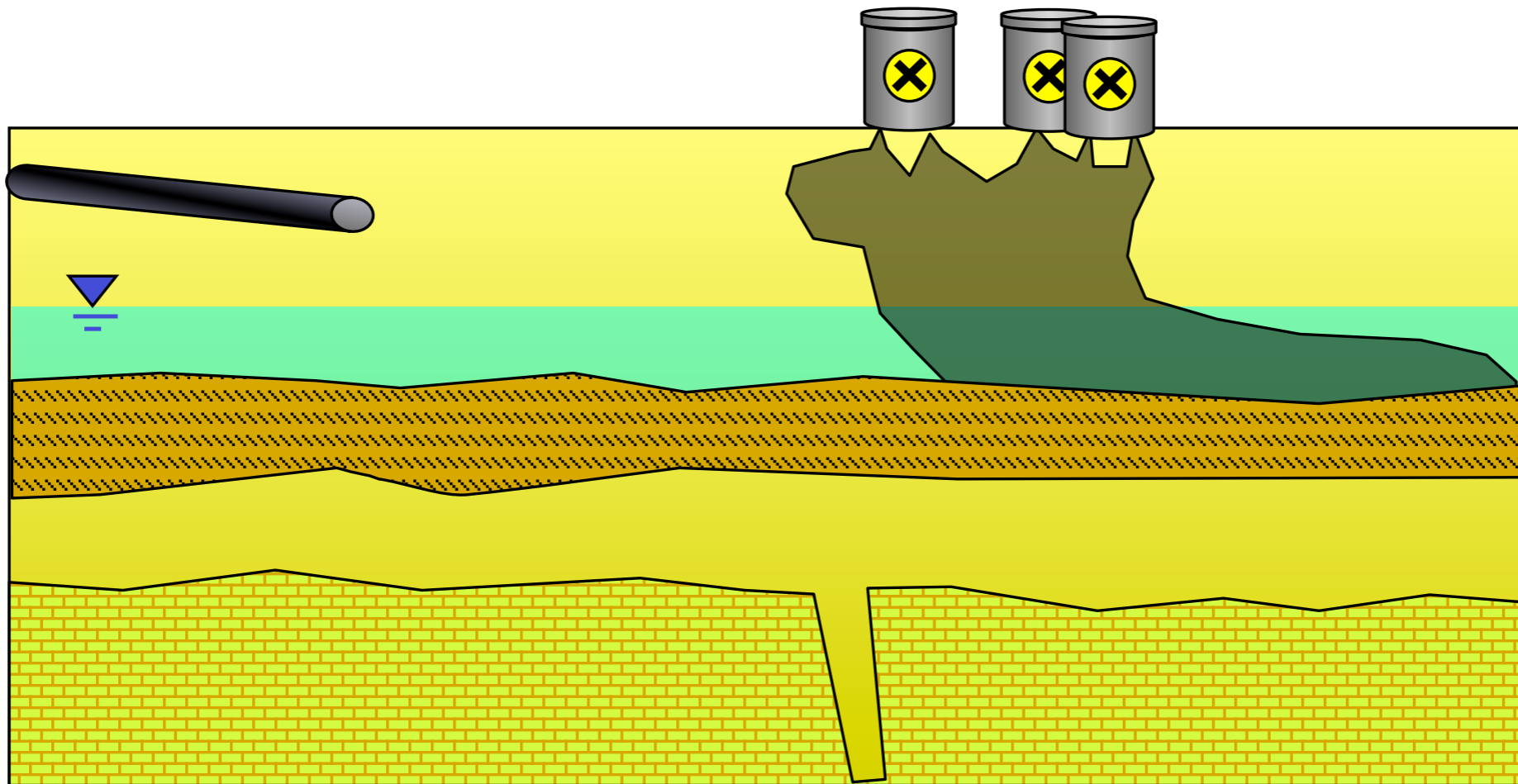


EM METHODS

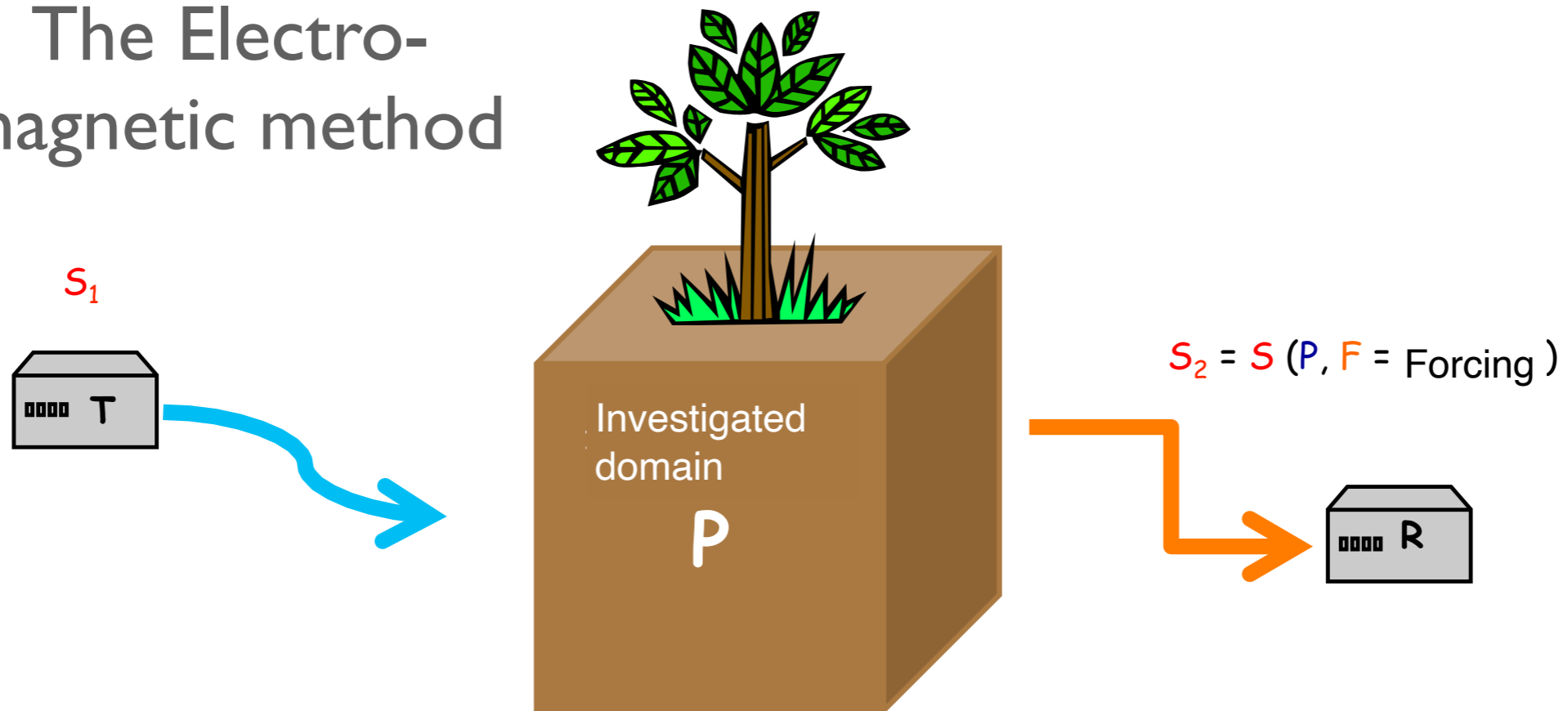
Electro-magnetic
properties

APPLICATIONS:

- Mineral exploration
- Groundwater
- Mapping contaminants
- Landfill surveys
- Cavities
- Location of faults
- Geological mapping
- Archeological



The Electro-magnetic method



S_1 = Signal =

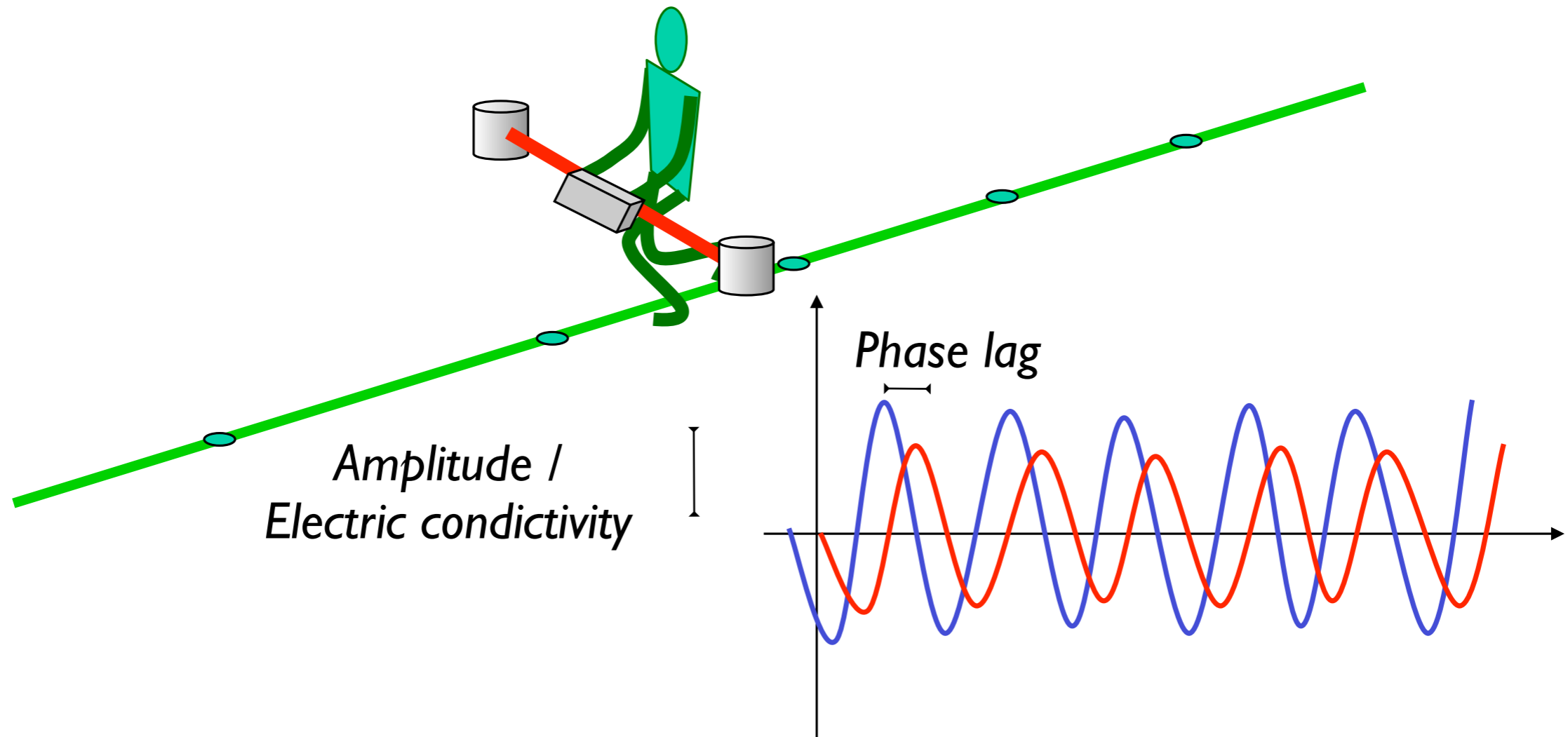
S_2 = Signal

P = Physical parameter

EM field

Induced EM field

Electrical conductivity

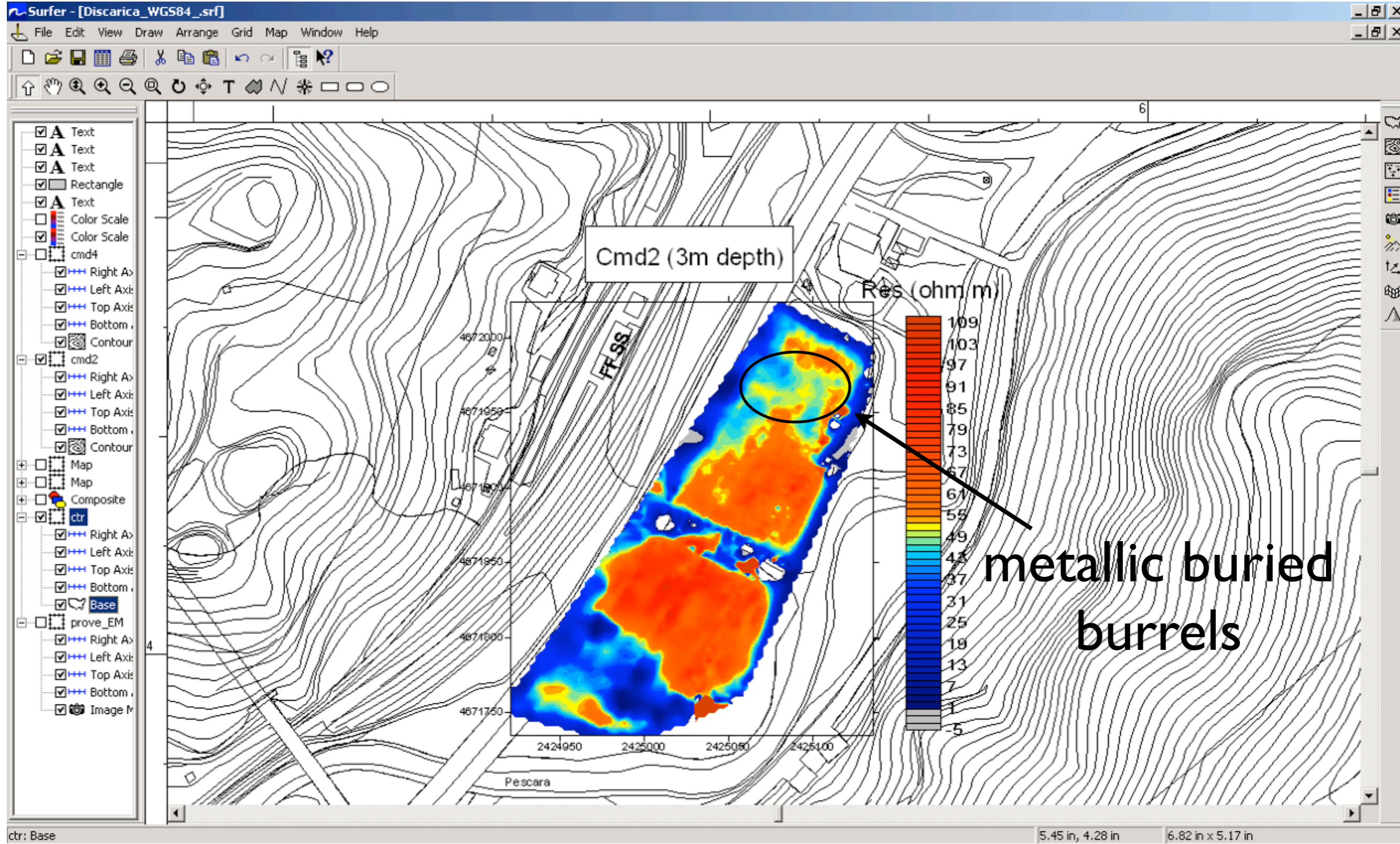


Relation between primary, secondary and resultant field: two values measured at each station

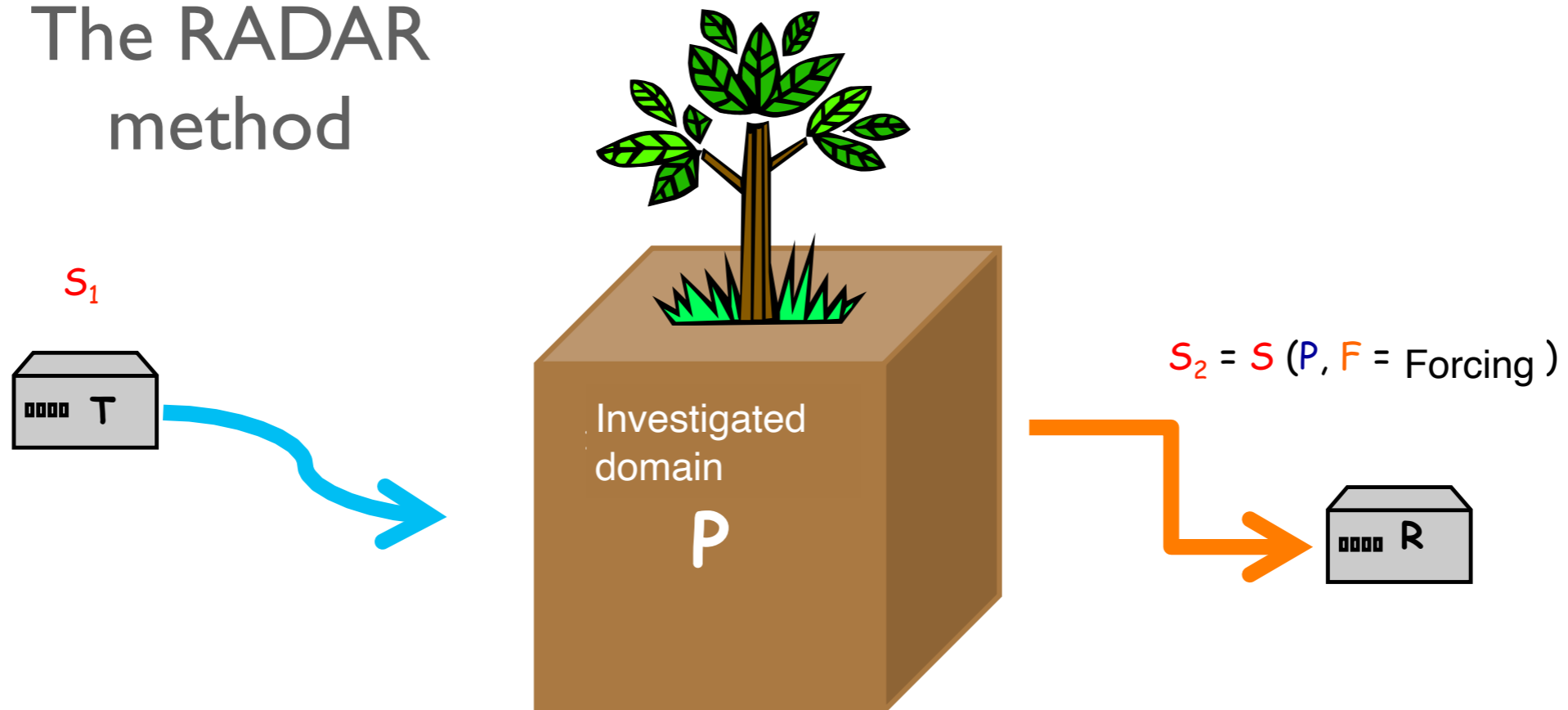
- Real (in-phase)
- Imaginary (quadrature, out-of-phase)

Amplitude
Phase lag

EM Example illegal landfill



The RADAR method



S_1 = Signal =

High frequency electro-magnetic waves

S_2 = Signal =

Reflected electro-magnetic waves

P = Physical parameter

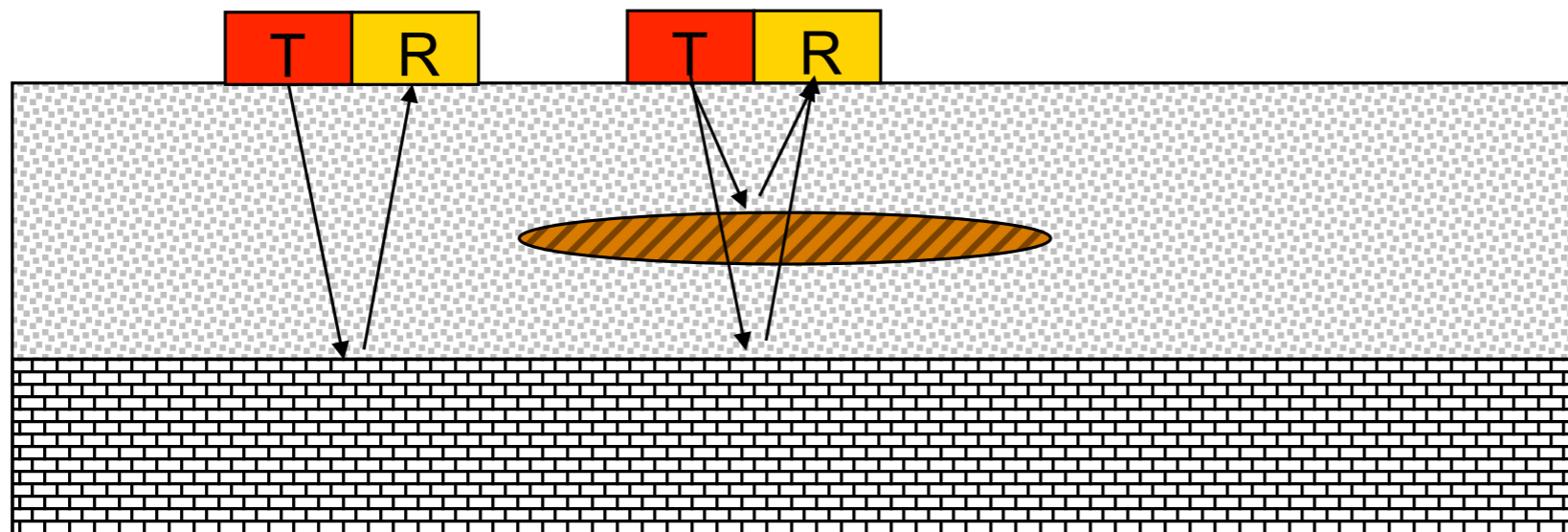
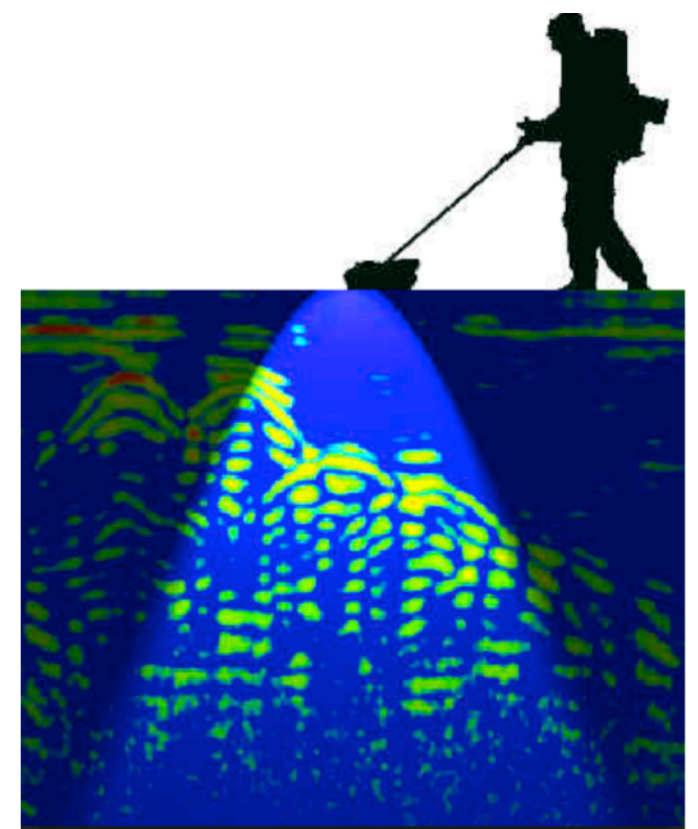
Dielectric contrast

Georadar

G P R

Ground Probing Radar (or Ground Penetrating Radar) is a high frequency EM technique (in the band 10-2500 MHz) based on the response of the subsoil to a short EM pulse.

A signal of short wavelength is radiated into the ground, is reflected, refracted, diffracted and hence detects the anomalous variations in the dielectric properties.



Anomalies can be soil horizons, the groundwater surface, soil/rock interface, man made objects (pipes, foundations, cables...)

HIGH RESOLUTION, LOW PENETRATION DEPTH

Georadar



Vertical Radar Profile VRP

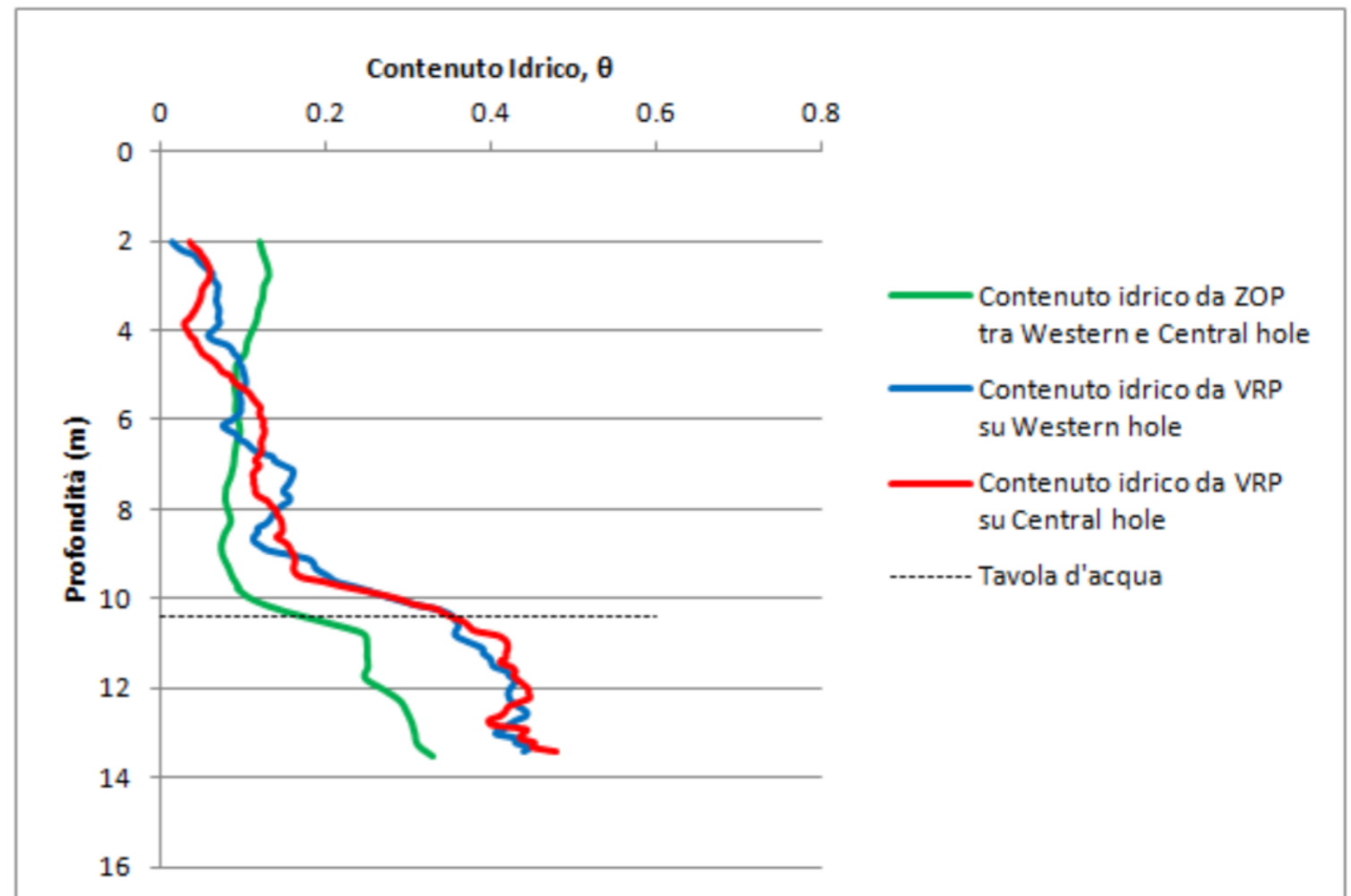
Δ Velocity of EM waves =

$\Delta \epsilon$

Topp Relation

$$\epsilon_r = 3.03 + 9.3\theta + 146\theta^2 - 76.7\theta^3$$

Water cont.



Geophysical methods

- Electric methods
- Seismic methods
- GeoRADAR
- EM methods
- Gravimetry
- Magnetism
- ...



Applications

?

- Oil and Gas exploration
- Minerals exploration
- Engineering studies
- Hydrogeological studies
- Pollutants identification
- Geological characterisation
- Legal problems
- Archeology
- ...

Take home messages



	S 1	P	S 2
Seismic methods	Elastic waves	Elastic moduli/ density	Induced vibrations
Electrical methods	Current injection	Resistivity	Voltage potential
Electro-magnetic methods	Electro-magnetic Field	Conductibility	EM secondary fields
Radar (EM)	Electro-magnetic waves	Dielectric contrast	Reflected EM waves

Geophysical
methods :



Applications

The choice of the methods must follow the criteria:

- The target must be suitable with the **Physical Parameter** measured
- The method must have enough spatial (or temporal) **RESOLUTION** and enough depth **PENETRATION**
- The cost
- Logistical issues
- Environmental impact

Not basing on the instrument I have.....

Geophysical
methods :



Applications

The choice of the methods must follow the criteria:

- The target must be suitable with the **Physical Parameter**
measured *INVERSION*
- The method must have enough spatial (or temporal)
RESOLUTION and enough depth **PENETRATION**
RESOLUTION
- The cost *Logistics !*
- Logistical issues
- Environmental impact

Concepts of Inversion in GEOPHYSICS

THE INVERSION

The geophysical measurement

An indirect measurement

Instrument



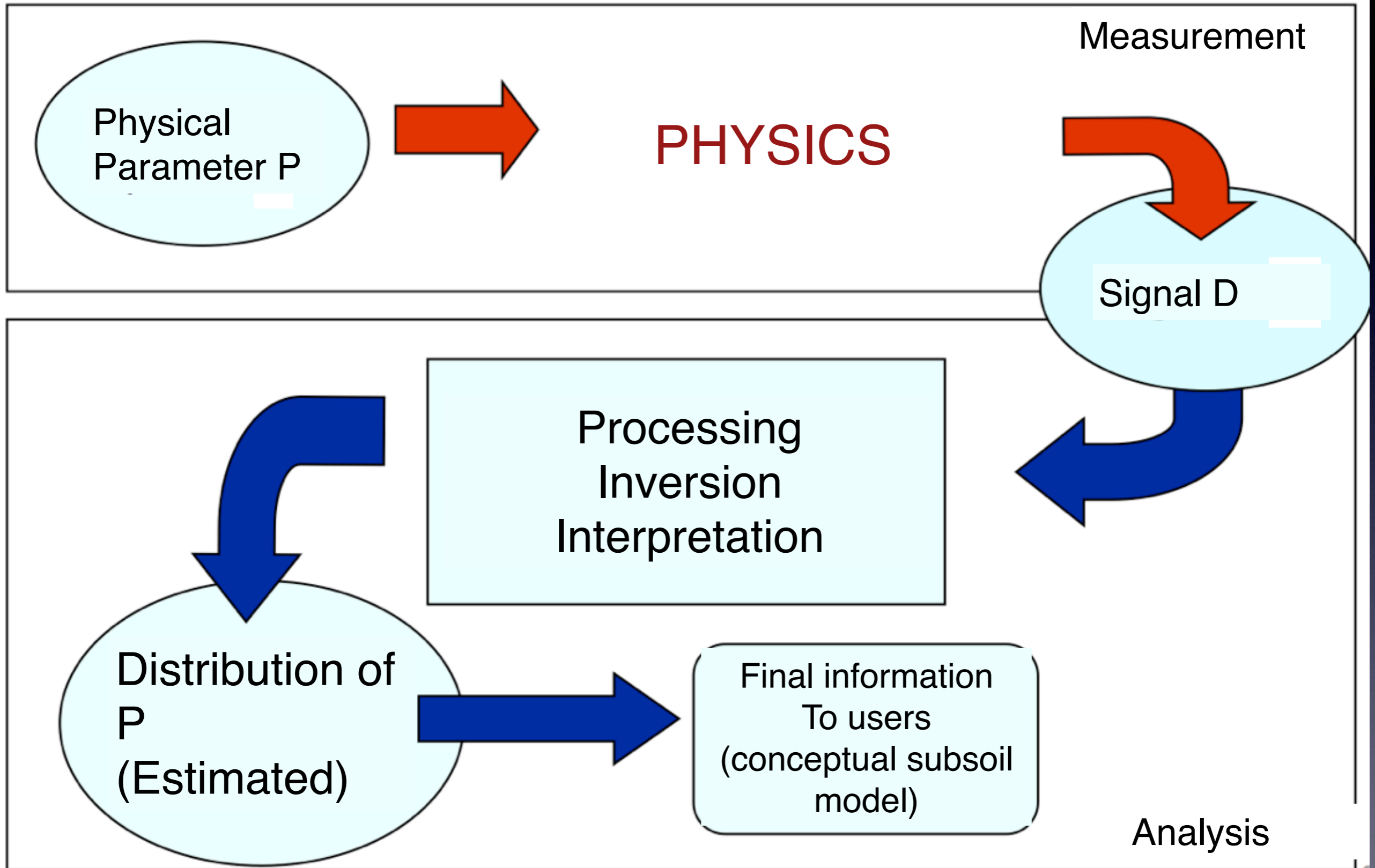
$D =$ geophysical quantity measured (DATA)

$P =$ geophysical parameter who rule D

Investigated domain

$D = D (P, F = \text{forcing conditions})$

Measurements and Analysis in Applied Geophysics



Forward and Inverse MODELS

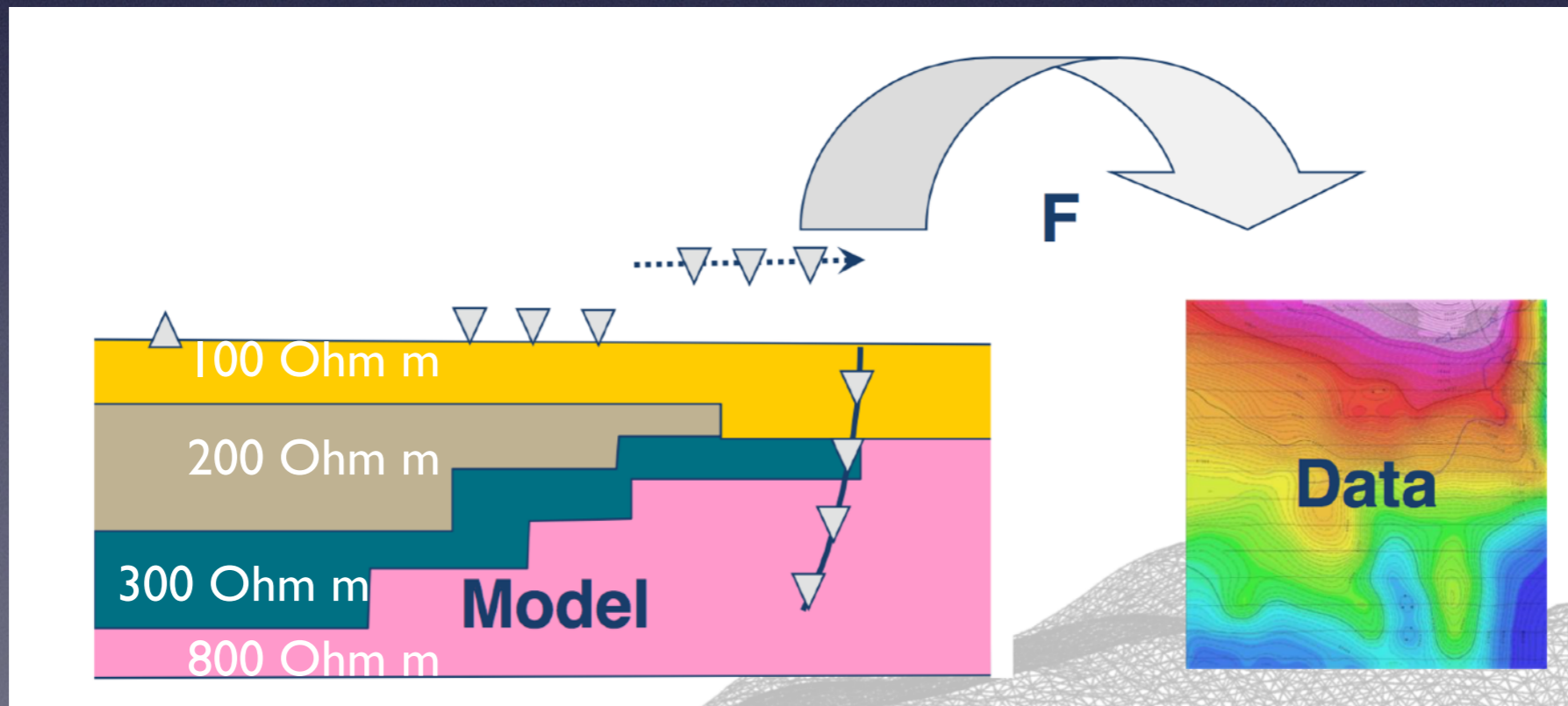
FORWARD MODEL

From a model **M**, I get a data distribution **d**

$$d = F(M)$$

output → *input*

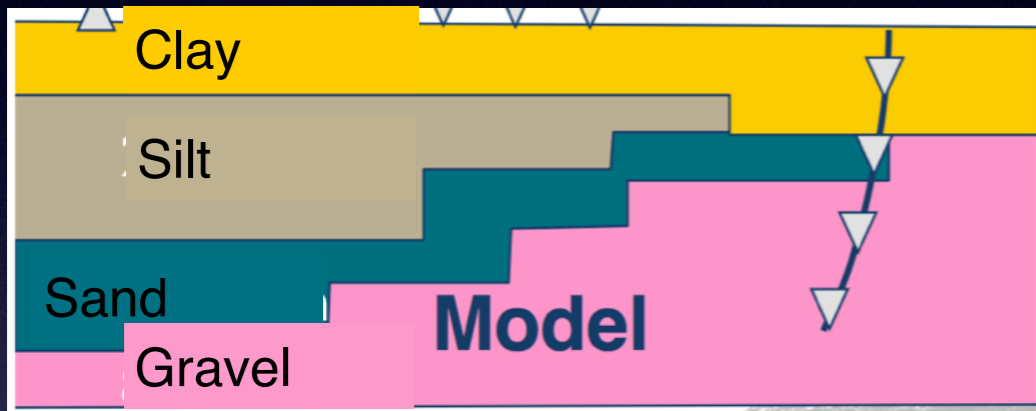
Where **F** is an operator which rules the relations between models and data



FORWARD MODEL

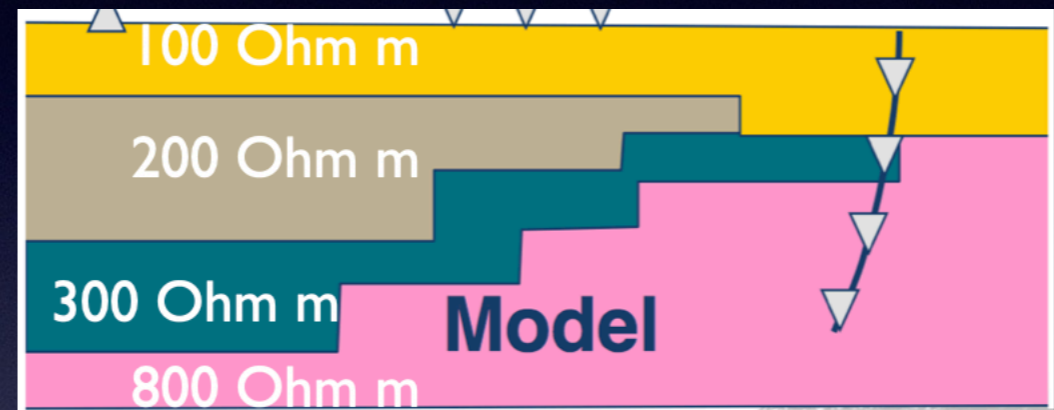
Example

1)



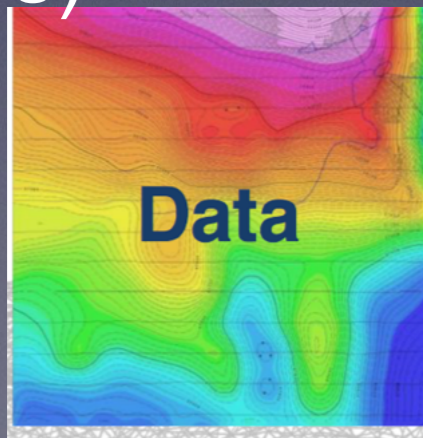
Model
of the subsoil

2)



Synthetical Electrical model
of the subsoil

3)

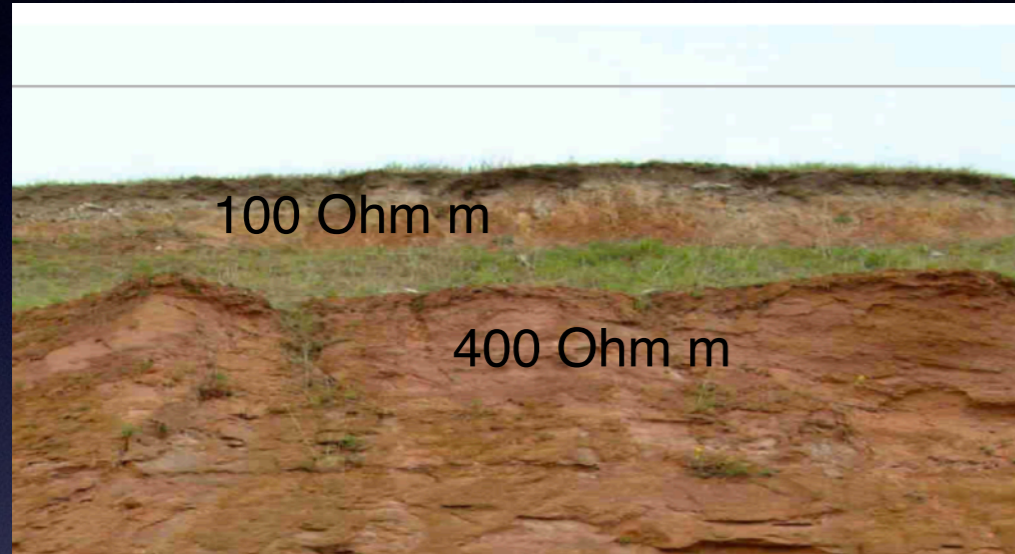


Physics laws who rules
electrical distribution

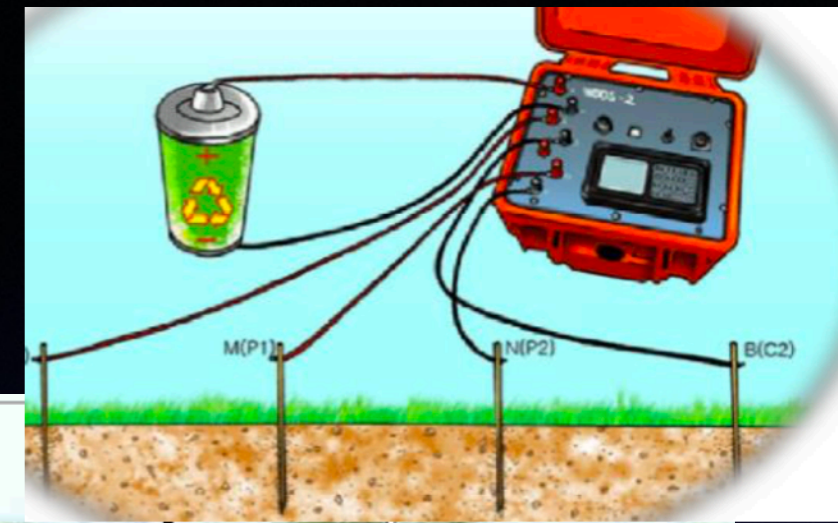
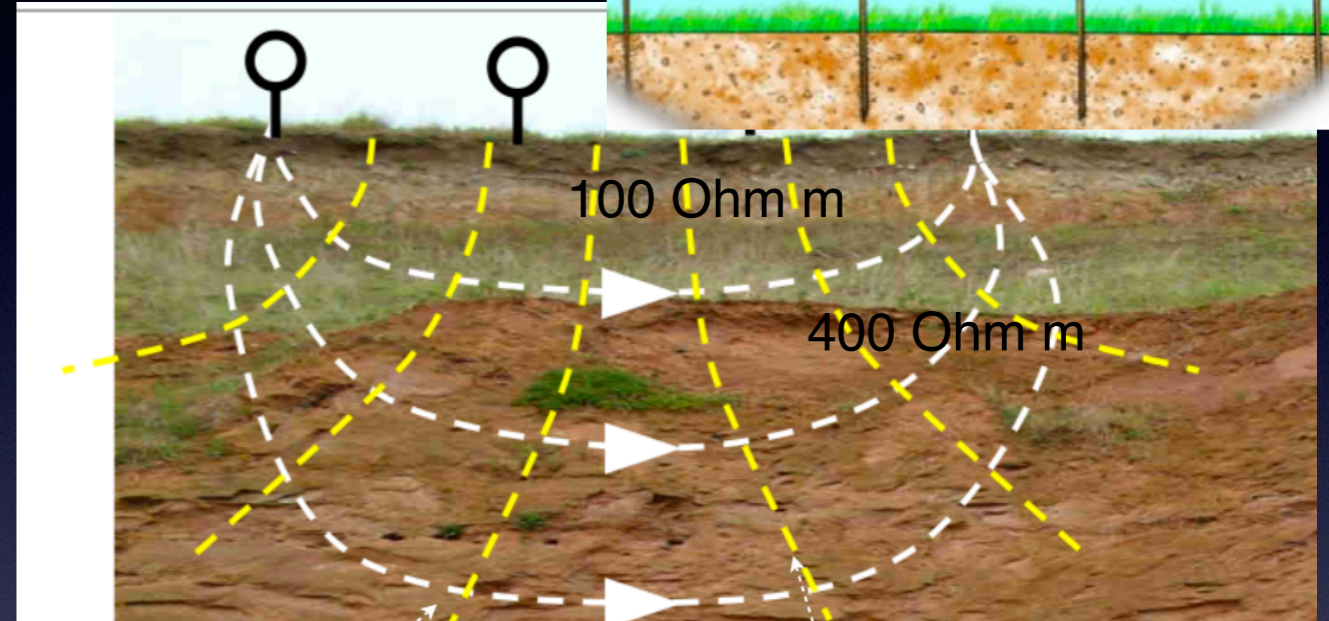
Knowing the physics,
I can simulate which DATA
I would collect in that subsoil

FORWARD MODEL

Model



Simulated Measurements



Instrument Data

A	B	M	N	r (0*m)	var	M (ms)	PS (mV)	V (mV)	I (mA)
1	6	2	7	95.12	0.06	0	-54.6	1223.95	51.06
1	6	7	12	107.52	0.11	0	13.88	-661.98	51.06
1	6	12	17	90.99	3.65	0	-6.16	-35.01	51.06
1	6	17	22	132.99	0.62	0	-15.07	-14.62	51.06
1	6	22	27	164.2	0.56	0	-6.92	-7.64	51.06
1	6	27	32	171.17	5.56	0	33.54	-4.11	51.06
1	6	32	37	184.15	0.57	0	3.16	-2.58	51.06
1	6	37	42	175.62	11.21	0	-18.03	-1.56	51.06
1	6	42	47	109.62	13	0	-108.03	-0.66	51.06
1	6	47	4	86.06	1.18	0	134.71	114.72	51.06
1	6	3	8	79.6	0.11	0	-72.83	338.88	51.06
1	6	8	13	79.26	0.09	0	7.23	-191.71	51.06
1	6	13	18	98.01	5.02	0	-2.69	-27.89	51.06
1	6	18	23	138.48	0.84	0	12.57	-12.54	51.06
1	6	23	28	168.14	0.52	0	9.14	-6.77	51.06
1	6	28	33	178.22	4.41	0	97.31	-3.81	51.06
1	6	33	38	174.61	6.65	0	-80.62	-2.22	51.06
1	6	38	43	172.48	10.18	0	-21.83	-1.41	51.06
1	6	43	48	86.77	18.83	0	-99.48	-0.48	51.06
1	6	48	5	106.96	0.28	0	139.72	649.74	51.06
1	6	4	9	86.73	1.84	0	-43.25	29.37	51.06
1	6	9	14	74.7	0.1	0	-1.46	-97.29	51.06
1	6	14	19	105.3	6.65	0	10.54	-22.86	51.06

Physics laws who rules electrical distribution

Knowing the physics,
I can simulate which DATA
I would collect in that subsoil

Forward and Inverse Models

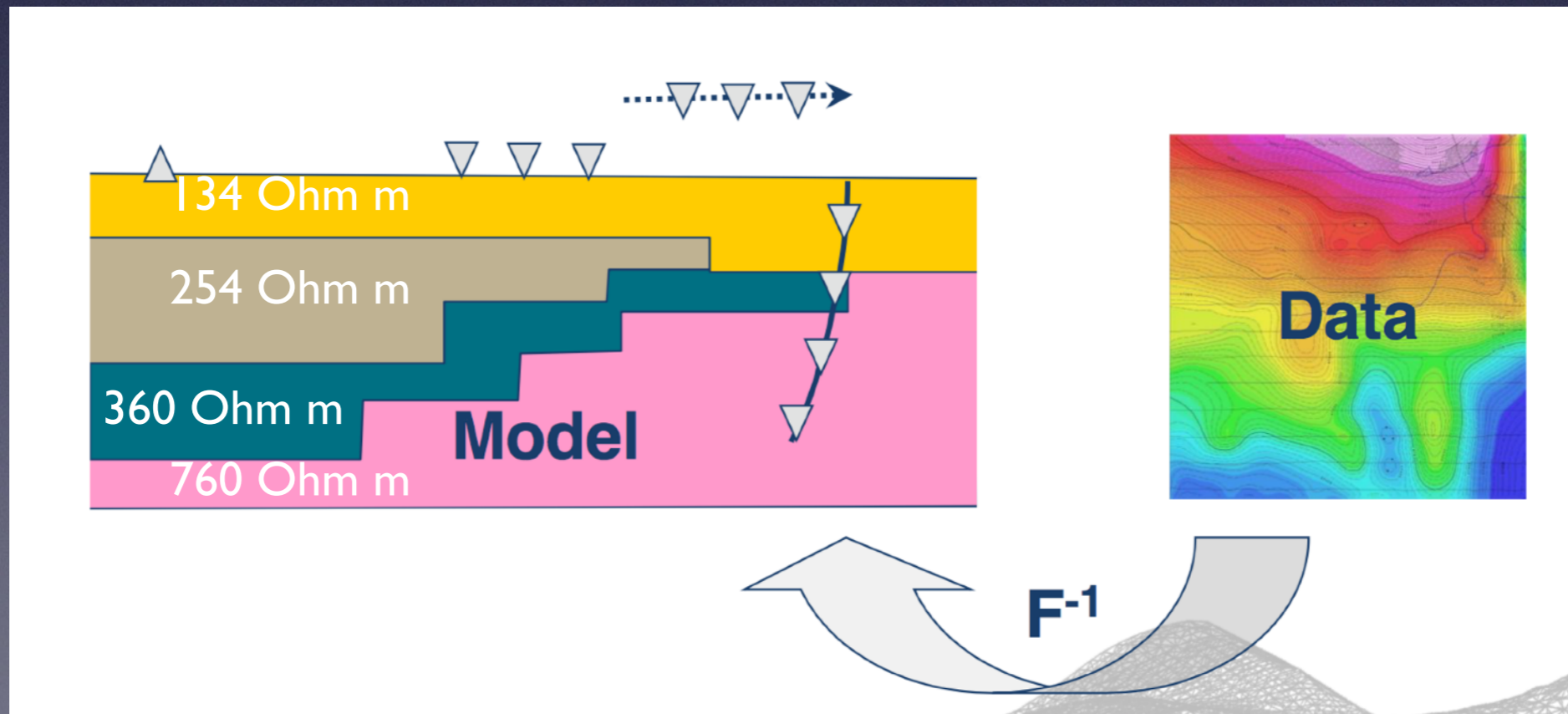
INVERSE MODEL

THE INVERSION in Geophysics are the mathematical and statistical techniques to determine the distribution of the Parameter P (e.g. *resistivity, density etc.*) starting from the observed DATA d

From the data collected d , I retrieve a subsoil model M

$$M = F^{-1}(d)$$

output \swarrow \nwarrow input



Forward and Inverse Models

DIRECT

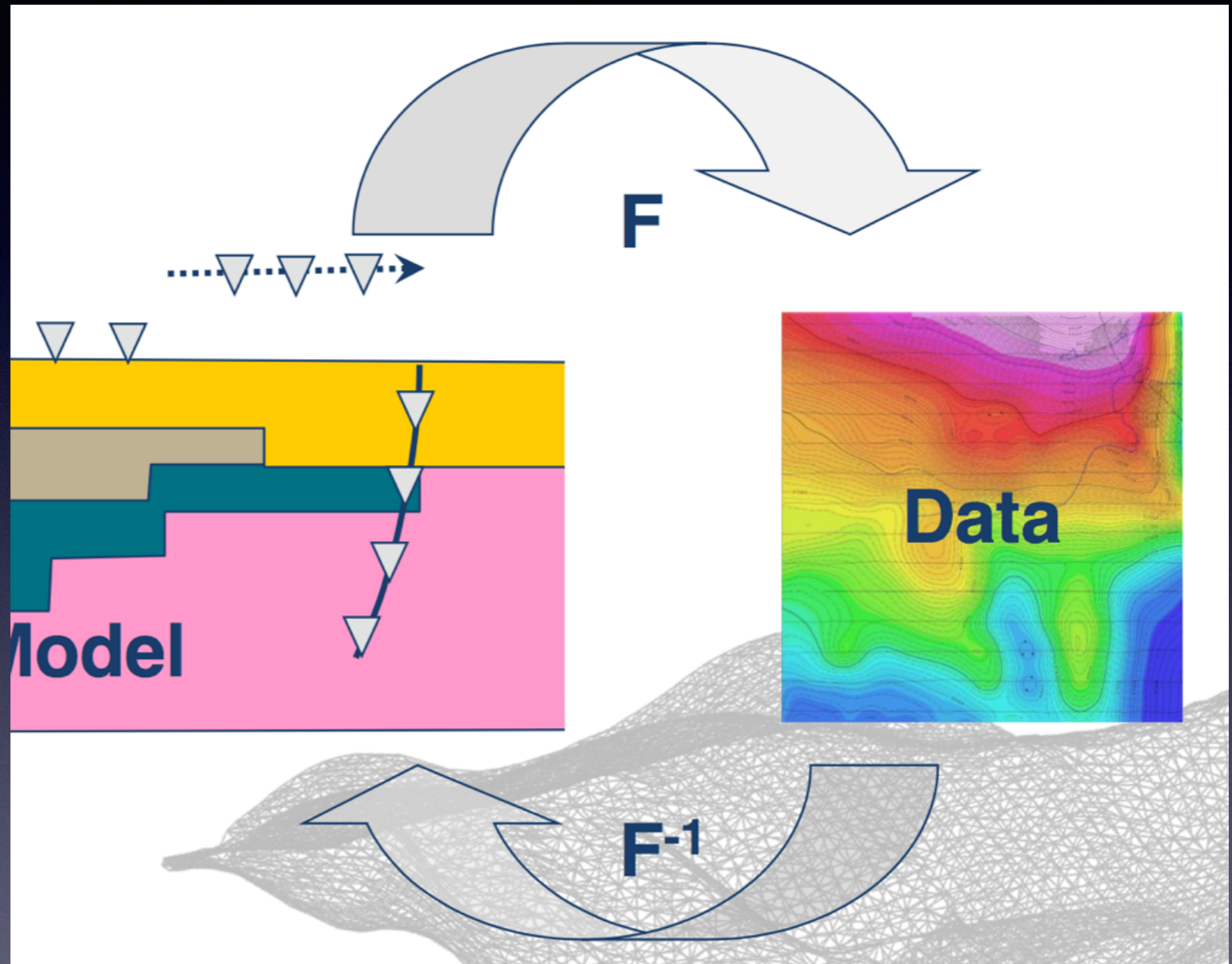
$$d = F(M)$$

output \nearrow d \nwarrow input M

INVERSE

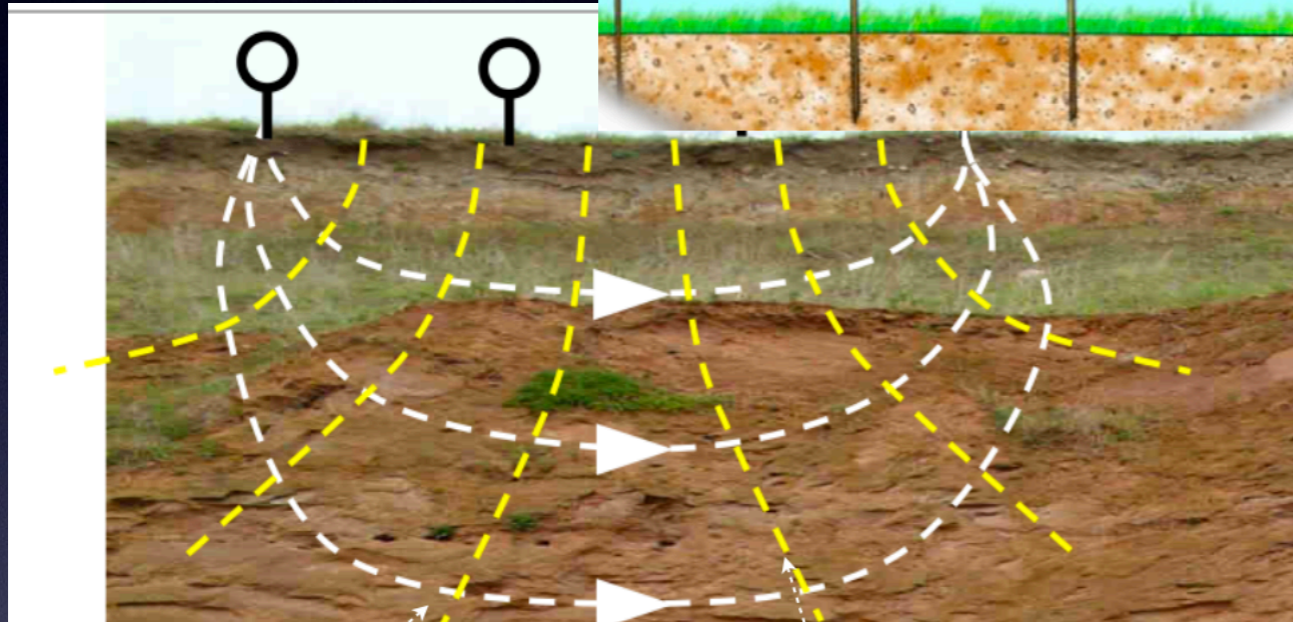
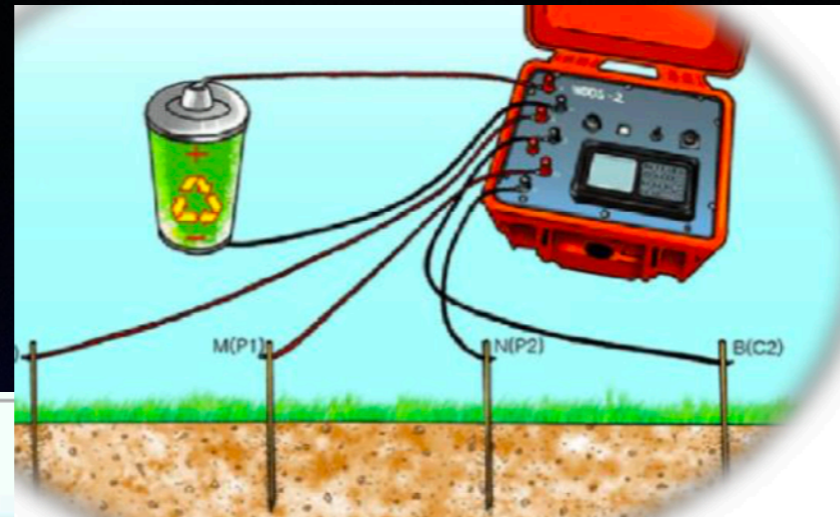
$$M = F^{-1}(d)$$

output \nearrow M \nwarrow input d



INVERSE MODEL

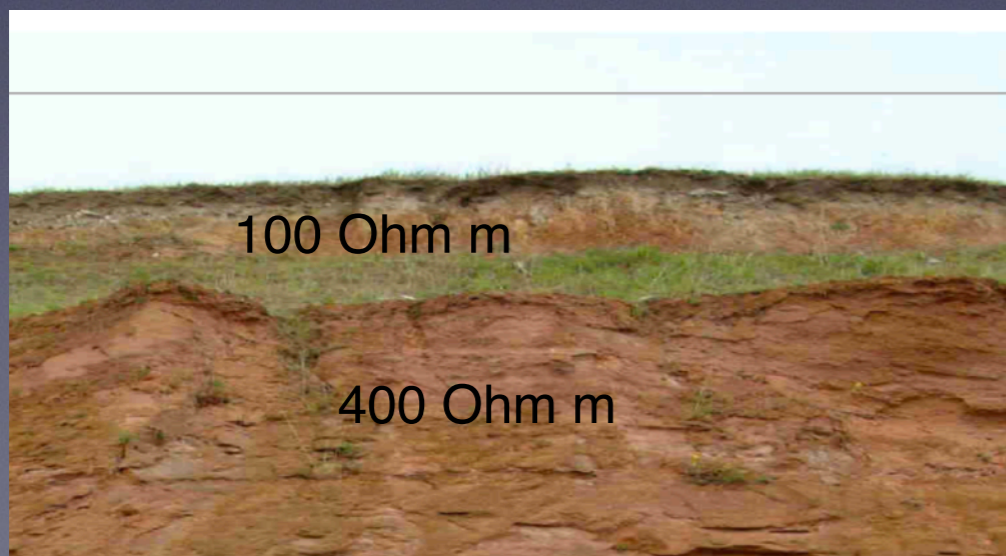
Field Measurements



Collected Data

A	B	M	N	r (0*m)	var	M (ms)	PS (mV)	V (mV)	I (mA)
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1	6	38	43	172.48	10.18	0	-21.83	-1.41	51.06
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1	6	48	5	106.96	0.28	0	139.72	649.74	51.06
1	6	4	9	86.73	1.84	0	-43.25	29.37	51.06
1	6	9	14	74.7	0.1	0	-1.46	-97.29	51.06
1	6	14	19	105.3	6.65	0	10.54	-22.86	51.06

Model



← INVERSION

From the collected DATA
I retrieve a model of the subsoil

Direct and Inverse models



$$d = F(m) = d(p, f)$$

Forward model

Forcing conditions

Examples

d= electric potential, P = resistive, F= injected current
(electric methods)

d= soil vibrations, P = elastic waves velocities, F= seismic source
(seismic methods)

d= EM waves, P = EM waves velocities, F= EM source
(GEORADAR)

We do not want **d**
(Physical quantity measured)

but **P**
(Physical earth Parameter)

We need the soil characteristics, retrieved by indirect not invasive methods

Direct and Inverse models



$$m = F^{-1}(d)$$

Inverse model

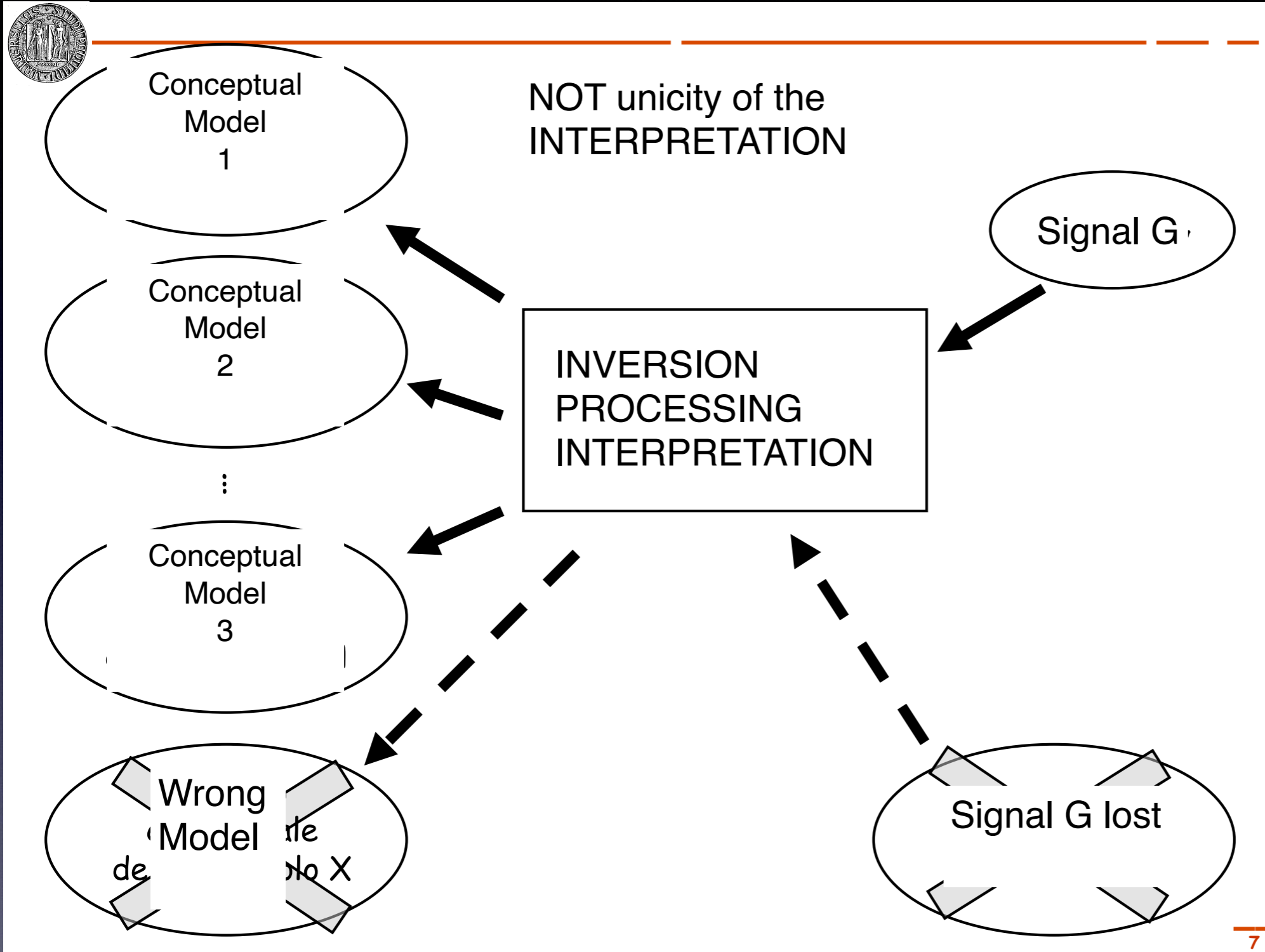
Examples

To retrieve the resistivity distribution in the soil
(electric methods)

To retrieve the seismic velocities in the soil
(seismic methods)

We need an
Inversion
Process

Direct and Inverse models



The indirect methods are easy, fast, cheap...

The price of is the **non unicity in the solution**

The inverse model is mathematically **ill-posed**, it does not respect: **unicity**, linearity, existence of the solution...

- inversion: an ill posed problem

Hadamard 1923,

Math problem is well posed if:

- 1) for all the admissible data a solution exists (existence)
- 2) for all the admissible data the solution is unique (unicity)
- 3) solution depends in a continuous way from the data (stability)

Otherwise it is ILL POSED !

- inversion: an ill posed problem

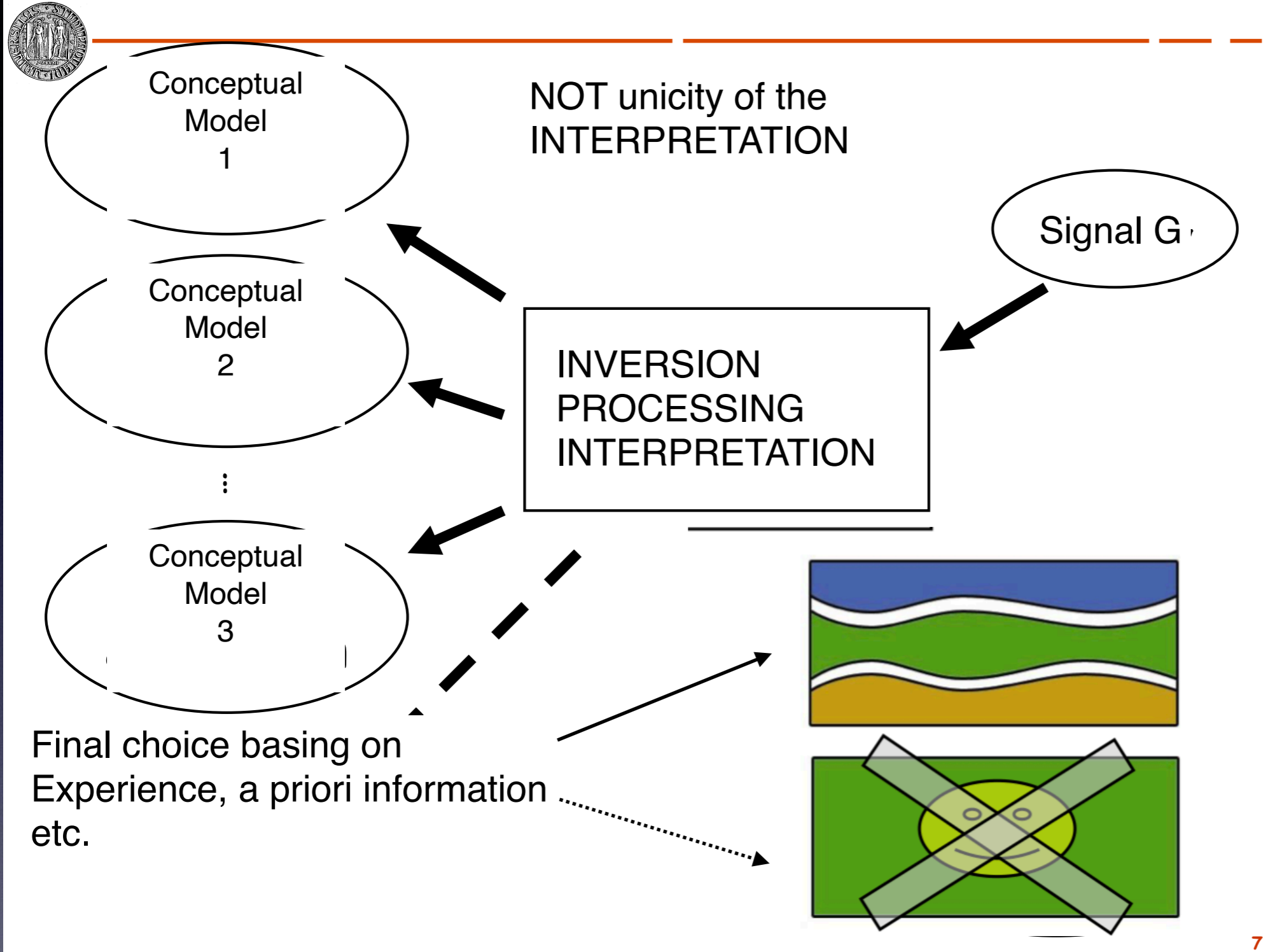
Hadamard 1923,

Math problem is well posed if:

- 1) for all the admissible data a solution exists (existence)
- 2) for all the admissible data the solution is unique (unicity)
- 3) solution depends in a continuous way from the data (stability)

Inverse problem is ILL POSED !

Direct and Inverse models



Correct Interpretation Can be Hard

- *Bad cases*

Bad data input to excellent inversion algorithm

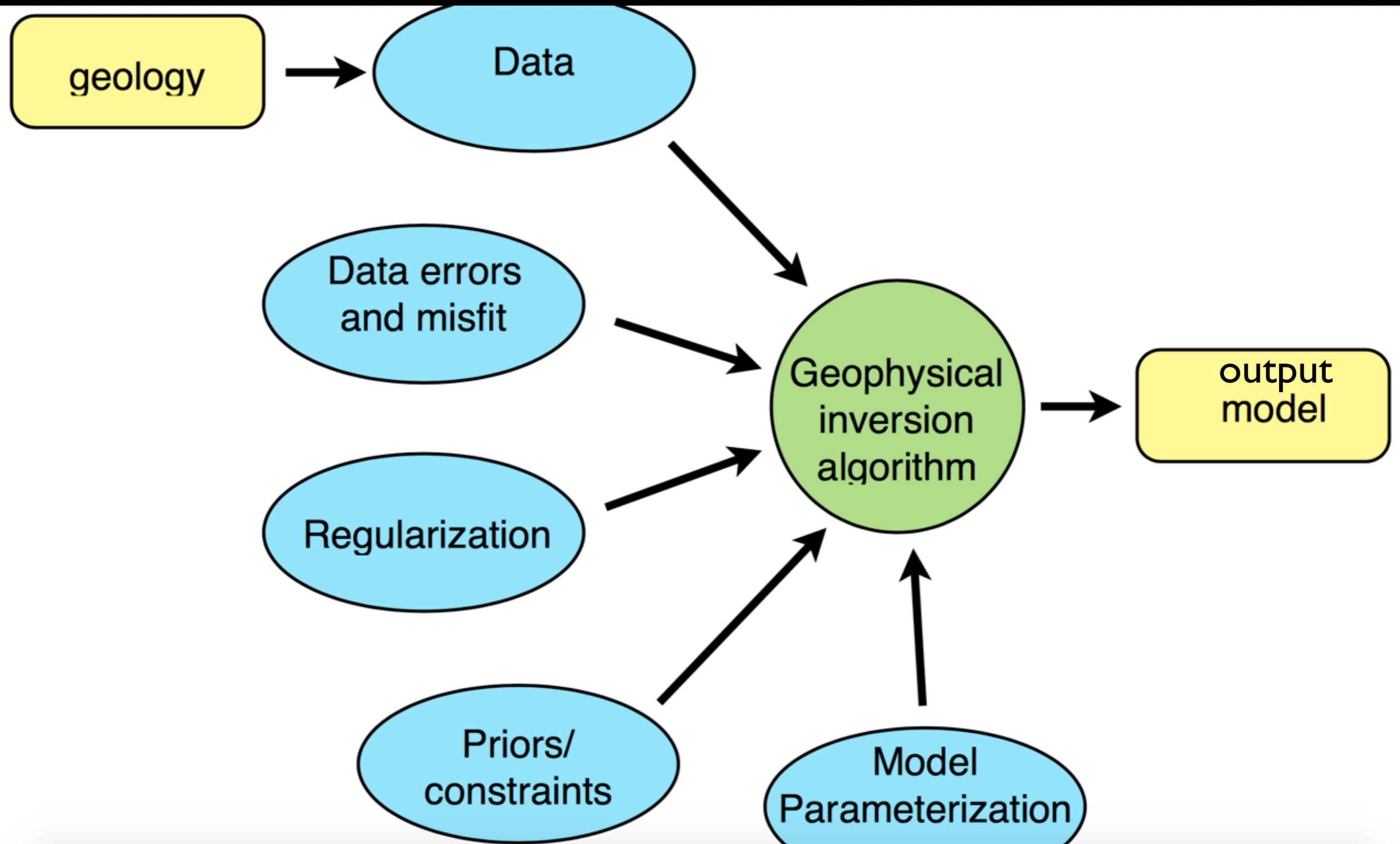


BAD OUTPUT

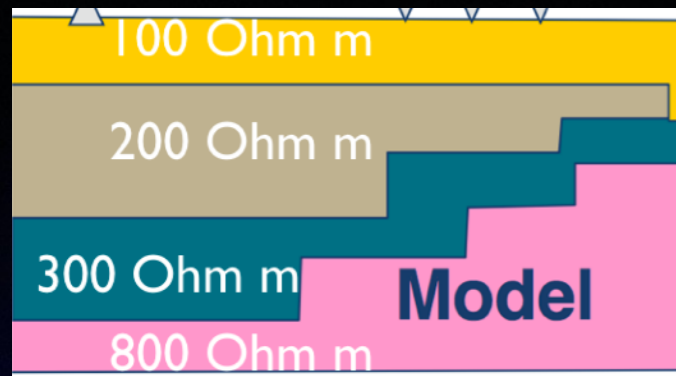
Garbage in, garbage out!

Inversion process

Find the model which minimise the data misfit



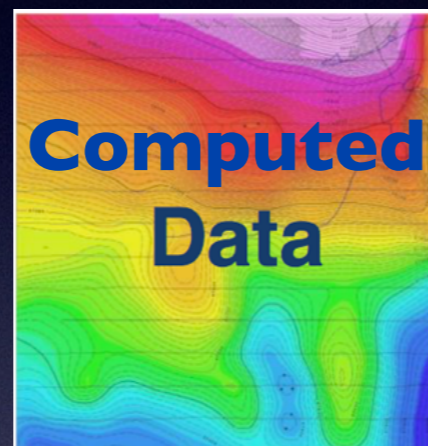
INVERSION PROCESS



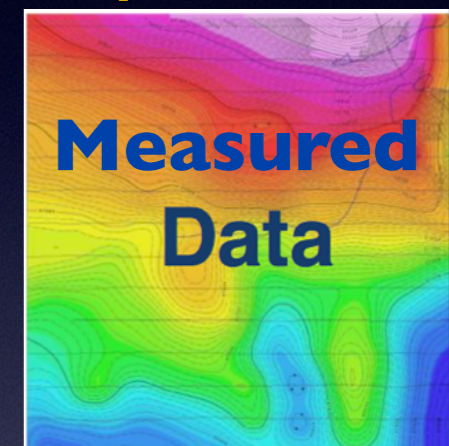
Forward model



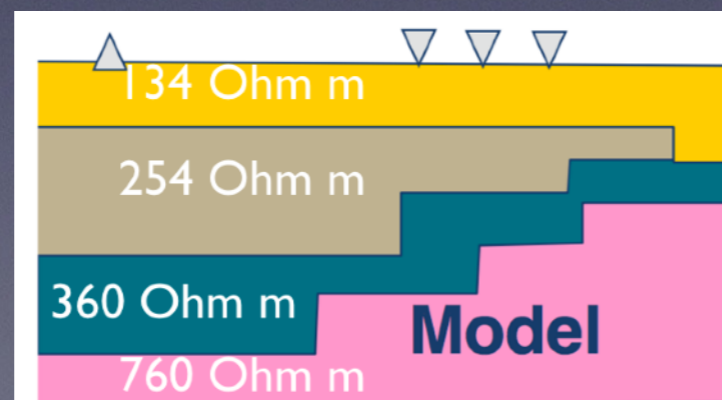
Input data



Comparing differences

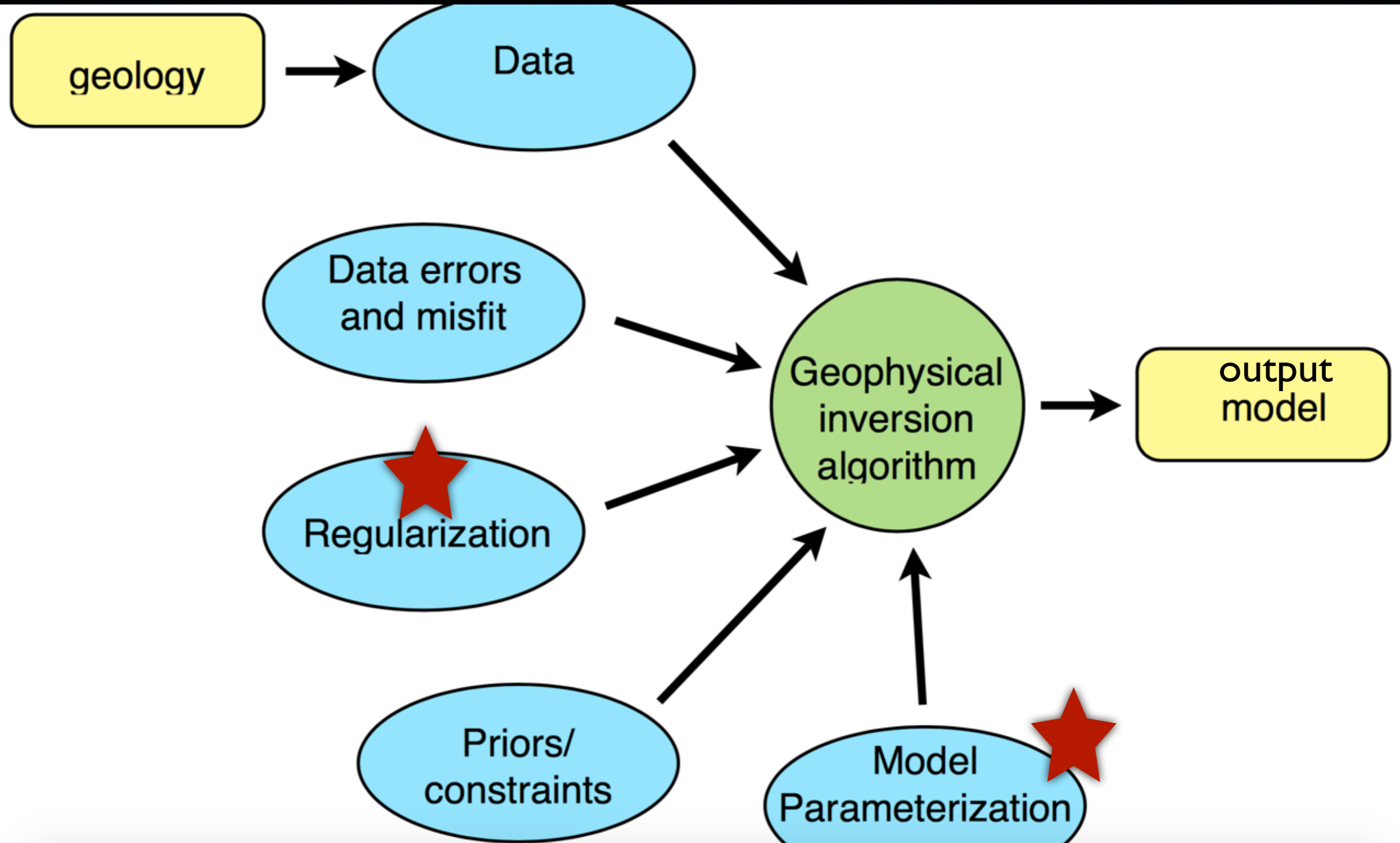


Find the model which minimize the differences



Output result

INVERSION PROCESS



Inversion

Regularisation

Essential in ill posed problems

find the smoothest model that fits the data
(e.g. Tikhonov Regularization)



Ockham Razor (William of Ockham XIV sec)

“It is vain to do with more that which can be done with fewer”

«A parità di fattori la spiegazione più semplice è da preferire»



Find the best simplest model that fit the data

Inversion

Regularisation

*Minimise the appropriate objective functions
with several strategies, able to solve the
given conditions*

(linear or not linear)



*(Minimise the difference between observed dataset and
calculated ones)*

Inversion

Parametrisation

The problem of identify parameters need an parameters optimisation (Parametrisation).

1) Discretize the space

2) Choose the physical parameters for the starting model

$$d = F(m)$$

Direct model

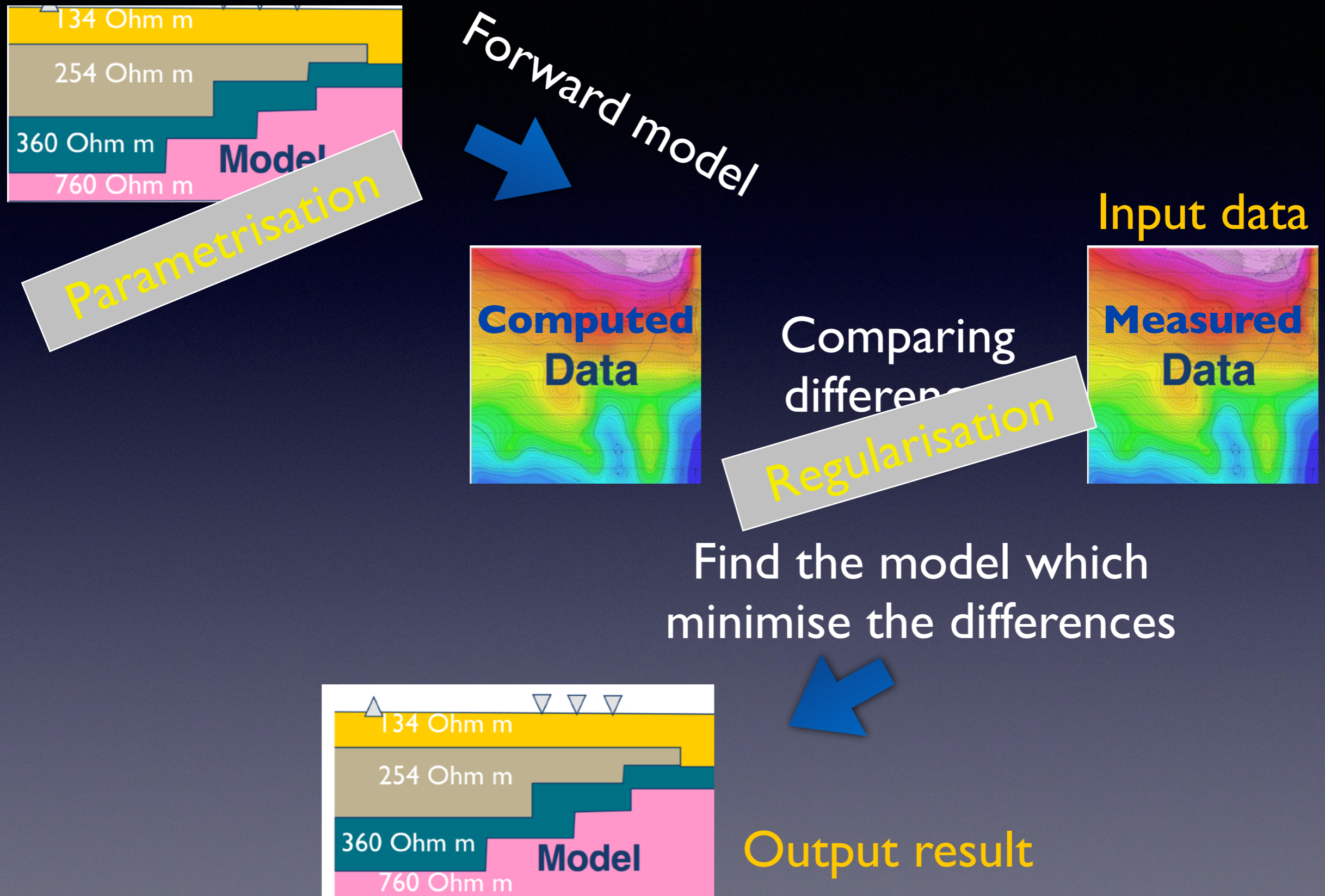
$$m = (m_1, m_2, \dots, m_N)$$

m_i models parameters
(usually positive)

e.g.

We can parametrize in $\log(m)$ to avoid negative values

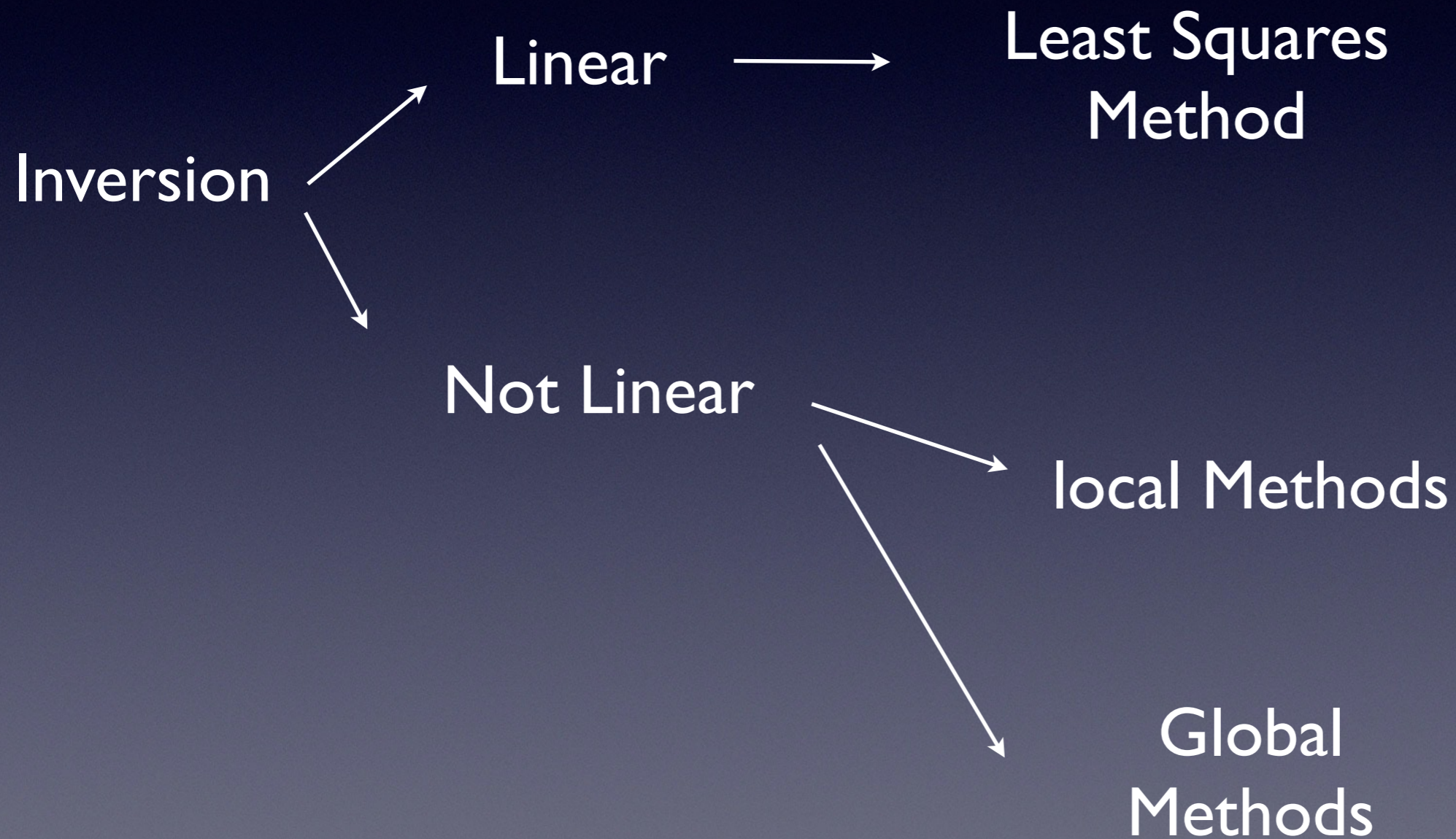
Inversion



EXTRA
MATERIAL

Inversion

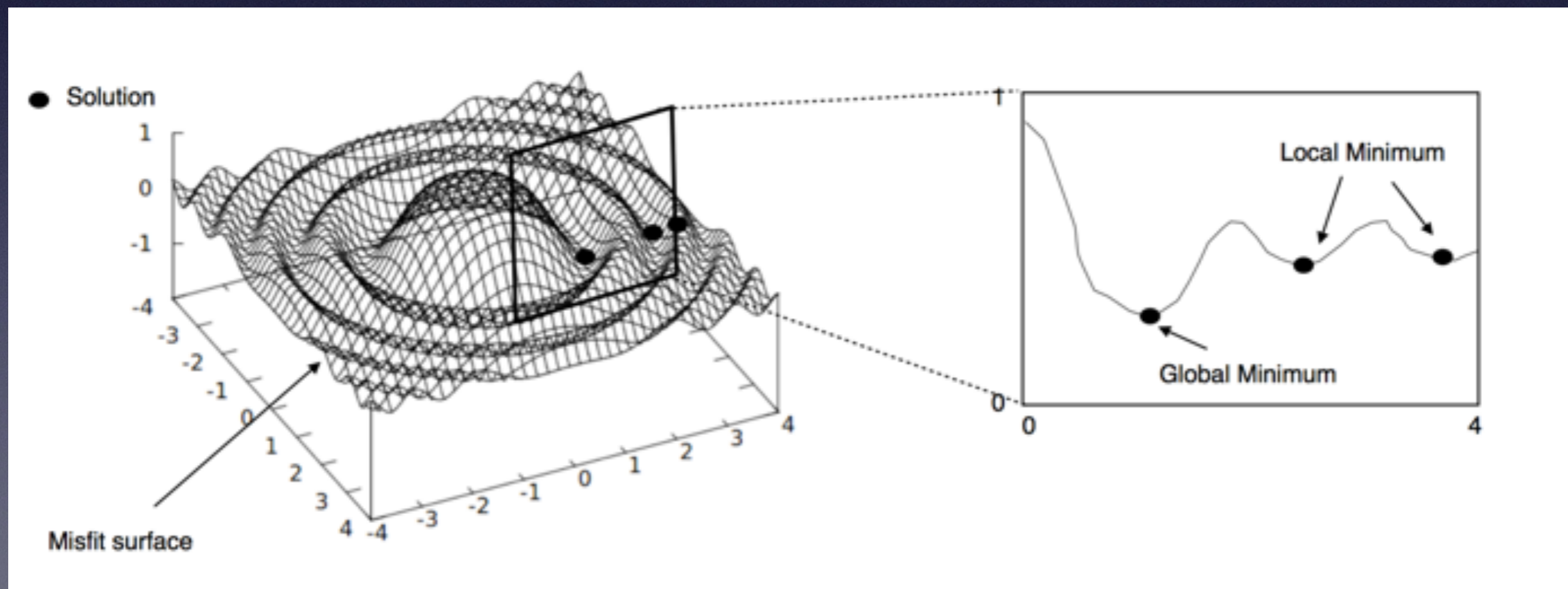
Find the model who minimise data misfit....



Inversion

Inversion find the optimum value of the *misfit*, a functional of the difference between observed data and the synthetic one calculated from a starting model correctly parameterised.

The minima of the *misfit surface*, as a function of the parameters model, are the solutions.

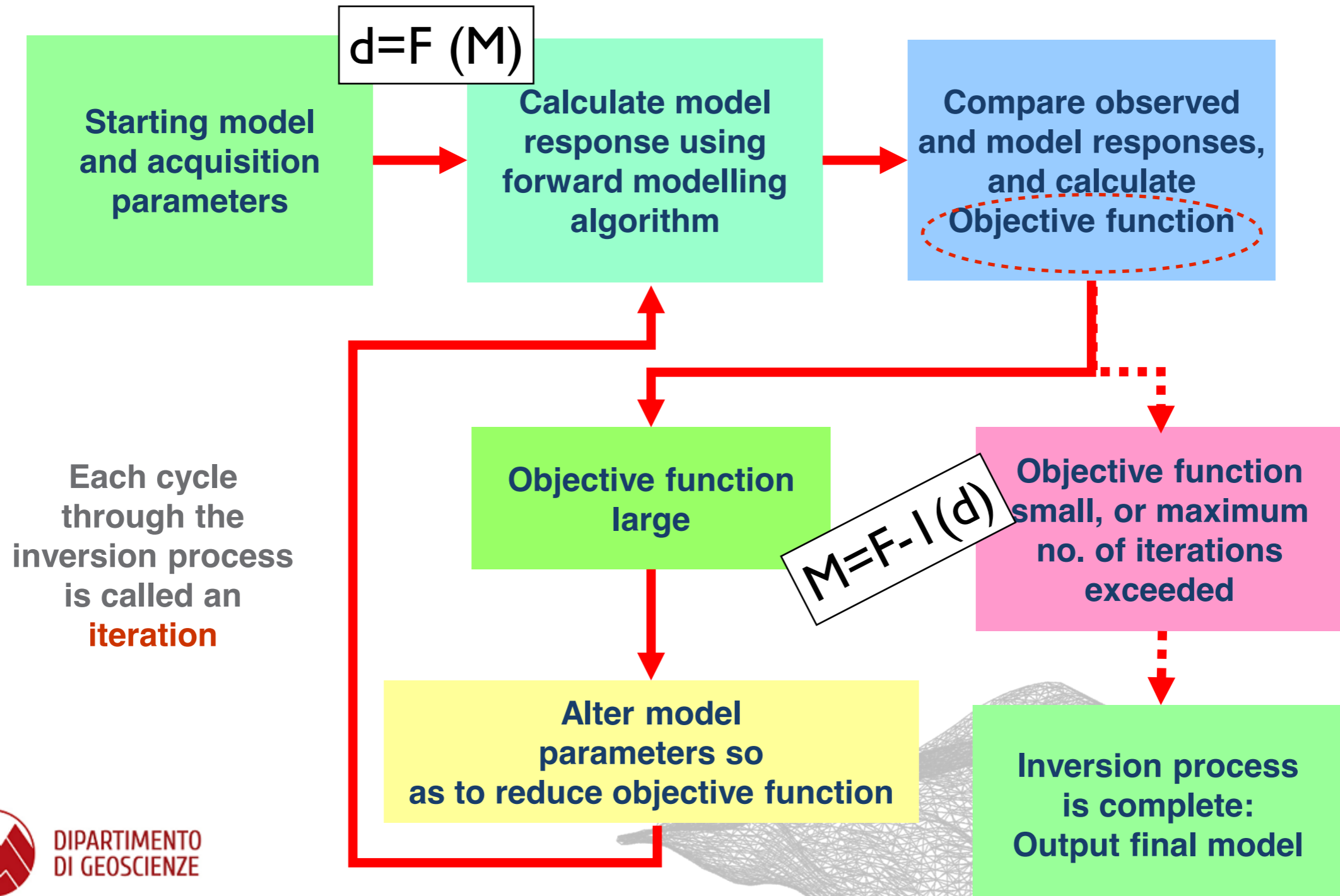


Solutions can be local or global (e.g. Montecarlo methods)

Find the model who minimise the data misfit

Iterative Process

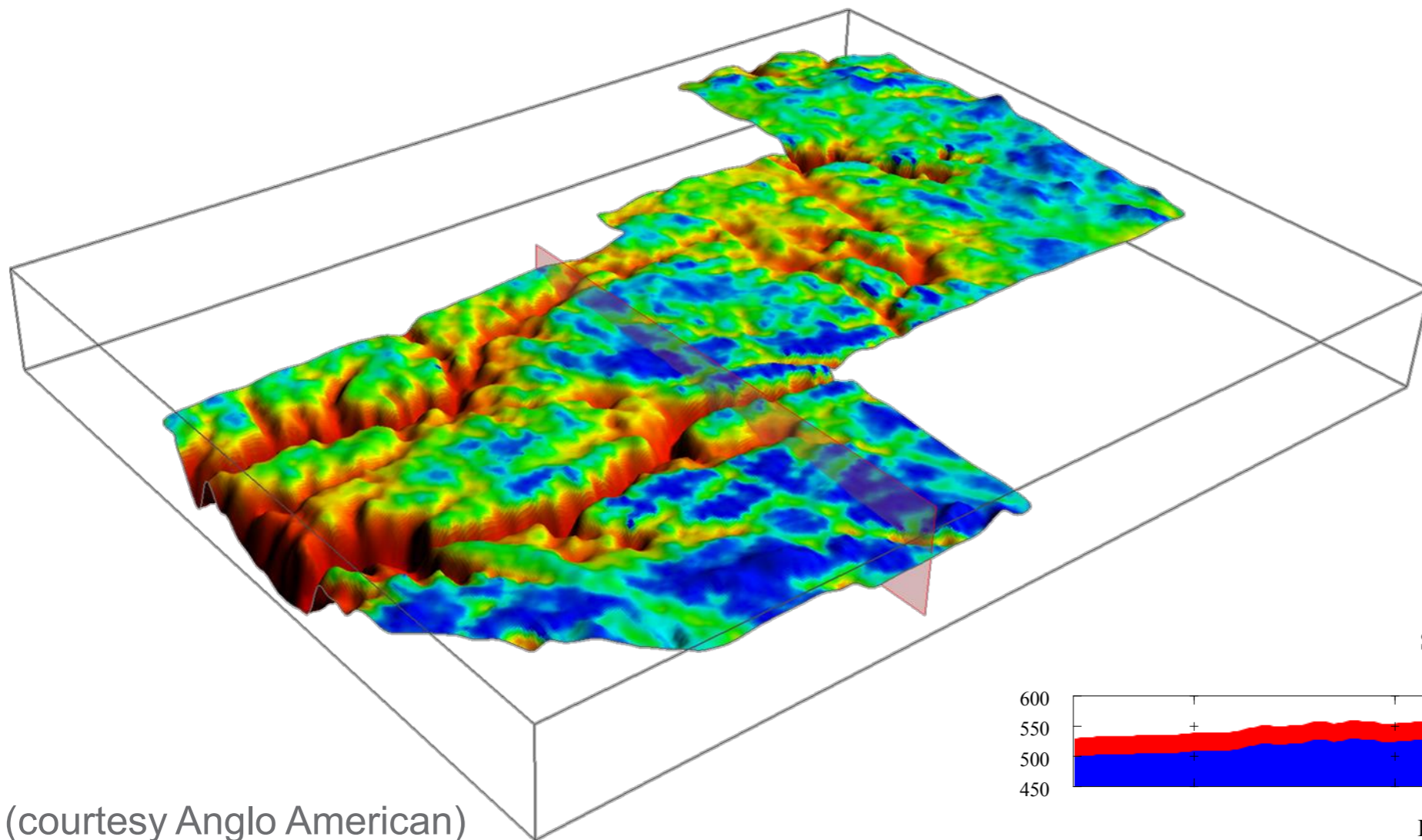
Iterative inversion



Inverse Modelling

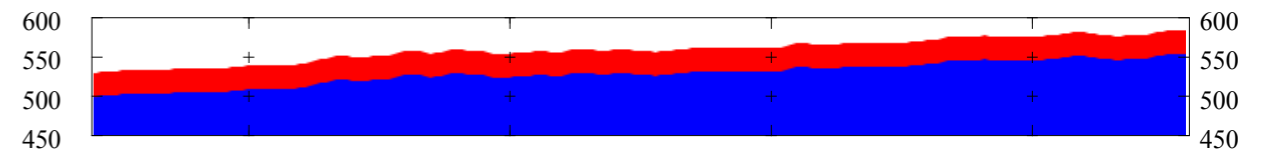
Lithology Based Modelling

- Provide physical properties (single value or distribution) for each lithology and adjust the geometry to fit the data.

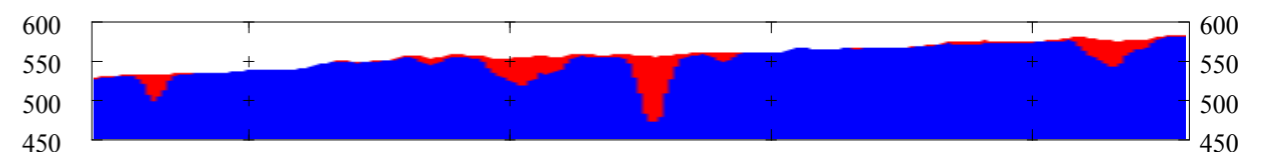


**RESULT IS A
GEOLOGICAL
MODEL**

Starting Model



Inverted Model



(courtesy Anglo American)



Mira Geoscience
...modelling the earth

Which Precision ??

The production manager asked a geologist, engineer, and geophysicist what $2 + 2$ was.

The geologist thought for a bit and then said “*somewhere between 3 and 5*”.

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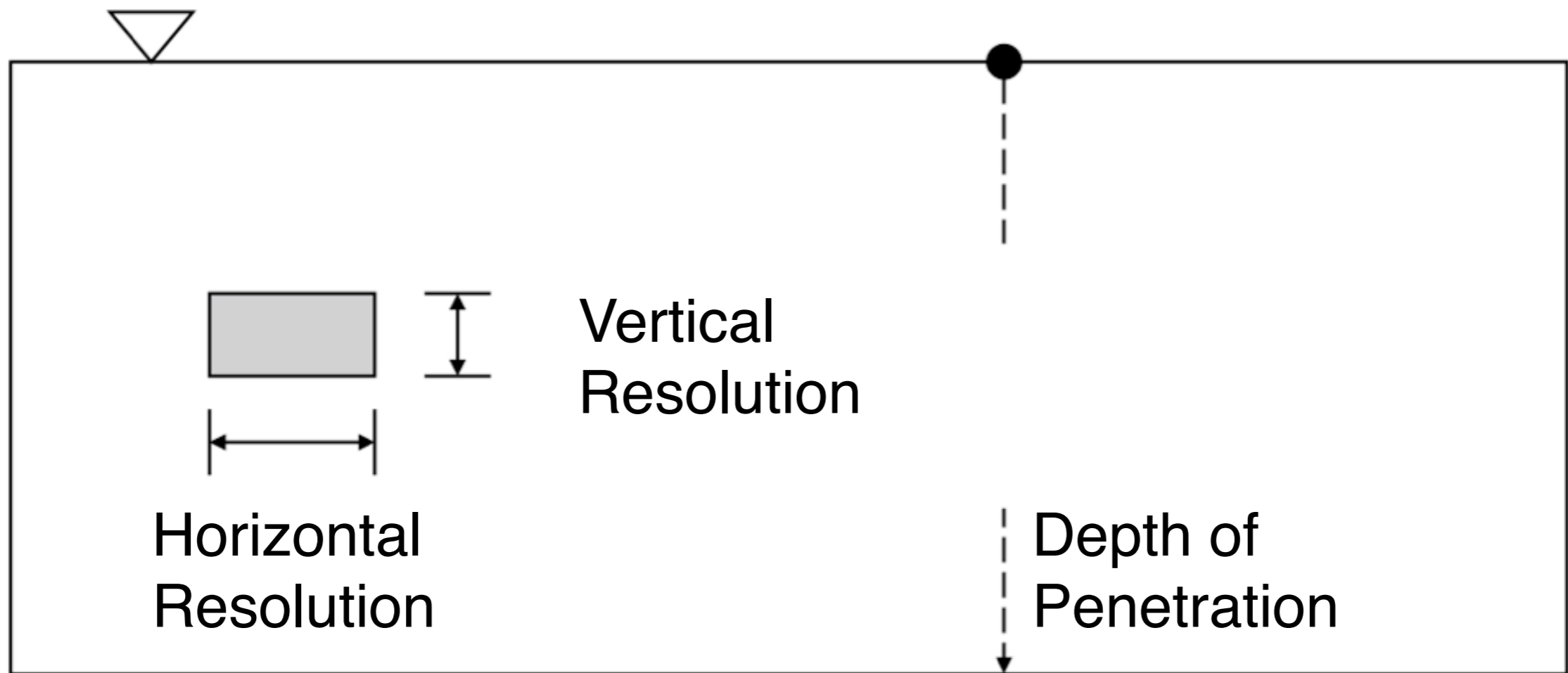
The engineer fiddled with a calculator and said “*3.9999999*”.

The geophysicist looked her in the eye and asked “*what answer do you want*” ??

Concept of Resolution

In the geophysical problem

The method should have enough **RESOLUTION** and **PENETRATION**



Resolution vs Penetration

Is always a compromise !

*You can go deep with poor resolution,
or have details imaging but in the near surface*

Elements of signal processing

Elements of signal processing

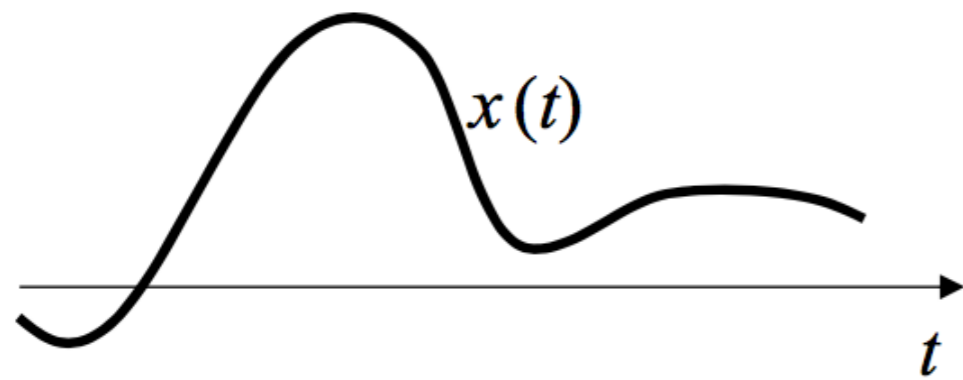
Every analog signal must be:

- recorded (then truncated)
 - sampled

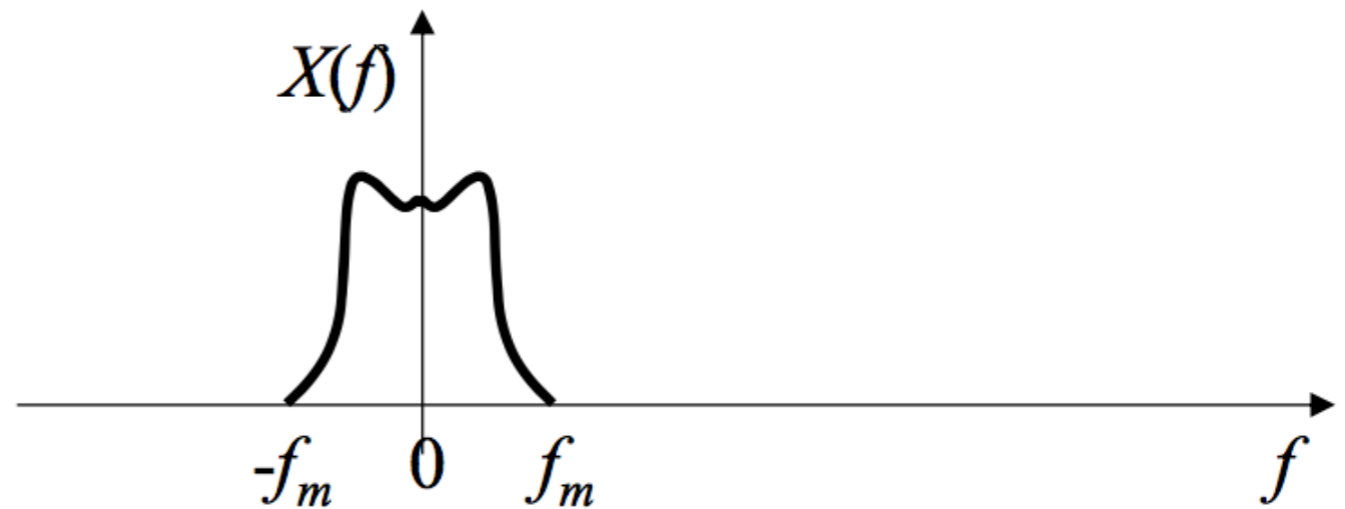
Elements of signal processing

The **ALIASING** problem

Time domain

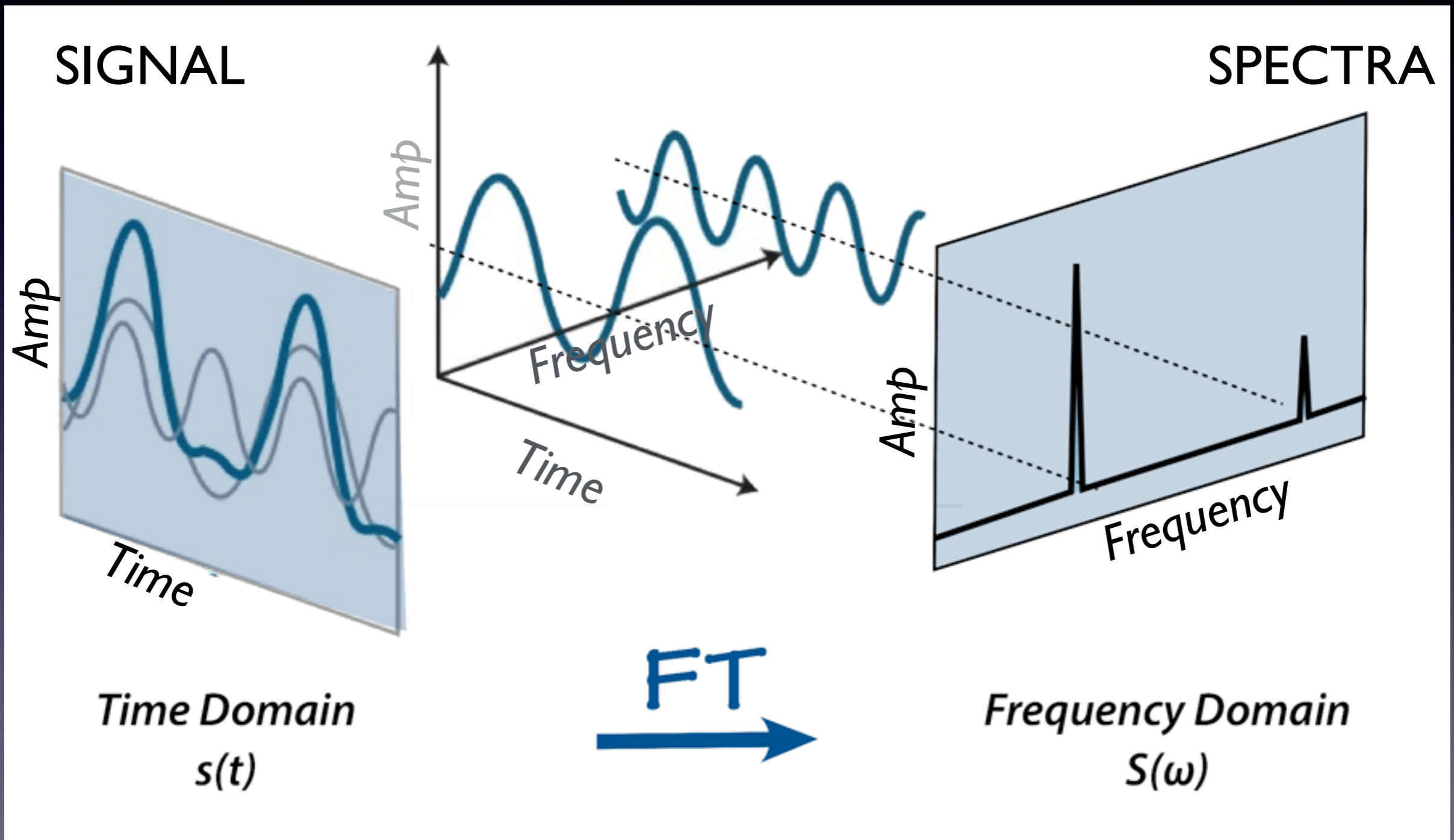


Frequency domain



Fourier time-frequency transform

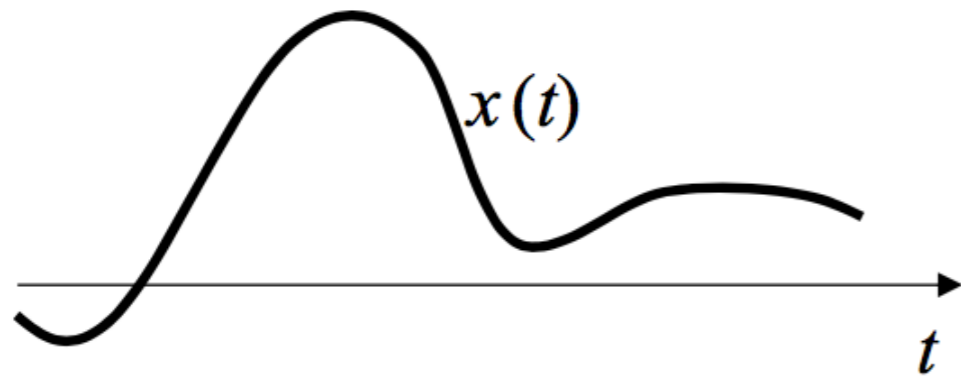
every signal can be decomposed in a sum of sinusoidal with different frequencies and amplitudes



Ideal Sampling

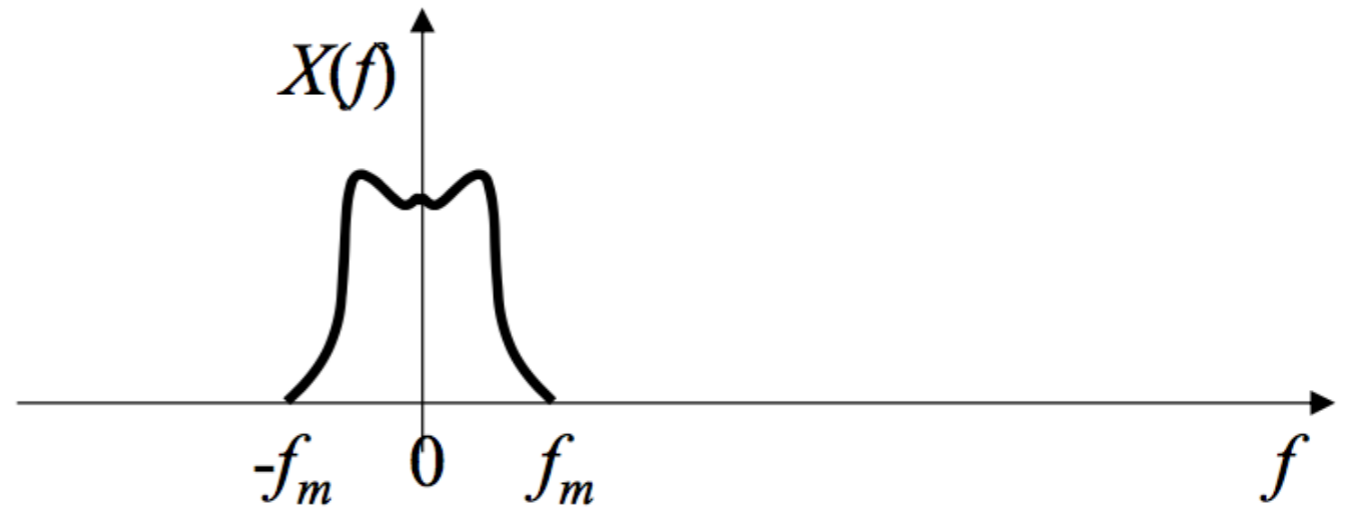
TIME DOMAIN.

Function $X(t)$

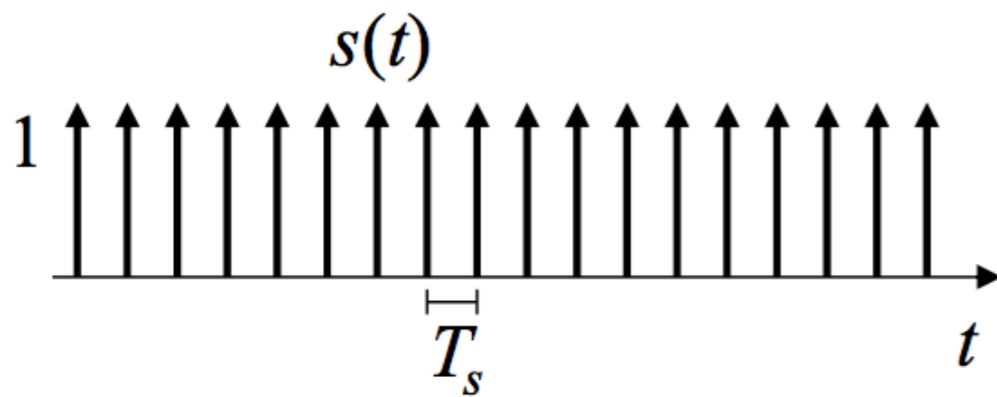


FREQUENCY DOMAIN

FFT transform

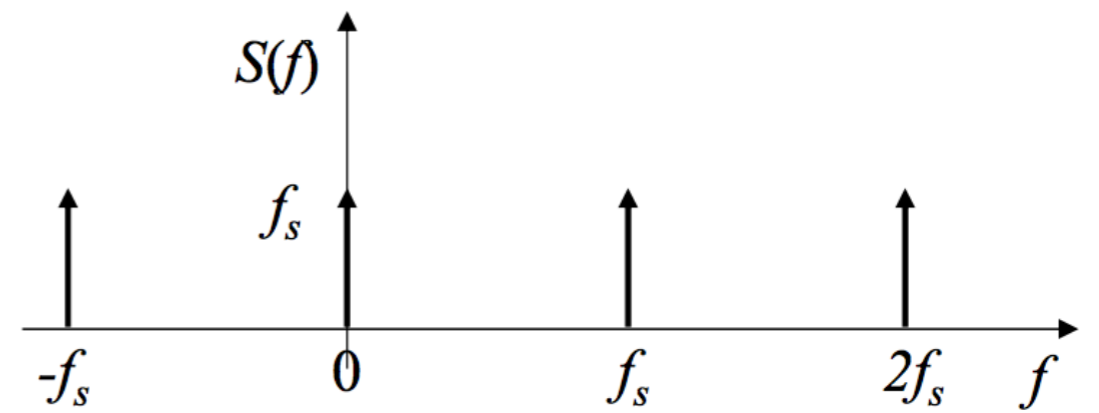


Impulse train $s(t)$



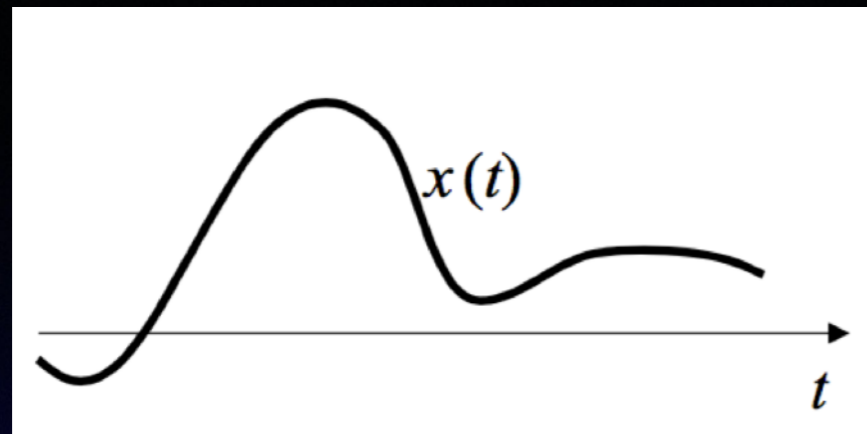
$T_s =$ sampling interval

Impulse spectra $s(f)$



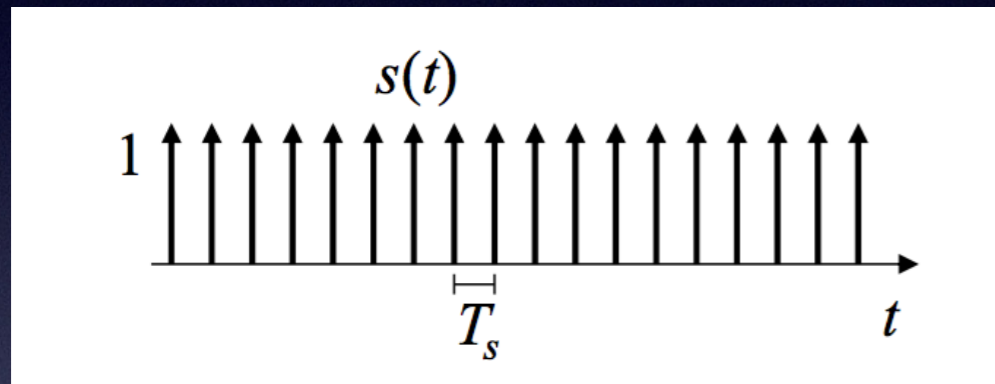
$f_s = 1/T_s$ sampling frequency

SAMPLING ideal

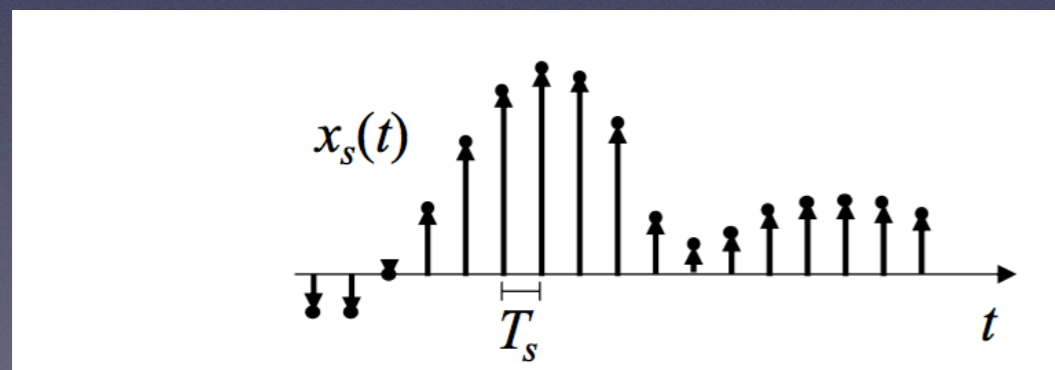


Signal $x(t)$

Multiplication



Impulse train $s(t)$ with
Sampling T_s



Function $x_s(t)$ SAMPLED

SAMPLING (ideal)

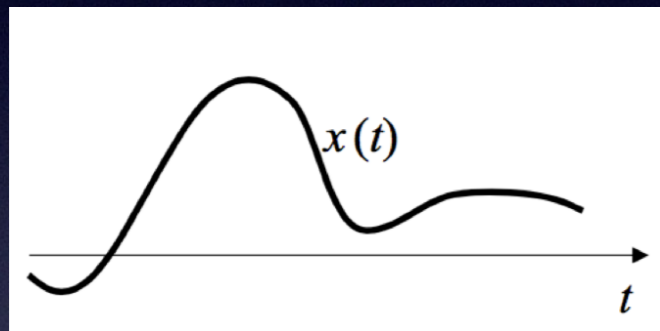
Convolution theorem

To algebraic multiplication in time domain corresponds to a convolution in frequency domain

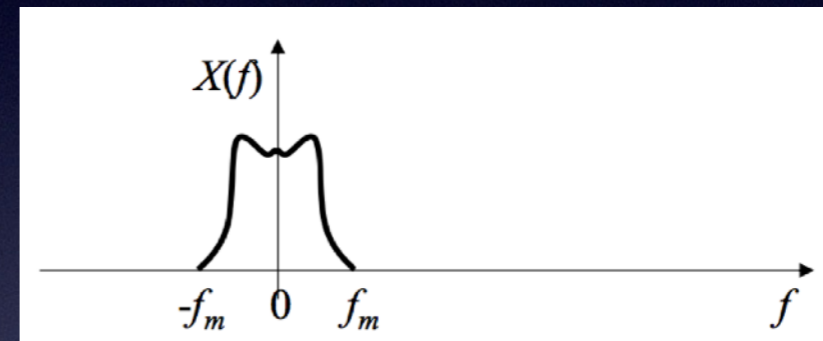
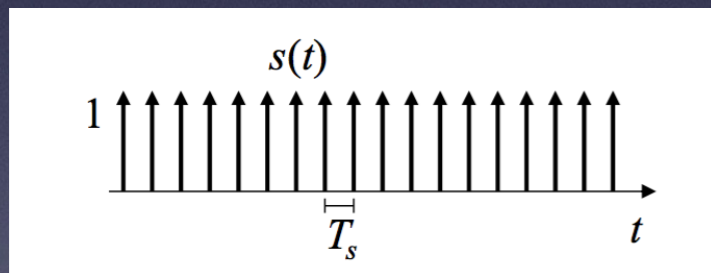
$$x(t) \cdot s(t) \Leftrightarrow X(f) * S(f)$$

Time domain

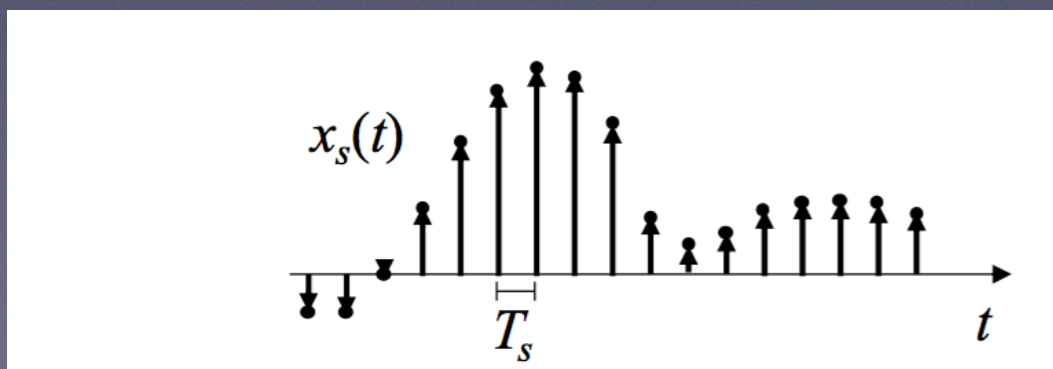
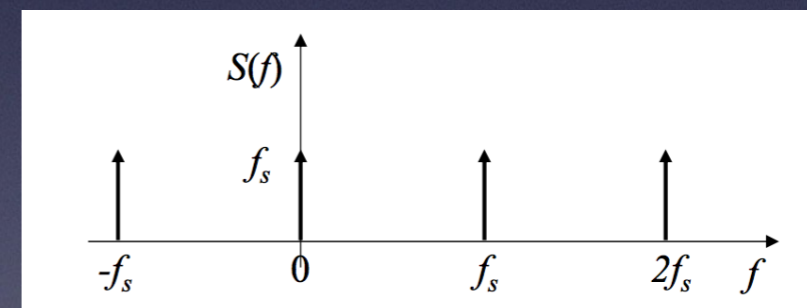
Frequency domain



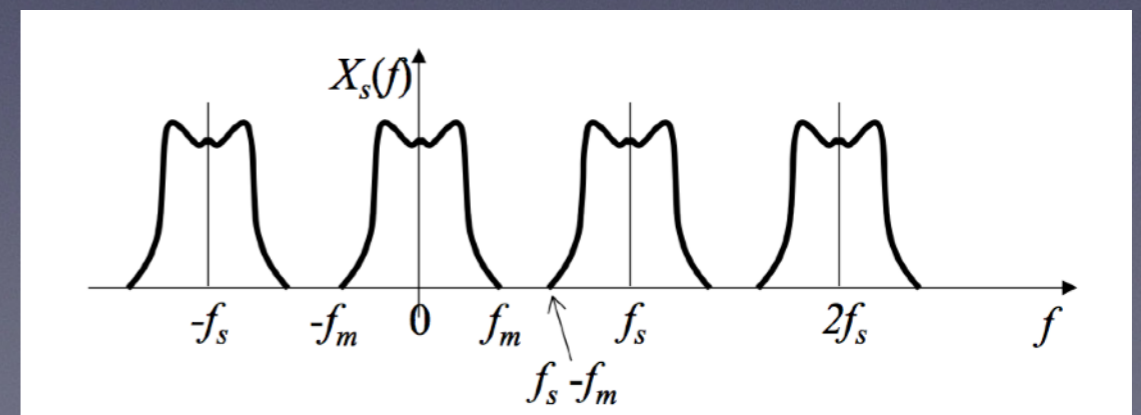
Multiplication \times



Convolution $*$

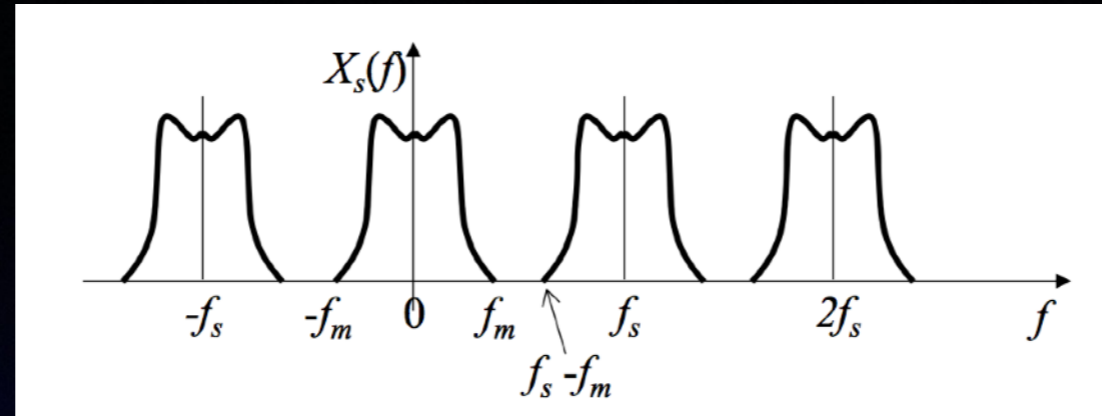


Sampled signal



Spectra of sampled signal

SAMPLING



f_s = Central frequency

f_m = Maximum frequency

To avoid overlap between functions, the sampling must be $\geq 2f_m$

Sampling THEOREM (or Nyquist theorem)

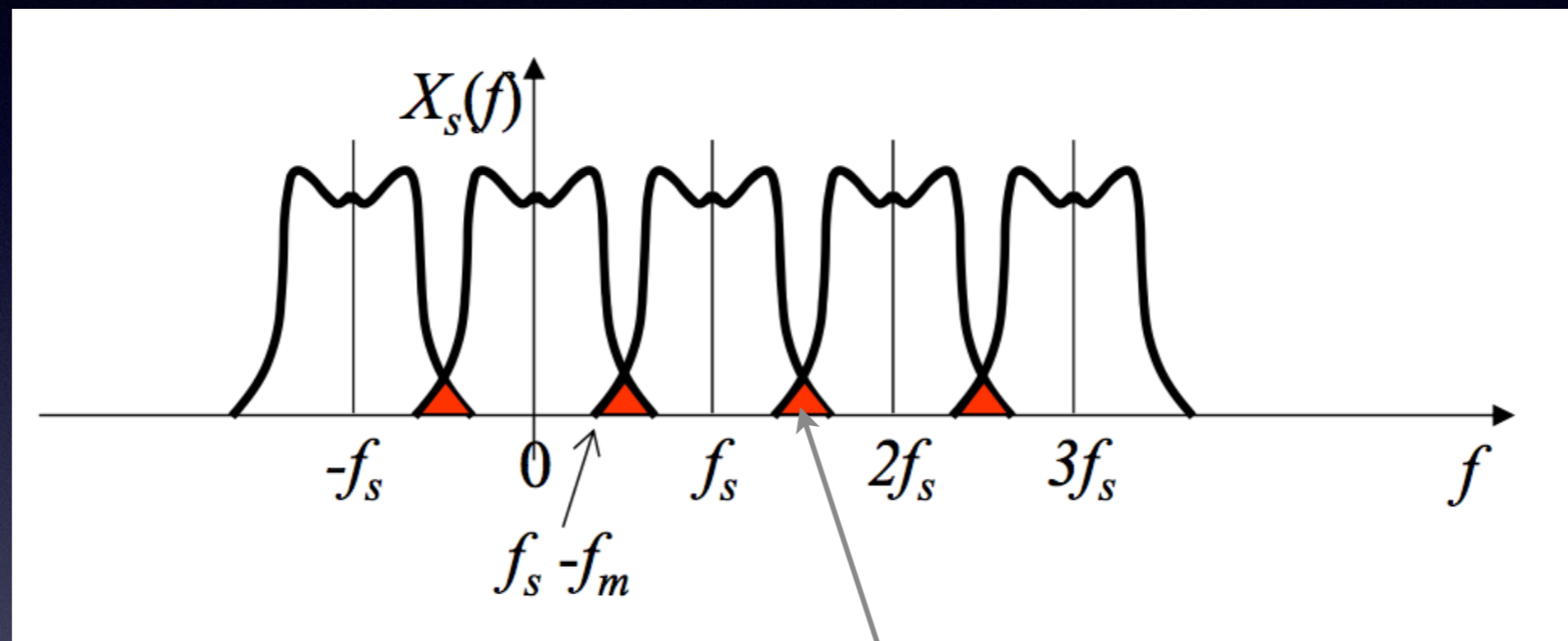
$$f_s = \frac{1}{T_s} \geq 2f_m$$

Given an analog signal $s(t)$, having maximum frequency F_m , the MINIMUM sampling frequency (f_s) must be $> 2 F_m$

SAMPLING

If $F_s < 2F_m$

You cannot retrieve correctly the starting signal $s(t)$



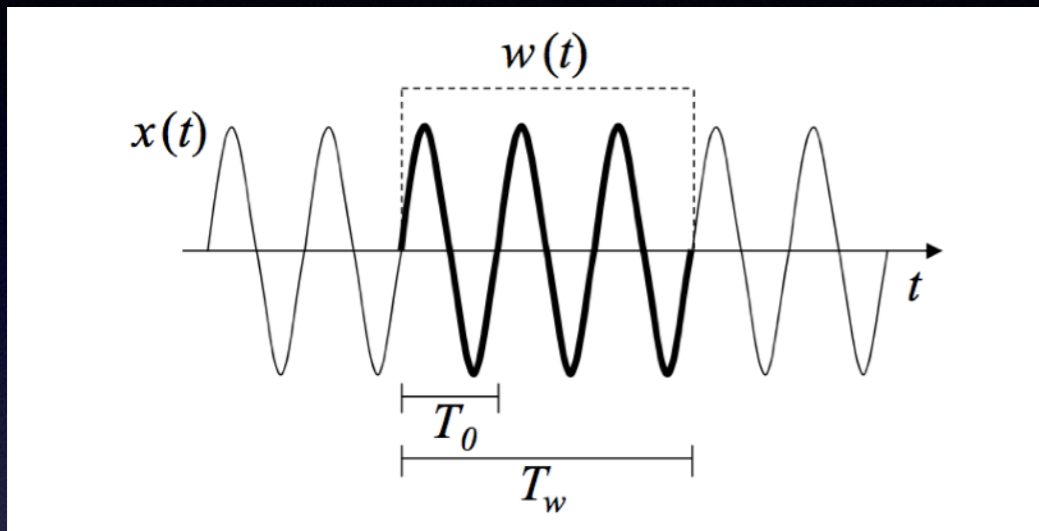
Because you have interferences due to overlapping

SAMPLING

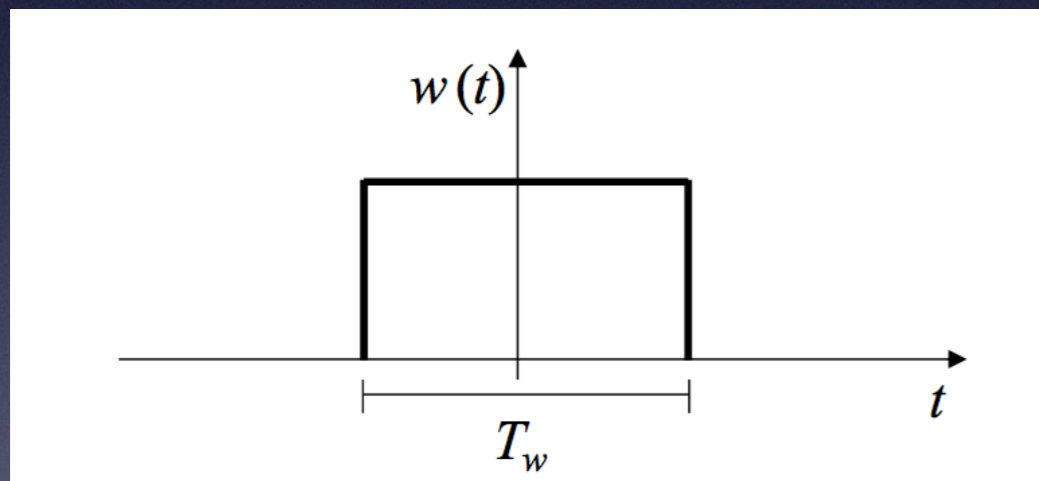
CUTTING PROBLEM

finite recording

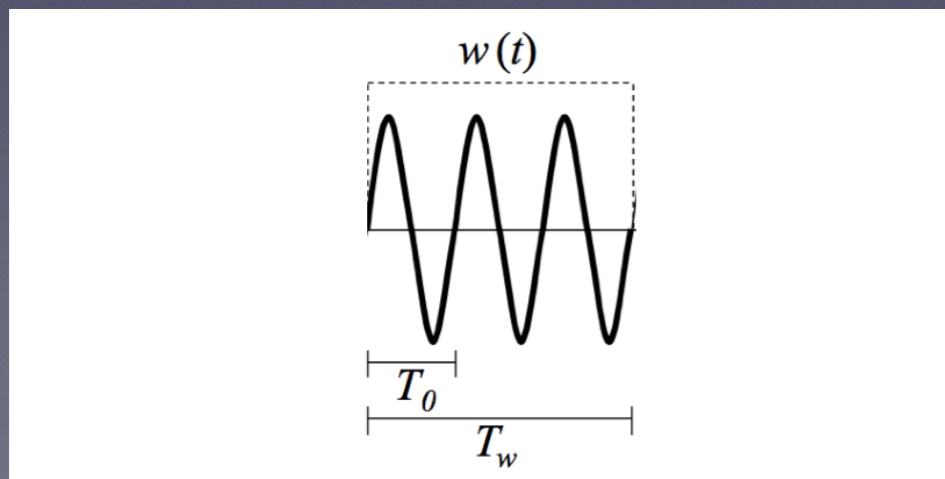
Multiplication



Signal continuous $x(t)$
($T_0 = \text{period}$)



Recording Window
With length ' T_w '



Signal $x(t)$
Cutter (recorded)

||

CUTTING PROBLEM

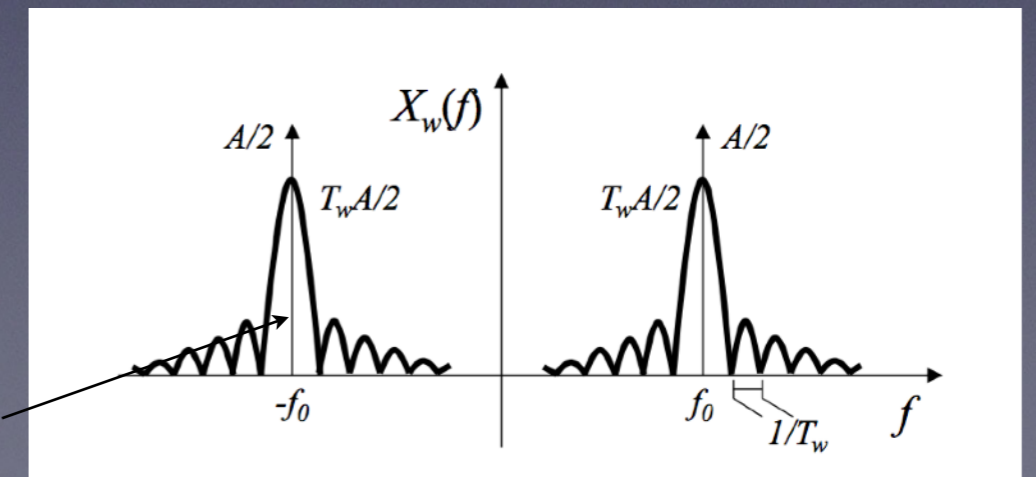
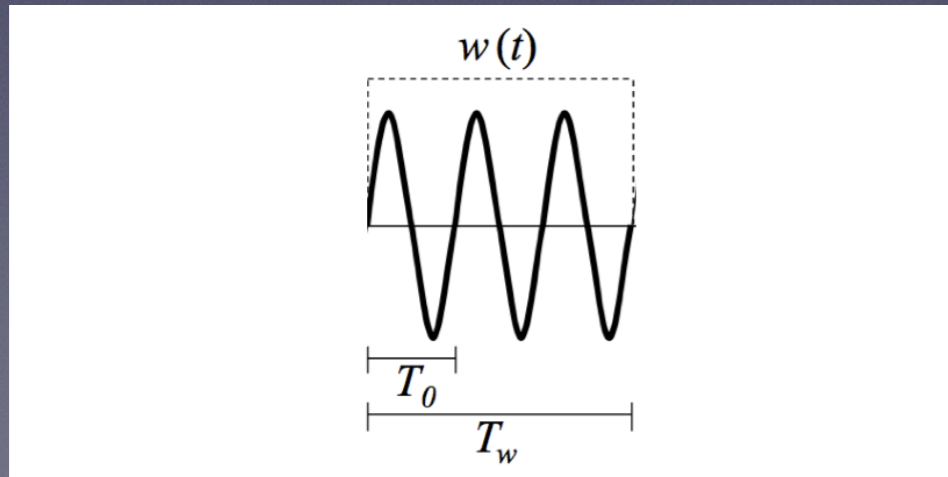
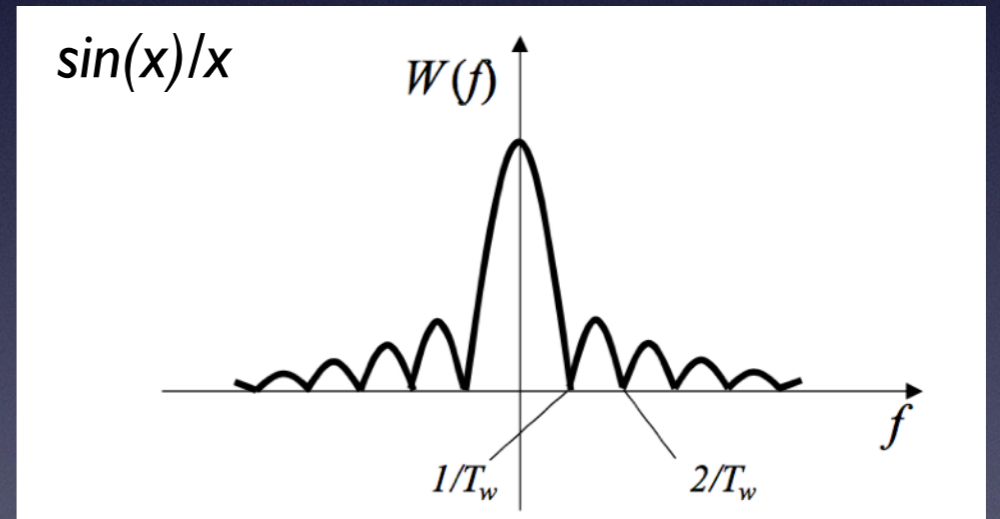
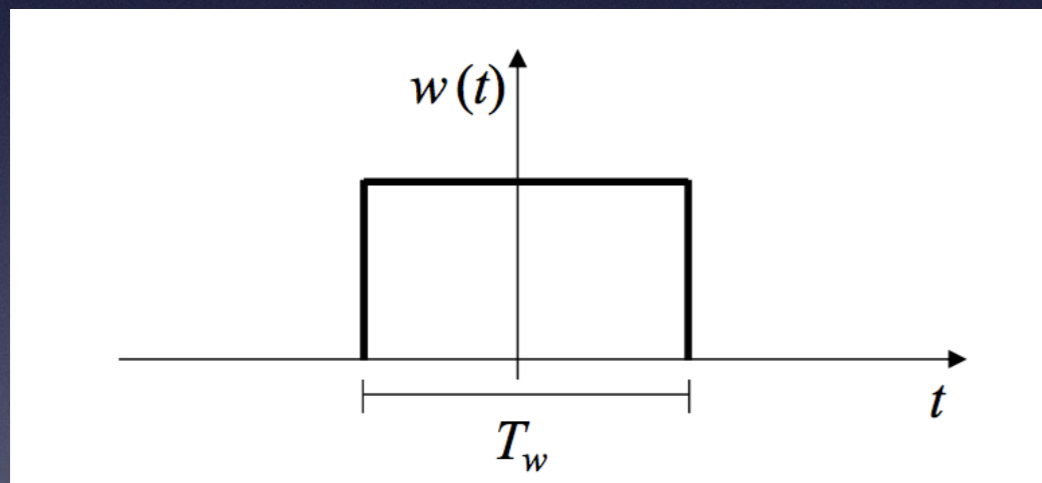
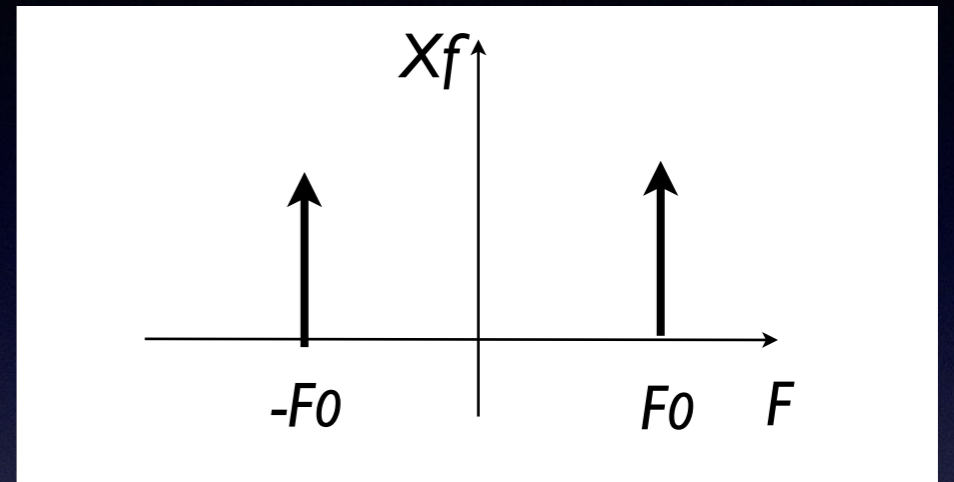
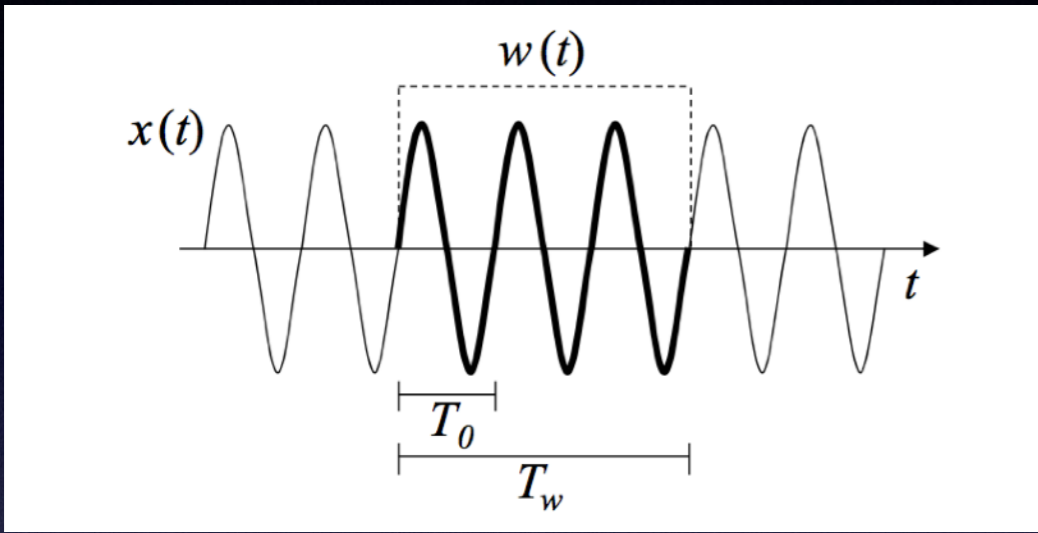
finite recording

Time Domain

Frequency domain

Multiplication

Convolution



Frequency Dispersed

CUTTING PROBLEM

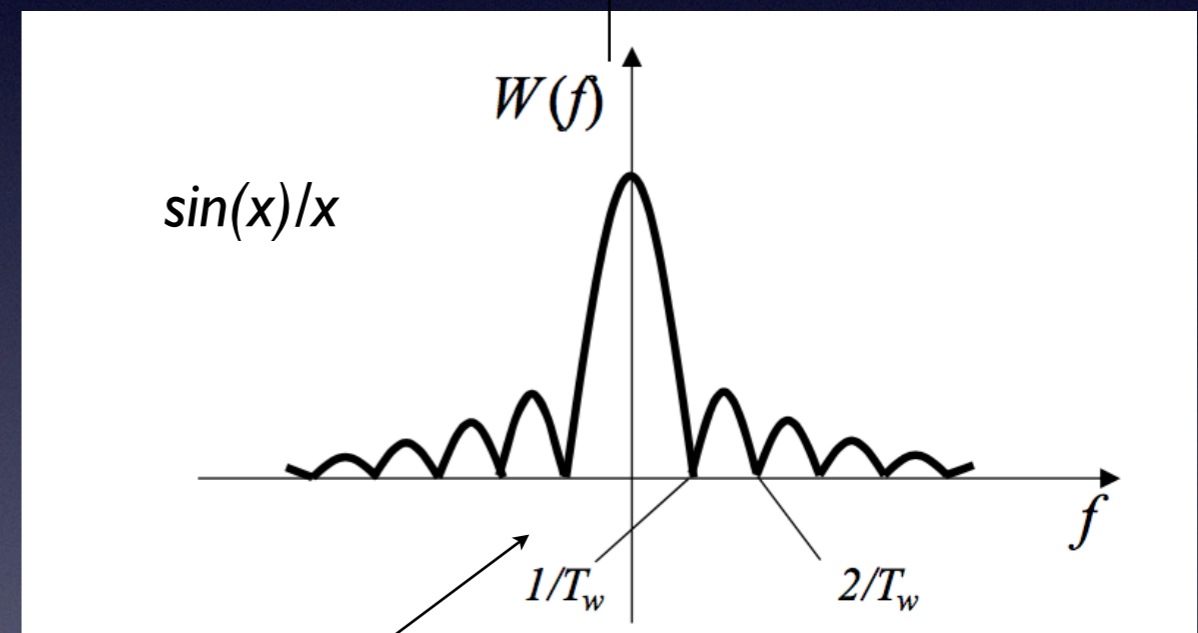
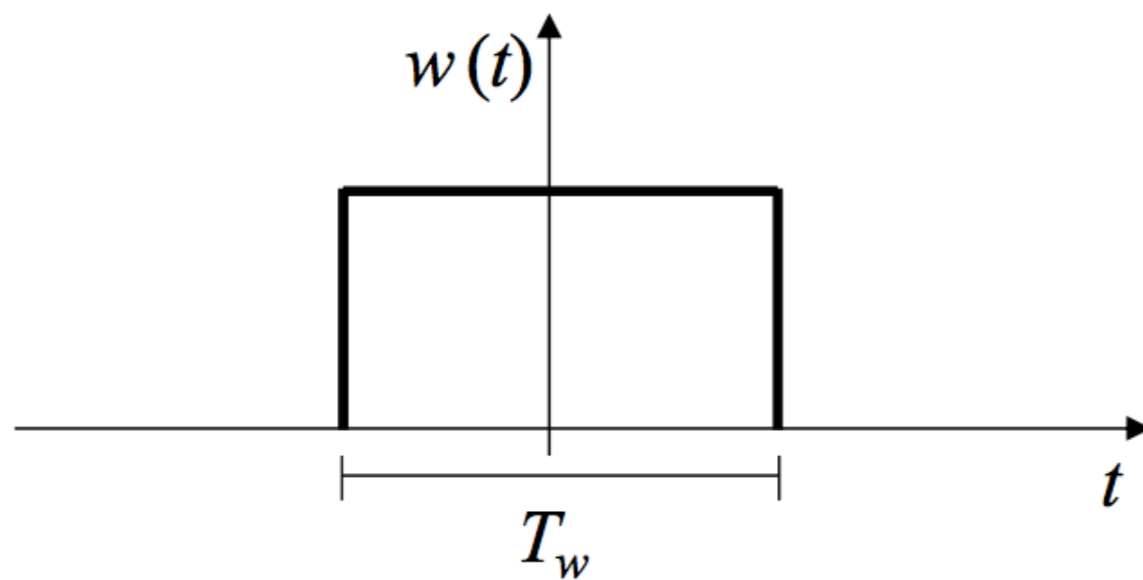
finite recording

Time domain

Frequency domain

Principle of indetermination in time/frequency

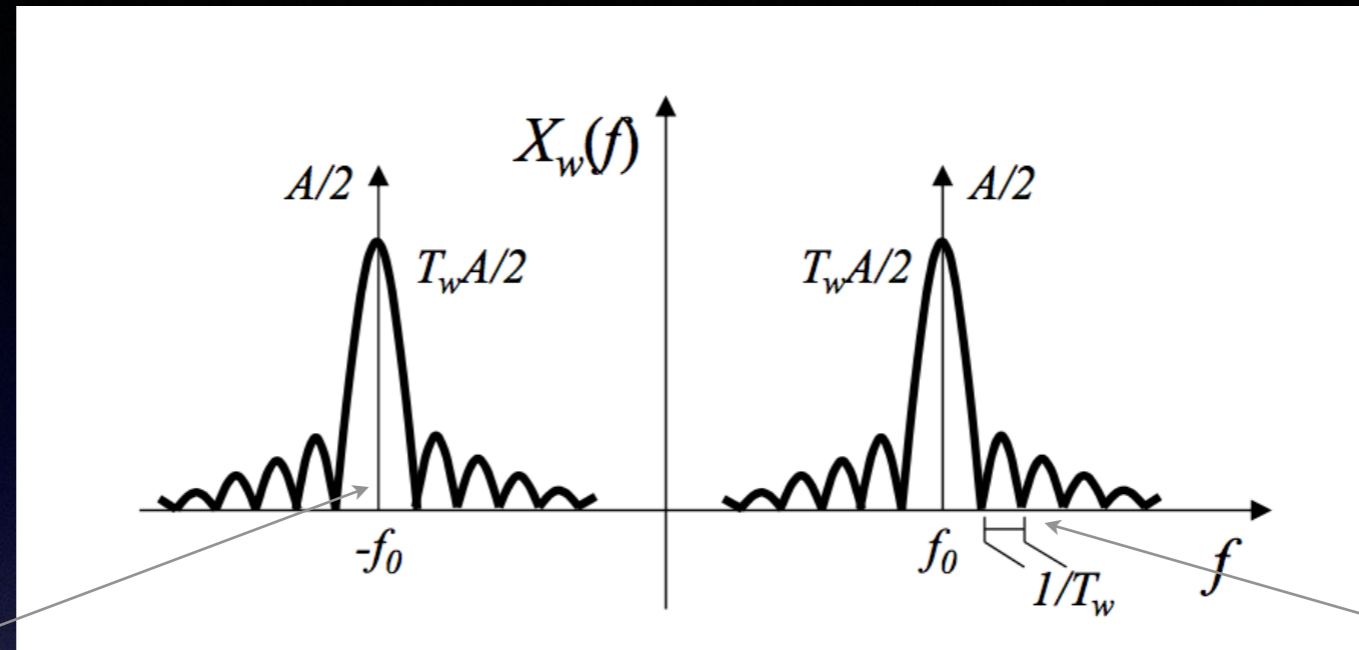
$\gg T_w \ll$ dispersion



*Dispersione
in Frequenza*

CUTTING PROBLEM

finite recording



*Frequency
Dispersion*

*Function of
 T_w
(time of recording)*

Frequency dispersion depends on the window recording T_w

$\gg T_w \ll$ dispersion

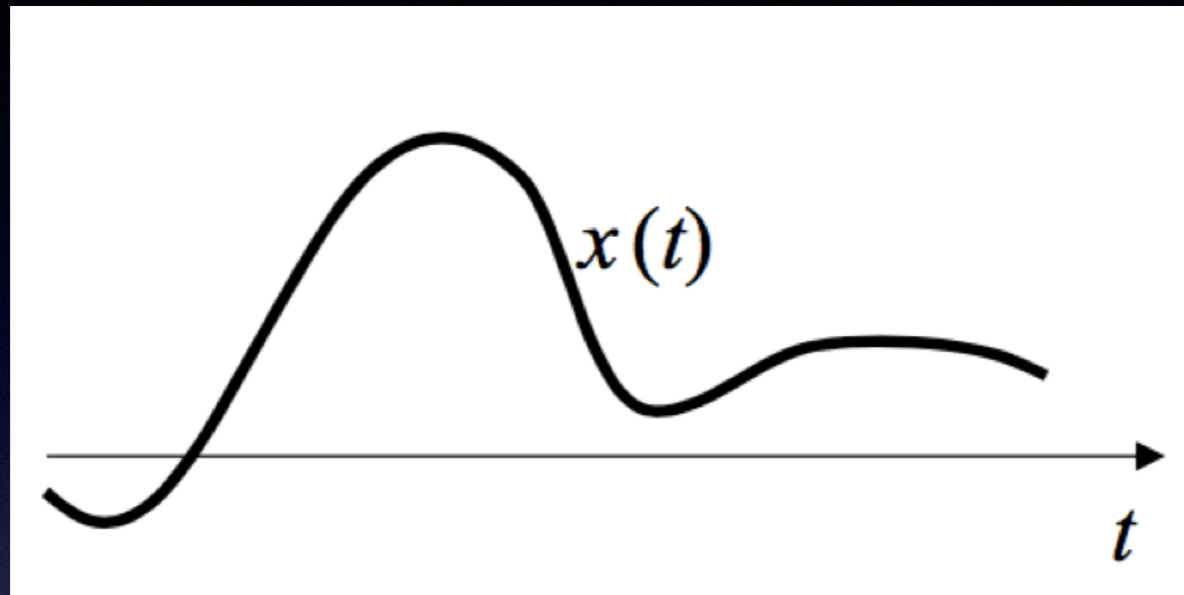
Rule of thumb

$$T_w > 8 T_{\max}$$

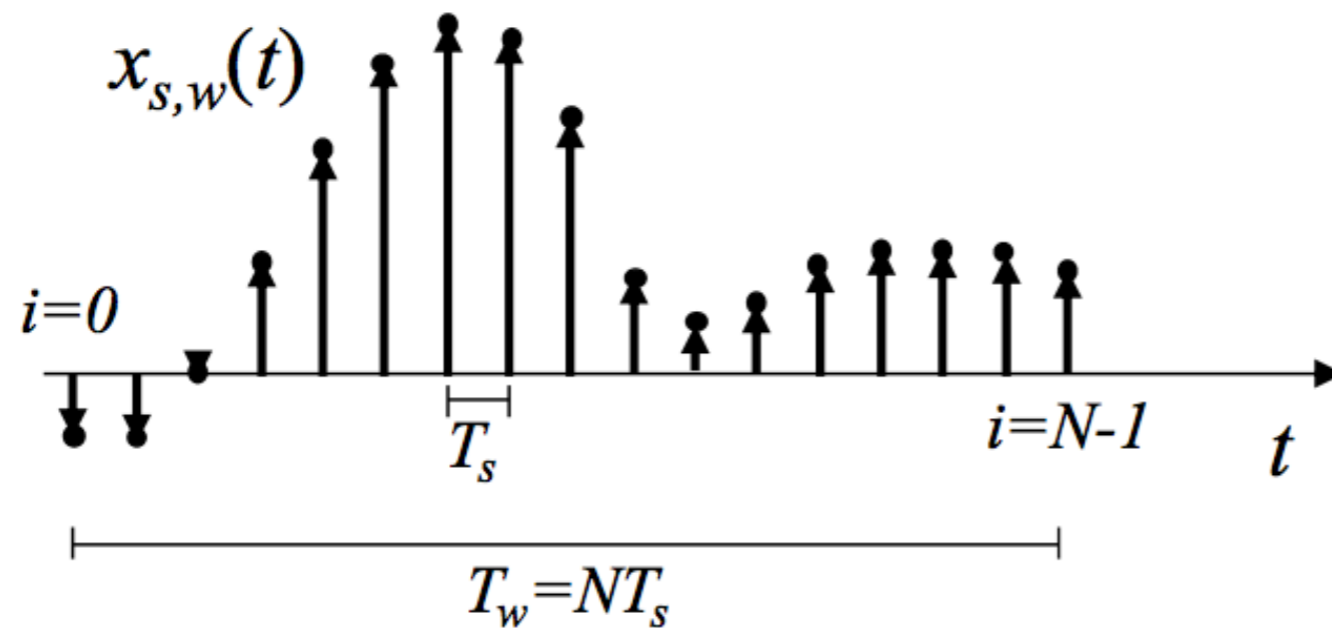
Recording time should be 8 times the Maximum Period of the signal

SAMPLING

Signal sampled and cutted



Continuous signal $x(t)$



Signal
Sampled with step T_s
And cutted with
period T_w
 $T_w = NT_s$

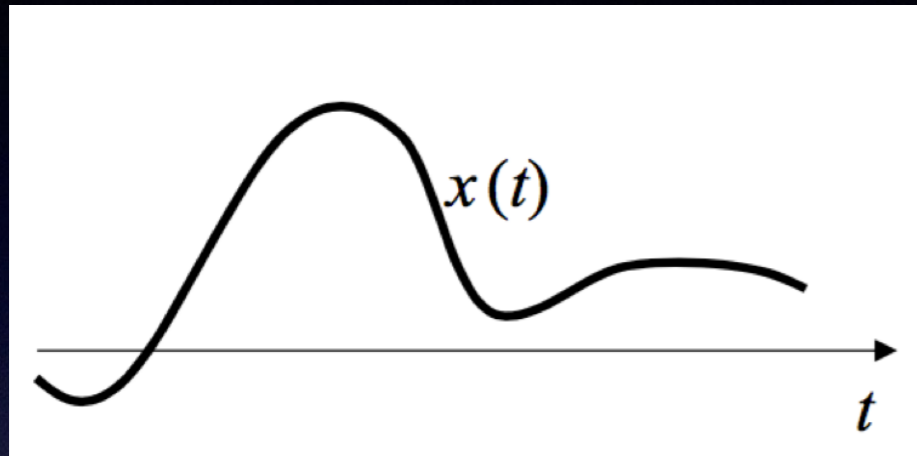
$N > 8$

SAMPLING

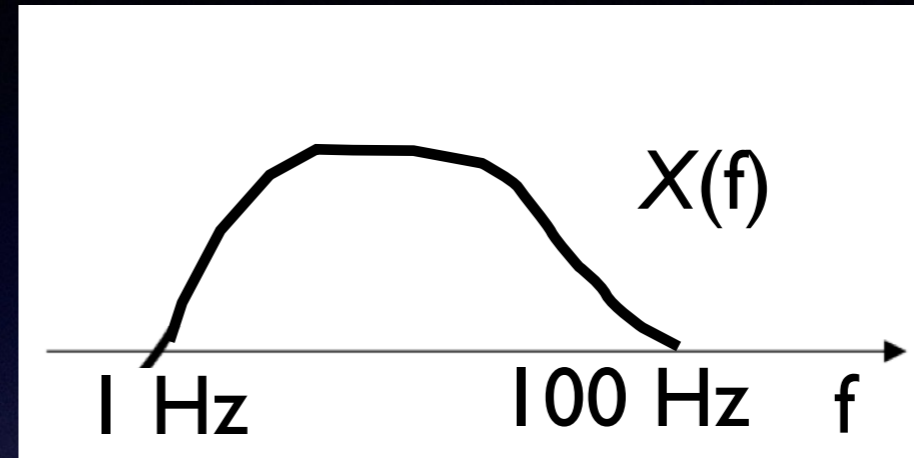
EXAMPLES

T = PERIOD
f = FREQUENCY

TIME DOMAIN



FREQUENCY DOMAIN



$F_{\min} = 1 \text{ Hz}$

$F_{\max} = 100 \text{ Hz}$

SAMPLING (Aliasing)

$$f_s = \frac{1}{T_s} \geq 2f_m$$

$$f_s = 2 f_{\max} = 200 \text{ Hz}$$

$$T_s = 1/200 = \underline{0,005 \text{ s}}$$

CUTTING

$$T_w > 8 T_{\max}$$

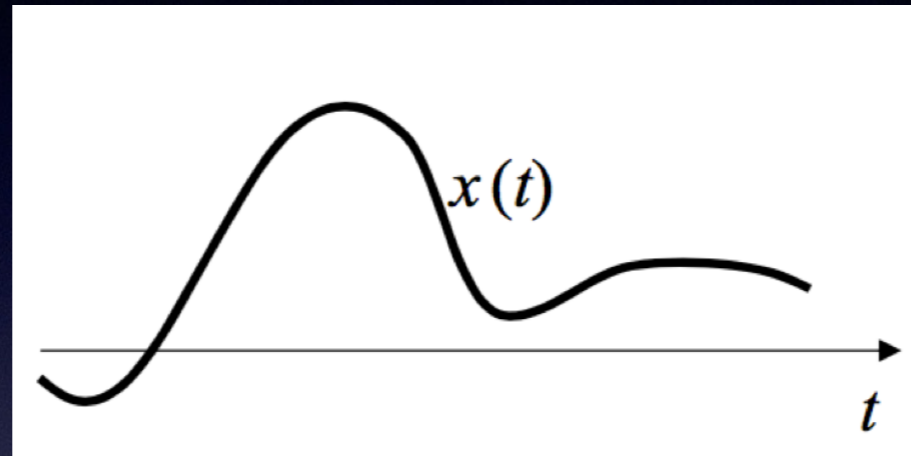
$$T_{\max} = 1/f_{\min} = 1/1 = 1 \text{ s}$$

$$T_w = 8 T_{\max} = \underline{8 \text{ s}}$$

SAMPLING EXAMPLES

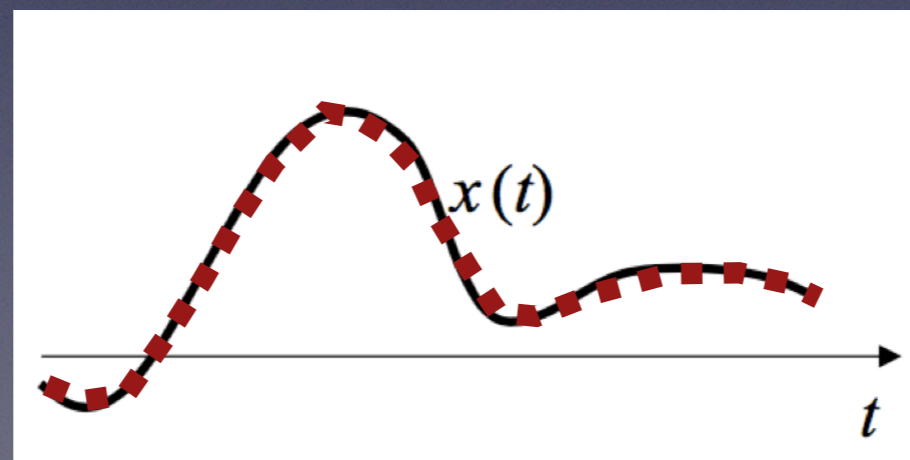
T = PERIOD
f = FREQUENCY

TIME DOMAIN signal



*Signal having min frequency
of 1 Hz (max period of 1 s)
and
max frequency of 100 Hz
(Min period of 0,01 s)*

If sampled at 200 Hz for 8 seconds



It is well reconstructed !

SAMPLING EXAMPLES

Even more easy...

When you are sending a WA vocal message



Voice is usually between 300Hz and 3400 Hz
($3 \cdot 10^{-3}$ seconds and $3 \cdot 10^{-4}$ seconds)

Your phone **MUST** have a sampler recorder that records
your voice at **7800 Hz** (and at least for 10^{-2} seconds)