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Surface-wave method for near-surface characterization: a tutorial

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ABSTRACT

Surface-wave methods (SWMs) are very powerful tools for the near-surface characterization of sites. They can be used to determine the shear-wave velocity and the damping ratio overcoming, in some cases, the limitations of other shallow seismic techniques.

The different steps of SWM have to be optimized, taking into consideration the conditions imposed by the small scale of engineering problems. This only allows the acquisition of apparent dispersion characteristics: i.e. the high frequencies and short distances involved make robust modelling algorithms necessary in order to take modal superposition into account.

The acquisition has to be properly planned to obtain quality data over an adequate frequency range. Processing and inversion should enable the interpretation of the apparent dispersion characteristics, i.e. evaluating the local quality of the data, filtering coherent noise due to other seismic events and determining energy distribution, higher modes and attenuation.

The different approaches that are used to estimate and interpret the dispersion characteristics are considered. Their potential and limits with regard to sensitivity to noise, reliability and capability of extracting significant information present in surface waves are discussed. The theory and modelling algorithms, and the acquisition, processing and inversion procedures suitable for providing stiffness and damping ratio profiles are illustrated, with particular attention to reliability and resolution.

INTRODUCTION

We consider that the aim of a tutorial is to provide the widest and most complete information about a topic, both for experienced users and beginners. Hence, one of our main goals is to give a general overview of the different approaches, possibilities, advantages and limitations of surface-wave methods (SWMs). Moreover, we also discuss in greater detail some specific ‘critical points’ of particular interest to beginners. Of course, a long list of references and some state-of-the-art descriptions are given, but the results of our experience and our points of view are the main themes of this paper. Therefore, we have tried to balance the ‘general’ and the ‘particular’. Some of the aspects that are only sketched in this tutorial are analysed in greater detail in other papers in this issue of *Near Surface Geophysics*.

The characterization of the subsoil using seismic techniques consists of observing a wavefield, measuring the properties of the propagation, and by means of an interpretation procedure, obtaining the distribution of the subsoil properties that influence the propagation: i.e. the deformability and the dissipative properties at very low strain. This can be achieved using different techniques and acquisition geometries, and analysing the propagation of different kinds of waves (P, S, Rayleigh, Love, Scholte, Lamb, Stoneley, etc.) in the subsoil and the related phenomena (reflection, refraction, diffraction, dispersion, etc.). The wavefield is sampled in space and in time, and from the records obtained, the properties of the propagation are inferred and used for an imaging or inversion procedure that leads to the geometrical distribution of subsoil characteristics and the dynamic behaviour of the site.

Seismic techniques that are traditionally more diffused are based on body-wave propagation, in particular P-wave reflection, and in this context, surface waves, which are the main component of the ground roll, are coherent noise to be removed or attenuated. Much effort has been put into developing tools to filter out surface waves from reflection data, thus enhancing the useful signal relating to reflections and disregarding surface-wave properties and information content. More recently, interest in the large amount of information contained in surface waves has increased. In fact, surface waves can be interpreted or even expressly acquired and analysed to characterize the shallow near surface.

However, the use of surface waves is not recent: seismological applications for the characterization of the crust and upper-mantle structure date back 50 years (Ewing *et al.* 1957; Dorman *et al.* 1960; Dorman and Ewing 1962; Bullen 1963; Knopoff 1972;

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Kovach 1978; Mokhart *et al*. 1988; Keilis-Borok *et al*. 1989; Herrmann and Al-Eqabi 1991; Al-Eqabi and Herrmann 1993). Conversely, on a very small scale, ultrasonic Rayleigh waves have also been used for material characterization and for the identification of surface defects (e.g. Viktorov 1967), while on an intermediate scale, possibly the most recent application is diffused engineering. Jones (1958, 1962) and Ballard (1964) developed the first measurement system, but a strong impetus came from the introduction of the spectral analysis of surface waves (SASW) method (Nazarian and Stokoe 1984, 1986; Stokoe and Nazarian 1985; Stokoe *et al.* 1988, 1994; Roesset *et al*. 1991; Gucunski and Woods 1991; Tokimatsu *et al*. 1992b), which has been used for a wide variety of applications (seismic site response assessment, evaluation of liquefaction potential, identification of soft layers, pavement system analysis, etc.). The use of multistation techniques has enabled a more robust and stable estimation of subsoil properties. Many authors developed the acquisition, processing and inversion techniques, and described applications for different purposes (McMechan and Yedlin 1981; Gabriels *et al*. 1987; Tselentis and Delis 1990; Tokimatsu 1995; Park *et al*. 1999; Xia *et al*. 1999; Foti 2000). The use of Rayleigh waves, at the small scale, for the determination of soil damping has been discussed by Lai (1998) and Rix *et al*. (2001). Passive methods for subsoil characterization using microtremors have been presented by Horike (1985), Tokimatsu *et al*. (1992a) and Zwicky and Rix (1999): the book by Okada (2003) provides a good review of the microtremor method. Jongmans and Demanet (1993) discussed the importance of surface-wave analysis for the estimation of the dynamic characteristics of soils. Applications of the analysis of Rayleigh, Stoneley and Love waves to a variety of engineering problems have been discussed by Glangeaud *et al*. (1999) and the use of Scholte–Rayleigh waves in marine applications was reported by e.g. Shtivelman (1999). Surface-wave results have been compared with other geophysical results by Abbiss (1981), Hiltunen and Woods (1988) and Shtivelmann (1999). The use of the multimode nature of surface waves was reported by Szelwis and Behle (1984), Gabriels *et al*. (1987) and Socco *et al*. (2002). The application of surface waves to the processing of body-wave data was discussed by Mari (1984).

Surface waves exist only in media with a free surface, and they propagate in a limited layer close to the surface, the layer having a thickness that is roughly equal to one wavelength. Hence, in the same medium, waves of different wavelength affect different depths; if the medium is not homogeneous they propagate with different velocities and different attenuations in different materials. Hence, the velocity of propagation can be strongly frequency-dependent (dispersion), according to the geometric distribution of the soil properties: this behaviour is called geometric dispersion, as opposed to the intrinsic dispersion, which depends on the intrinsic attenuation. The geometric dispersion is the physical principle on which the SWM test is based: it is possible to measure the dispersive characteristics at a site and invert them to estimate the soil properties.

The propagation is mainly influenced by the shear-wave velocity of the materials, and hence in SWMs, the shear properties are obtained, thus providing parameters that are more useful in geotechnical engineering, and avoiding the water-masking effect in saturated media. Using standard seismic equipment for acquisition, it is easy to gather high S/N surface-wave data in records that can be also used for P-wave refraction or reflection. The inherent limits of other shallow seismic techniques can be overcome: there is no need for abrupt variations in seismic properties and virtually any property distribution can be investigated, without problems of velocity inversions or hidden layers.

The surface-wave method consists of three phases: *acquisition*, *processing* and *inversion*, the latter based on forward *modelling*, and this tutorial is focused on the analysis of these three phases. Acquisition consists of recording surface waves with high S/N in the appropriate frequency band, processing involves determining the properties of surface waves and inversion aims to estimate the soil properties related to the measured propagation, by means of comparison with the results of a simulation. Besides these main steps, the forward modelling has at least two functions: it is the kernel of the inversion, and it can be used to evaluate the properties of the method, i.e. to assess the resolution, reliability and sensitivity.

Since the different phases of the test contribute in different ways to the quality of the final result, the requirements for each step can be obtained, starting from the desired qualities of the final result itself and backward propagating them through inversion, processing and acquisition. The final result should be as precise and reliable as possible and this produces requirements for the inversion process regarding uniqueness, resolution and uncertainty propagation. To obtain a result with the previously mentioned characteristics, the inversion should use a data set containing a large amount of independent information and characterized by small uncertainties and no bias, and these needs define the main requirements for the processing. Finally, to extract a data set to be inverted with the above qualities without losing information, the processing needs high-quality raw data that adequately sample the propagation and this leads to the requirements for the acquisition. The first step, the acquisition, is required to gather data with a high S/N and with reduced coherent noise; the sampling should recognize the different propagation phenomena. This view affords a comparison of the different approaches to the test: considering the properties of the results, one may evaluate, for instance, which approach produces the lowest uncertainty result.
Moreover, if the above requirements are fulfilled, the characteristics of the surface-wave propagation can be exploited, providing a robust, fast and powerful characterization technique.

Several kinds of surface wave exist, their importance depending on the site, on the distribution of properties and on the acquisition. In the following, Rayleigh waves will be discussed (or pseudo-Rayleigh, as this kind of surface wave is called in vertically heterogeneous media), but the considerations about the

Rayleigh waves will be quite general and applicable to other surface waves. The main aspects of the modelling, the acquisition, the processing, the inversion and the result will be discussed. For each of these features, a critical description of the theory, the procedures and the algorithms is given and an example of its application is presented.

In addition, some intrinsic limits depending on the physics of the propagation exist and have to be considered, for instance, a physical limit for the resolution and the investigation depth. However, an accurate testing procedure can improve the quality of the results, taking into account limitations and uncertainties. Following the approach described above, the limits of the SWM will be discussed with particular attention to some critical aspects:

- the depth at which reliable information can be obtained can be limited by the acquisition, or by the lack of resolution or sensitivity to a certain target, for instance a soft layer, at a certain depth;
- most approaches are based on a one-dimensional model, and if this hypothesis is not verified at the scale of the observation, model errors can affect the result;
- the propagation is a multimode phenomenon: the presence of different modes and the modal superposition can introduce ambiguity and complicate the interpretation.

MODELLING

The forward modelling is the simulation of the seismic propagation in a medium of known characteristics: this is the kernel of the inversion procedure, in which the experimental data gathered at a site are compared to simulated data until a satisfactory match is reached. The modelling is also useful because it offers the possibility of simulating the test response, affording an evaluation of the sensitivity, the resolution and the investigation depth. In the following, after an outline of the main characteristics of Rayleigh waves and a short overview of the different possible approaches to surface-wave modelling, the simple modelling algorithm proposed by Thomson (1950) and Haskell (1953) will be illustrated and used to obtain some details of the 1D propagation of Rayleigh waves.

The main properties of Rayleigh-wave propagation

Rayleigh waves are surface waves; they propagate close to the surface, affecting a limited depth depending on the wavelength. This depth–wavelength relationship is not linear in vertically heterogeneous media. There is no radiation towards the earth's interior and wavefronts are cylindrical in laterally homogenous media.

The propagation velocity depends mainly on the shear-wave velocity V_S: in a homogeneous half-space the Rayleigh-wave velocity V_R is slightly lower than V_S ($0.87V_S<V_R<0.96V_S$, depending on Poisson's ratio ν (Richart *et al.* 1970)).

In a vertically heterogeneous medium, i.e. a layered medium, V_R becomes frequency-dependent. Since the propagation depth depends on the wavelength, the high frequencies (short wavelengths) propagate in thin top layers and their velocity depends on the shallow soil properties, the low frequencies (long wavelengths) propagate in thicker layers and their velocity is therefore also influenced by the properties of deeper layers. This is the geometric dispersion that describes the dependence of phase velocity on the frequency, given the thickness and the shear-wave velocity of a layered stack. Figure 1 shows a sketch of the vertical displacements as a function of depth for two wavelengths.

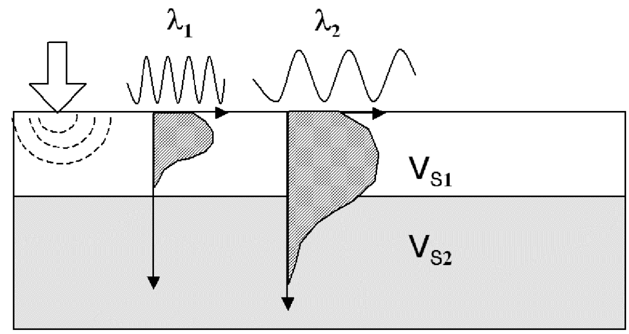

FIGURE 1
Schematic representation of geometric dispersion of Rayleigh waves: the vertical displacement associated with a short and a long wavelength.

Seismic sources at the free surface impart most of their energy in surface waves: in a homogeneous half-space, approximately 67% of the energy of a vertical circular footing is radiated in Rayleigh waves (Woods 1968). This is one of the reasons why surface waves often dominate seismic records. The geometric attenuation due to the energy spreading on cylindrical wavefronts is much lower than that of body waves, which also spread energy in depth; the Rayleigh waves then become predominant at large distances from the source (Rayleigh 1885). The intrinsic attenuation depends on both the shear and compressional dissipative properties; the shear damping is dominant, and is more influential at high Poisson's ratios. In heterogeneous media, the attenuation of the dispersion of V_R is frequency-dependent. It is related to the layers that are involved in the propagation of a certain wavelength.

The Rayleigh-wave propagation depends on the global behaviour of the site. The intrinsic attenuation acts as a lowpass filter, but the site strongly influences the amplitude at different frequencies. In the farfield, the frequency response of the site dominates and leaves a strong signature on the amplitude spectrum. As a result the low frequencies can be strongly attenuated and are difficult to detect at the free surface. Furthermore, depending on the soil property distribution, especially with strong velocity contrasts, resonance phenomena can occur.

One of the particular aspects of S-waves is that in vertically heterogeneous media the propagation is a multimode phenomenon: different modes of propagation can exist at the same frequency, having different distributions of the particle displacements and stresses and having different propagation velocities (Aki and Richards 2003). The relative importance of the different modes depends on the stratigraphy and on the source: modes can exhibit simultaneously and the modal superposition, with a given receiver array, produces apparent properties that can be problematic to interpret.

The basics of surface waves

In 1885, John Strutt, Lord Rayleigh, introduced the waves that nowadays bear his name and that are "propagated along the plane surface of an elastic body"; then Lamb (1904), discussed the forced "tremors over the surface of an elastic solid". Later, many solutions of the problem involving different soil models and different distributions of the soil properties have been used to understand the nature of ground motion, to simulate the effects of different sources of vibration at different scales, to understand the properties of surface waves and to use them for subsoil characterization.

Different constitutive laws can be used to describe the soil mechanical behaviour and different distributions of the soil parameters can be assumed, allowing them to vary as a function of one, two or three spatial coordinates. The choice of a one-dimensional model that describes the soil as a stack of plane, laterally homogeneous layers is often justified by the geological feature (a sedimentary basin) and allows for a solution of the forward problem: this is why the 1D model is the most widely used in seismological and characterization problems.

By writing the equation of motion for a laterally homogeneous medium, assuming a plane strain field, imposing the boundary conditions of the waves in a half-space with a free surface (no stress at the free surface and no stress and strain at infinity) and imposing the continuity of strain and stress at layer interfaces, a linear differential eigenvalues problem is obtained (Aki and Richards 1980). The vector **f**, formed by two displacement eigenfunctions and two stress eigenfunctions, and the 4x4 matrix **A**, depending on the vertical distribution of the soil properties, are related by the equation,

$$\frac{\mathrm{d}\mathbf{f}(z)}{\mathrm{d}z} = \mathbf{A}(z)\cdot\mathbf{f}(z), \tag{1}$$

expressing a linear differential eigenvalue problem that has a non-trivial solution only for special values of the wavenumber.

Assuming a layered model, a solution can be found, and many solution techniques have been proposed: numerical integration, finite differences, finite elements, boundary elements, spectral elements. The propagator matrix methods (Gilbert and Backus 1966) are the most frequently used: the Thomson–Haskell method of the transfer matrix (Thomson 1950; Haskell 1953) is a special case of the propagator matrix method applicable to a stack of homogeneous layers overlying a half-space. The dynamic stiffness matrix method, proposed by Kausel and Roesset (1981), is a finite-element formulation, derived from the Thomson and Haskell algorithm. The method of reflection and transmission coefficients (Kennet 1974) is an efficient iterative algorithm for computing the Rayleigh dispersion equation and explicitly models the constructive interference that leads to the existence of surface-wave modes.

The original numerical problems of the Thomson–Haskell approach have been solved by Knopoff (1964) and Dunkin (1965), and many authors have worked on the optimization of the algorithms (Watson 1970): the numerical aspects have been discussed and solved, and a complete implementation has been given by Herrmann (1996).

The transfer matrix approach has a conceptual simplicity and is used below to obtain the principal relationships that describe Rayleigh-wave propagation in layered media and to discuss their main properties.

The subsoil is modelled as a stack of linear elastic or linear viscoelastic plane layers, each characterized by four parameters: V_S, V_P, ρ and H. In general, the velocities can be complex to account for the dissipative phenomena (Schwab and Knopoff 1971): the use of the two (P and S) quality factors or the damping ratio is an equivalent alternative. The coupling between velocity and attenuation in weakly and strongly dissipative media has been discussed by Lai and Rix (2002). Using the Helmoltz decomposition in each layer, the Fourier-transformed displacement potentials can be written: they are related to stresses and displacements in the layer by means of the material parameters. Hence, it is possible to form a layer matrix, containing the geometric and mechanical characteristics of each layer, and to propagate the solution across the layers using the continuity at the interfaces: a single matrix equation for the whole system can be assembled, in which the boundary conditions at the surface and at infinity can be imposed.

A non-trivial solution for the displacement and stress in the layered system can be found by imposing a special relationship between frequency and wavenumber: the resulting equation is known as the Rayleigh secular equation, and can be written in an implicit form as

$$F(k,f)=0, \qquad F_R\left[\lambda(z),G(z),\rho(z),k_j,f\right]=0\,, \tag{2}$$

where k is the wavenumber, f is the frequency, λ and G are the Lamé parameters and ρ is the mass density.

For a given frequency, a solution can exist only for special values of the wavenumber $k = k_j(\omega)$: in vertical heterogeneous media this is a multivalued function of frequency that represents the modal curves. Figure 2 shows an example of modal curves for a simple three-layer model with the velocity increasing with the depth.

The leaky modes, arising from the possible complex-conjugate roots of the Rayleigh equation, are inhomogeneous waves propagating along the surface with a phase velocity greater than the shear wave and smaller than the P-wave: they decay in the direction of propagation and are usually negligible.

Modal curves and apparent dispersion curve

Modal curves are possible solutions: in a layered medium with a finite number of layers in a finite frequency range, there is a

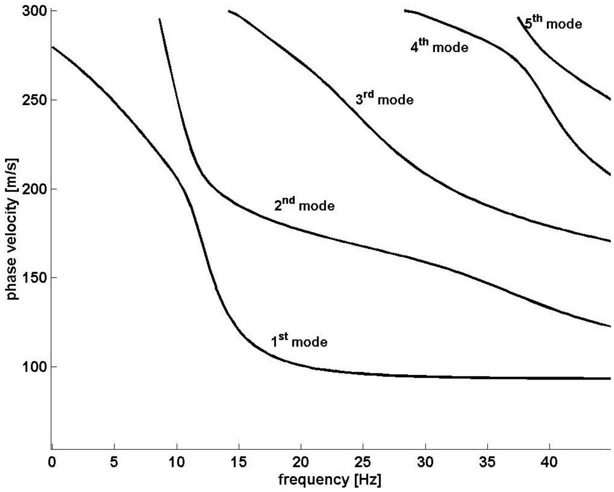

FIGURE 2
Modal curves for a three-layer model (3 m at 100 m/s, 3 m at 200 m/s, half-space at 300 m/s).

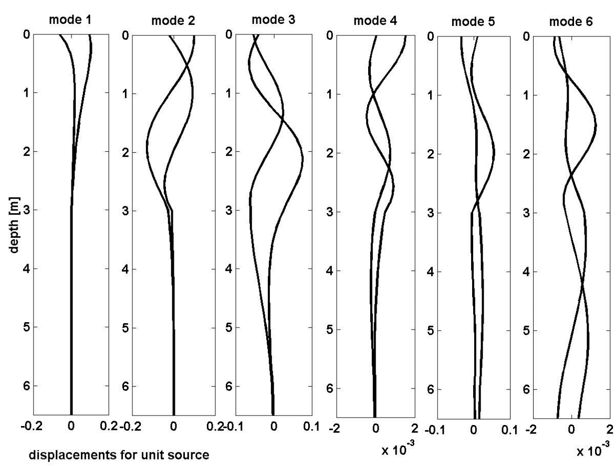

FIGURE 3
Horizontal and vertical displacements for different modes propagating in the model of Fig. 2.

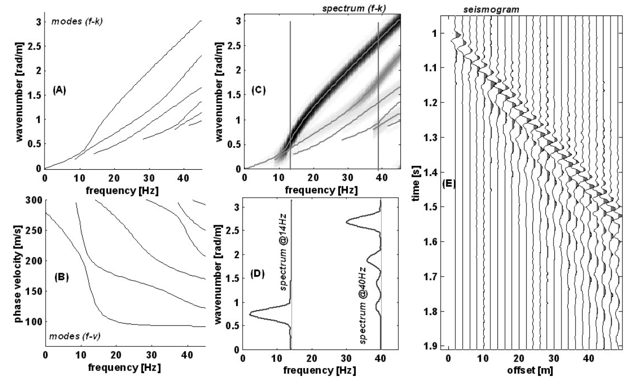

FIGURE 4
Synthetic data for the model of Figs 2 and 3. Modal curves in *f–k* (a) and *f–v* (b), synthetic *f–k* spectrum (c) with a dominating first mode (Hanning windowing). The two sections of the spectrum (d) show the lobes produced by the finite array, and the lower energy of higher modes. The synthetic seismogram is shown in (e).

finite number of modes. Each mode, apart the first one (also known as the fundamental mode), exists only above a cut-off frequency, at which the modal phase velocity is the maximum shear-wave velocity of the system: higher modes have higher velocities and there can be common roots and apparent intersections of modes.

The computation of modes is not enough to describe the propagation completely: they represent only the kinematic description of the possible velocities, and not displacements and stress, so they do not carry information about the energy propagating along each mode. The assumption that the first mode is dominant is often not verified at the engineering scale and higher modes can be dominant or superimposed on one another. Hence the resulting apparent velocity that can be observed in an ideal experiment can be different from the first-mode velocity. Moreover, the energy distribution along a single mode, i.e. the amplitude as a function of the frequency, depends strongly on the stratigraphy and is therefore additional information.

A complete modelling should then go on to simulate the solution in terms of displacements or stresses. For each mode, at each frequency, the four stress and displacement eigenfunctions can be computed. In particular, the displacement versus depth plots can be very useful for understanding the main features of the propagation: the amplitude at the surface and the amplitude as a function of depth reveal the penetration of the different modes (Fig. 3 shows the horizontal and vertical displacements as a function of depth for the first six modes at a frequency of 50 Hz for the same model used in Fig. 2).

The eigenfunctions are then used to compute energy integrals, the displacement Green's function can be computed, and the geometric parameters that describe the measuring array can be introduced.

The position and the spectrum of the source influence, via the Green's function, the energy partitioning on modes. The receiver array has strong effects on the observed wavefield: the sampling in space produces limits in the wavenumber domain, and the spectral leakage due to a finite-space window produces spreading of the energy in the spectrum (see e.g. Bracewell 1986). Although, at a single frequency, the eigenvalues imply a series of discrete possible solutions, each one with a spike of energy concentrated exactly at a modal wavenumber, the spreading due to the finite window will produce lobes. Figure 4 illustrates these aspects, by considering the model of Figs 2 and 3. The modal curves are shown in the *f–k* domain in Fig. 4(a) and in the *f–v* domain in Fig. 4(b): the energy should be concentrated exactly on modes, but the spectral leakage produces a spectrum as in Fig. 4(c). The cross-sections of the spectrum show the lobes of the array response function depicted in Fig. 4(d). The inverse transform of the spectrum gives a seismogram (Fig. 4e). Spectral lobes are a minor concern if a single mode is present or if the difference in wavenumbers between modes is large compared to the lobes of the spatial-window spectrum; otherwise, the energy of modes can be superimposed, producing a spectral maximum that does not correspond to a possible solution, and this can prevent the unambiguous identification of modal curves that track the energy maxima in the spectrum.

The approach briefly described above allows a simulation test: the site, the source and the receiver array are considered, but only Rayleigh waves are simulated. In actual seismic data other phenomena are present, but Rayleigh waves are dominant due to their energy, and this approach to the modelling suggests a possible analysis of experimental data. Rayleigh waves can be acquired in a full-waveform record in the time–offset domain, transformed into the frequency–wavenumber domain, where the energy associated with Rayleigh waves is easily recognized, and then the kinematics and energetic properties can be identified. In many cases, only the position of the energy maxima at each frequency is considered and interpreted. However, the kinematics information, known as the 'dispersion curve', is an apparent property, depending both on the model and on the observation layout.

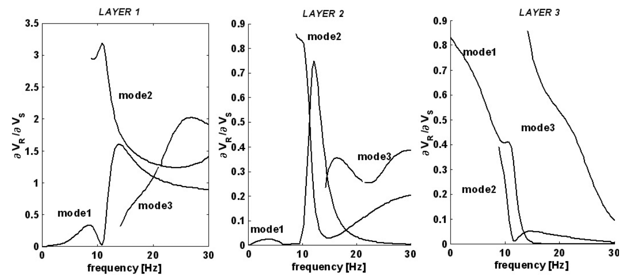

FIGURE 5
Example of sensitivity of modal curves for the model of Fig. 2. The three plots depict the partial derivatives of the modal Rayleigh phase velocities as a function of frequency, with respect to the shear-wave velocity of the three layers (the third layer is the half-space).

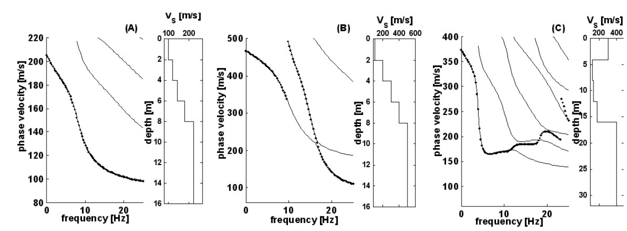

FIGURE 6
Examples of modal and apparent curves (tracking of energy maxima) for three different models: in (a) the apparent curve lies on the first mode, in (b) the velocity contrast of the deeper layer makes the apparent curve jump on the second mode at low frequencies. In the well-known situation in (c), the stiff top layer makes the apparent curve move towards higher modes at high frequencies.

Some properties of the solution

Those properties of the Rayleigh-wave propagation that need to be analysed for site characterization are discussed in relation to the properties of the solution of the forward problem (synthetic data) and verified with experimental data.

The Rayleigh-wave velocity mainly depends on the shear-wave velocity of the layers: hence the bulk density and Poisson's ratio have less effect. This is an important consideration for the inversion, in which the number of unknown model parameters can be reduced by identifying the less significant ones and assuming them *a priori.*

The shallow layers influence all the wavelengths and thus all the frequencies, while the deep layers influence only the long wavelengths, i.e. the low frequencies. This means that the information about the shallow layers is carried by all the frequencies while that concerning the deep layers is carried by only a few data: in the inversion this means a mix-determined problem. The analysis of the sensitivity of Rayleigh phase velocity to layer shear-wave velocity can help in designing the tests and assessing the quality of the final result. The partial derivatives can be used with this aim: the application of the variational principle enables a closed-form solution for the partial derivatives of the Rayleigh phase velocity with respect to the body-wave velocities of layers to be obtained (Lai 1998). In Fig. 5, this approach is used to assess the sensitivity of different modes to the velocity of the three layers at different frequencies and it clearly seen that the first mode is not the most sensitive to the model parameters.

The modes are simulated in theory as possible solutions, but are detected experimentally as energy maxima: so a simple kinematics approach to the sensitivity analysis neglects the fact that modes can be undetectable or that an apparent dispersion can result from an experiment. The simple selection of the position of energy maxima can produce discontinuities and jumps, and the importance of higher modes cannot be excluded *a priori* on the basis of general knowledge about the site. In Fig. 6, the apparent curve related to energy maxima is plotted with the modal curves for three models: higher modes can be dominant even at low frequencies and at normally dispersive sites. The model in Fig. 6(a), with a gradual increase in the velocity, has a dominant first mode, while the examples in Figs 6(a) and (b) show the importance of higher modes, both at sites with increasing velocity and at sites with velocity inversions.

The presence of several modes, their superposition and all the factors that influence the energy distribution make a straightforward interpretation of data difficult, especially at a small scale. The first mode has the lowest possible velocity, and if the energy associated with higher modes and detected experimentally is assumed to be a first mode, the velocities of the model are definitely overestimated.

As mentioned above, the dispersion curve is not an intrinsic property of the site, but also depends on the observations: the modal superposition is affected by the array, and the dispersion curve is simply an apparent property. In Fig. 7, the influence of the measuring array is shown and the dispersion curve obtained for the same model with two different array lengths is depicted: it can be seen that short arrays only allow the acquisition of an apparent dispersion that is not coincident with any modal curve over a wide frequency range. Also, the attenuation, which modifies the spectral amplitudes with the offset, can influence the modal superposition. This is one of the main reasons why a complete modelling, consistent with the acquisition, is often necessary, and why the acquisition has to be designed to ensure adequate sampling of the wavefield.

An interesting property (that can be used to speed up the inversion) is the scaling of the solution with the wavelength: the velocities and the frequencies scale simply if all the layer velocities are scaled; the frequency scales if all the layer thickness are scaled.

Another feature that can be simulated by the modelling is the frequency response of the site, which affects the energy distribution along modes. The energy can be concentrated in a narrow frequency band, and a strong attenuation of the source energy can be produced below a certain frequency: for a soft layer on a stiff bedrock, these site effects can make the acquisition of data below the natural frequency of the site difficult.

The solution of surface-wave propagation in non-1D media has been discussed less in the literature and used less in the characterization. The Rayleigh-wave scattering by different objects is used in the non-destructive evaluation of materials. The theory of surface waves for media involving smooth lateral variation was presented by Keilis-Borok *et al*. (1989), however most applications use finite-difference or finite-element methods. The effects of obstacles on the SASW techniques have been investigated for instance by Gucunski *et al.* (1996), the application of S-waves to

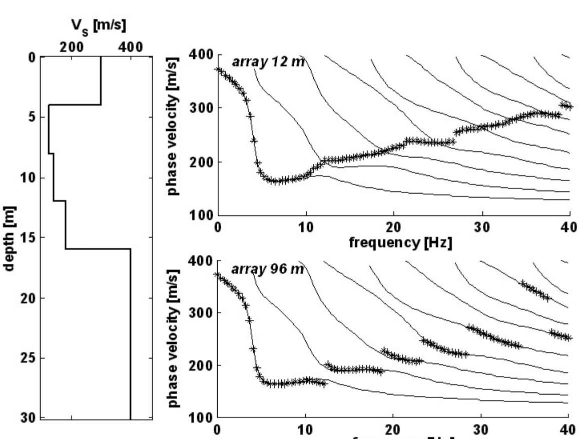

FIGURE 7
Modal and apparent curves obtained with two different arrays (12 m and 96 m with geophone spacing of 1 m) for the depicted model. The modal superposition, which depends on the response of the array, is responsible for the differences in the high-frequency band.

the detection of cavities has been presented by Leparoux *et al*. (2000), and the effects of inhomogeneities is discussed by Abraham *et al.* (this issue).

ACQUISITION

The main task of the acquisition is to measure surface waves and thus produce information about the dispersion and the attenuation characteristics. The requirements to be fulfilled by the acquired data are achieved by 'backward propagation' of the requirements imposed on the final result, over the different steps of the procedure. The data to be processed and inverted should then have a high S/N over a wide frequency range, should allow for modal separation and recognition, should allow for separating and filtering out of coherent noise and should allow estimation of uncertainties.

After describing the different approaches that can be used for data acquisition, the role of the parameters that define the time and space sampling of the surface-wave propagation and of the hardware used for acquisition will be analysed with respect to the above-mentioned requirements.

Many acquisition techniques have been used in surface-wave surveying, depending on the type of application, the depth of investigation and the scale of acquisition. The surface waves propagating in the crust and in the upper mantle, often referred to as long-period oscillations, involve periods between 5–10 s and 300–500 s, sometimes up to 800 s (Keilis-Borok *et al*. 1989). They are observed and acquired with wide arrays of seismometers at nearly continental scales. In seismological applications, active surface-wave measurements have also been used for the study of the shallow structure (the first hundreds of metres of the crust). Examples of acquisitions have been given by Mokhart *et al*. (1988), Herrmann and Al-Eqabi (1991) and Al-Eqabi and Herrmann (1993). In this type of application, the surface-wave data consist of some tens of traces (with a time window of tens of seconds) and are generated by an explosive source. They are gathered with standard low-frequency vertical geophones (less than 2 Hz), with an array length of some tens of kilometres, and wavelengths of some kilometres are obtained.

For engineering problems the scale is smaller: from centimetres up to some tens of metres. The equipment has to be light, portable and cheap: the receivers are geophones, the sources are sledgehammers, weight drops, small vibrators, explosions, even noise can sometimes be used. The two-station acquisition pioneered by Jones (1958, 1962) and Ballard (1964) and developed to become the SASW test (Nazarian and Stokoe 1984; Stokoe *et al*. 1994) has been widely used for pavement assessment and for geotechnical characterization. The acquisition of both the components of Rayleigh waves has also been proposed (Tokimatsu *et al.* 1998). Active multistation techniques are based on arrays of geophones (12– 48 or more) and different sources, without any substantial differences from the shallow crust acquisition (Gabriels *et al*. 1987; Tselentis and Delis 1998; Park *et al*. 1999; Shtivelman 1999; Foti 2000; a,b; 2002; Foti *et al.* 2003). Passive techniques usually observe the vertical motion of microtremors, using a two-dimensional array of sensors distributed on the ground surface in the attempt to go deeper, using the great wavelengths of microtremors (Okada 2003). Multistation applications have also been performed in marine environments, with the use of hydrophones to detect the pressure field related to the surface-wave propagation on the sea-bottom, and explosives or airguns as sources (Mari 1984; Shtivelman, this issue). On a smaller scale, the use of ultrasonic surface waves and the techniques to generate and acquire surface waves are described by, for example, Viktorov (1967).

The multichannel approach should be preferred to the two-station approach because it averages and attenuate errors, improves the possibility of mode separation and identification, allows for the recognition and the interpretation of other seismic events present in the data (such as refracted and reflected body waves), allows for coherent noise filtering and, moreover, does not require complex acquisition procedures to sample the propagation over an adequate frequency band. The most usual array is a linear, evenly spaced array of vertical low-frequency geophones (4.5 Hz can be a good compromise) with an in-line end-off configuration.

As illustrated when describing the modelling of surface-wave propagation, the modal velocities are a characteristic of the layered system and the modal curves are continuous and regular functions of the model parameters. The ideal surface-wave method would record only surface waves to produce and invert experimental modal curves, in order to obtain the model parameters, but the limits introduced by the discrete time and space sampling, within a limited time and space interval, prevent the achievement of this ideal task. The effects of the acquisition on the data lead to an experimental 'apparent' dispersion curve, which is the consequence of the difficulties in separating the energy associated with different modes. This effect is particularly relevant for the small scale involved in engineering problems. In the following, the limits introduced by the acquisition are investigated by analysing the properties of data in the *f–k* domain, but the conclusions are general. The main aspects to be considered are related to the spatial sampling of surface waves (the receiver spacing and the total array length), while time sampling is less critical. The analysis of the influence of the sampling parameters on the data quality, in terms of information content and uncertainties, permits a better understanding of the surface-wave testing procedure and also supplies some practical rules for a 'best practice' of acquisition.

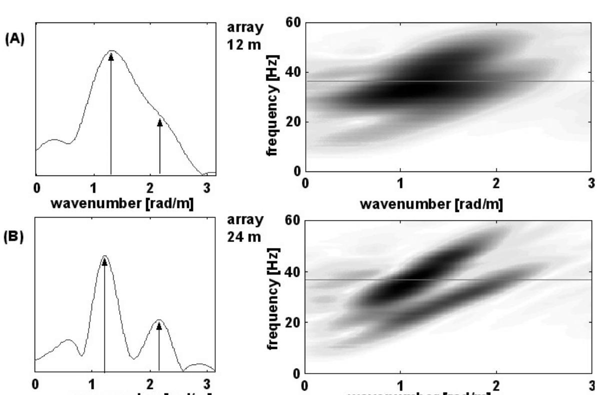

FIGURE 8
The effect of array length on wavenumber resolution and on modal separation capability is shown on field data sets: at the same site two arrays, (a) 12 m long, (b) 24 m long, are recorded, and the corresponding *f–k* spectra are shown, together with a section at 35 Hz.

Space sampling

Array length

The array length affects the wavenumber resolution Δk and therefore the possibility of mode separation. The windowing produces leakage in the *f–k* spectrum, due to the main lobe of the spatial-window spectrum (Fig. 8), and creates ripples that often prevent higher modes being identified. Some problems related to spectral discretization (this affects the accuracy of the identification of the spectrum maxima) and ripples can be partially resolved by windowing and zero padding procedures performed during data processing, but the loss of information caused by the chosen acquisition parameters cannot be recovered during processing.

Another aspect to be considered with respect to spatial-window length is the maximum observable wavelength. No theoretical upper limit on the wavelength is imposed by the array length: wavelengths longer than the array can be observed, and the maximum wavelength depends mainly on the site global behaviour and on the frequency content of the propagating signal. However, long arrays should be preferred because they improve the modal separation and because, as will be shown later, when dealing with the number of channels, they reduce the data uncertainties. On the other hand, short arrays are less sensitive to lateral variations, produce a better S/N ratio, are less affected by high-frequency attenuation and, given the number of channels, produce less severe spatial aliasing. These considerations stress the need to find a good compromise between all these aspects: we have used arrays with lengths from 50 to 100 metres that often allow the use of a light source and are adequate for the characterization of the first 20–30 metres.

Receiver spacing

The *f–k* transform of a *t–x* signal is symmetric with $A(f,k)=A(-f,-k)$. As stated by the Nyquist sampling theorem, the maximum wavenumber that can be identified depends on the spatial sampling rate (the receiver spacing is ΔX):

$$k_{\mathrm{Nyq}} = \frac{1}{2}\frac{2\pi}{\Delta X} = \frac{\pi}{\Delta X} .$$

All the energy associated with $k^* > k_{\mathrm{Nyq}}$ will be aliased in $k^* - 2k_{\mathrm{Nyq}}$. In end-off configurations, all the coherent energy travels in the positive direction and is associated with positive wavenumbers. In the negative quadrant of the spectrum, noise and aliased events are present and it is then possible to recover the aliased information with a $2k_{Nyq}$ horizontal unwrapping. The minimum detectable wavelength is then $\lambda_{min} = \Delta X$.

The above-mentioned limitation has particular influence on the possibility of inferring information about the characteristics of the uppermost layers from the shorter wavelengths. However, the analysis of field data shows that the main limitations on the highest detectable frequency is, as for the lowest one, caused mainly by attenuation and site response, and the spatial sampling rate can become critical only at very low-velocity sites.

Receiver number

The number of receivers, obviously related to array length and receiver spacing, affects the propagation of the uncertainties over the data. The uncertainty in the estimated wavenumber (and hence in the phase velocity) depends on the uncertainty in the phase of each frequency component, but also on the number and position of receivers. To evaluate the propagation of the uncertainty in the phases of the single traces on the final estimate of the velocity, *the unit covariance matrix* (UCM), as defined by Menke (1989), can be used by applying the MOPA algorithm (Strobbia and Foti 2002) described later. The UCM allows the variance of the estimated wavenumber to be expressed as a function of the total array length and of the number of receivers, for a given value of the phase uncertainty. Figure 9 shows the uncertainty in the wavenumber, for different array lengths and numbers of receivers, given a fixed uncertainty in the phase. For a given array length, increasing the number of receivers reduces the amplification of the uncertainty. For instance, a 24-receiver array reduces the uncertainty by a factor of four with respect to a two-receiver array, and enables a solution for the trade-off length-spacing to be found.

Source offset

To plan an optimum source offset, two main aspects have to be considered: at small distances, the near-field effects contaminate the signal at low frequencies, while the attenuation reduces the S/N of traces at large distances, especially in the high-frequency band. These two phenomena are strongly dependent on the site and the experimental conditions, and in general cannot be predicted to determine the best source-offset. Possible solutions are the acquisition with different source-offsets to recognize the near-field, or the use of a small offset and the filtering of the near-field during processing. On the other hand, some rule of thumb has been suggested: for instance, a source-offset equal to the desired investigation depth has been suggested by Park *et al*. (1999).

Time sampling

The time-sampling parameters have a minor effect with respect to spatial sampling. The sampling rate is chosen, depending on the highest frequency that will be acquired according to Nyquist

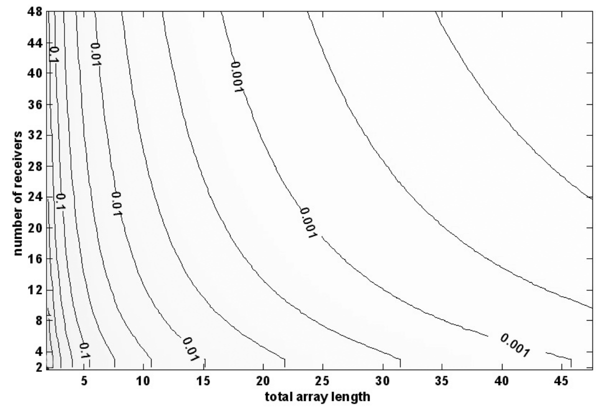

FIGURE 9
Uncertainty in the wavenumber as a function of the array length and the number of receivers, computed for unit uncertainty in the phase.

sampling theorem. The time-window has to be long enough to record the whole surface wave on all traces: with long arrays (more than 100 m) at low-velocity sites, several seconds can be needed. A long window with a pre-trigger can be used to evaluate the signal level during the acquisition and to improve the spectral resolution: the portion without signal will be muted, to reduce the effect of the noise that is present.

Acquisition strategies and technologies

Many different strategies can be employed to improve the efficiency of spatial sampling. The source and receivers can be moved to increase the spatial window and the number of receivers (Gabriels *et al*. 1987) and unevenly spaced array can be designed to optimize the sampling of different frequency components (2002a,b).

Stacking can improve data quality and it is usually performed during acquisition: if different shots are acquired separately, stacking can still be carried out during processing and a statistical estimate of the data uncertainty becomes possible.

Regarding the equipment, seismic instrumentation commonly used for engineering surveying is normally utilized. The main difference is that the transducers, which are usually low-frequency geophones (f<4.5 Hz), have a flat amplitude and a stable phase response within the frequency band of interest: the phase deviations of single receivers have a direct effect on the estimated velocity. Regarding the source, it should have an adequate level of energy in the frequency band of interest, bearing in mind that the site response can dominate, in spite of our efforts. Impulse and vibrating sources can be used: it is worth mentioning that the sweeps are not optimal signals for the maximization of the S/N in the frequency domain.

PROCESSING

The objective of the processing is to derive from full waveform records all the information about the surface-wave propagation without bias, to estimate the uncertainty, and to test whether the site fits the assumed 1D model: the processing, in fact, extracts from data the factors to be compared with the simulations within the inversion procedure. Thus it is the link between the real world of acquired data, containing surface waves together with other seismic events and noise, and the ideal world of simulated data containing only surface waves propagated in an ideal model.

To increase the reliability of the inversion, we have to sample the data space with the aim of increasing the independent information, without replicating it, and reducing its uncertainty.

Different approaches have been developed and used to process surface-wave data: multiple-filter analysis, introduced by Dziewonski *et al*. (1969), has been used for the determination of the group velocity as a function of the frequency from a dispersed wavefront. Herrmann (1973) studied the use of multiple-filter analysis to estimate the spectral amplitudes of various models. The use of the cross-power spectrum of two-station data (Dziewonski and Hales 1972) has been adopted by many authors working with the SASW approach. Nolet and Panza (1976) presented the *f–k* wavefield transform for an unambiguous investigation of higher modes. McMechan and Yedlin (1981) discussed the use of the τ–p transform obtained from a slant stack. Frequency–time analysis (e.g. Keilis-Borok *et al.* 1989), correlation (Park *et al*. 1999), and other approaches have been used for the processing of surface-wave data (Glangeaud *et al*., this issue).

Many of the approaches are formally identical or linearly dependent, and the results are identical using pure surface-wave synthetic data: the comparison should consider, in general, sensitivity to noise, stability, robustness, capacity for extracting information and for identifying different events, filtering and weighting of information. Probably, there is no *best technique*: it is however important to use a tool that has been purpose-designed for surface-wave analysis, and not simply for filtering of surface waves: for example, the *manual* picking of energy maxima of the *f–k* spectrum can introduce dramatic errors in the identification of the velocity.

Wavefield transforms are widely used to perform the analysis in domains where surface waves are easily identified and their properties are estimated: many of these domains are related by linear invertible transforms, and the aspects that are discussed below about the *f–k* transform can be considered quite general. The frequency–wavenumber (*f–k*) transform has the advantage of being a natural approach to the analysis of the seismic event described above. In fact, the ideal wavefield related to surface waves can be described with its energy located on the eigenvalues, which are lines in the *f–k* domain; the observation produces a spreading of energy that is constant in the whole *f–k* domain. A real wavefield will have, in addition to the above-mentioned contents, the energy of random and coherent noise: this additional energy can be separated in the *f–k* domain or it will produce a distortion in processed data if is superimposed on surface-wave energy. It is then possible to identify the surface waves as the dominant events or energy density maxima in a wide frequency

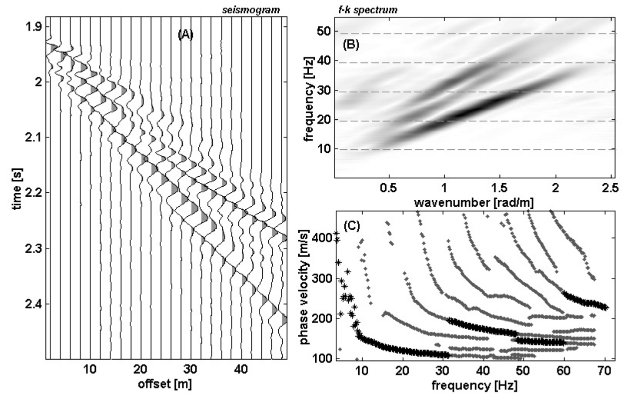

FIGURE 10
Example of experimental data: seismogram acquired with 24 vertical geophones; *f–k* spectrum and spectral maxima (relative maxima: dots; absolute maxima: asterisks).

band as can be easily verified on experimental data. However, it has to be said that other transforms, for instance the *f–p* or the *f–v* transform obtained from the slant stack, can be completely equivalent, and the application of a wavefield transform is only the first step of the processing.

The computation of the spectrum needs some care in order to allow proper identification of the maxima: some preprocessing (muting, filtering in the frequency–offset domain, etc.) is performed to remove eventual low-quality portions of the data. Then windowing and zero padding in the *t–x* domain are used to improve the resolution of the spectrum, and different windows can be used to discriminate between ripples and higher modes.

When a good image of the energy density of the propagation is obtained, the analysis of the energy maxima has to be performed, with a automatic search of maxima. When maxima have been identified in *f–k*, the velocities are computed simply as $v=2\pi f/k$.

The set of all the positions of the absolute maximum at each frequency, known as the *dispersion curve*, can sometimes have most of the required information, but the curve obtained is limited to a frequency range, may not be continuous, may not coincide with any branch of the modal curves, and can often be related to higher modes. The identification in the same frequency band of relative maxima with lower amplitudes can be very useful: sometimes they can be used to provide continuity of the frequency range of a curve, when the maximum jumps from one mode to another. An example is shown in Fig. 10, where the seismogram, the spectrum and the maxima are depicted: the absolute maxima (asterisks) lies on the first mode from 5 to 30 Hz, then jumps on a higher mode, without preventing the identification of the first mode at higher frequencies.

Sometimes secondary maxima appear to replicate the information of the first mode, but it must be remembered that the uncertainty in the velocity information depends on the wavenumber uncertainty: the higher wavenumbers, related to

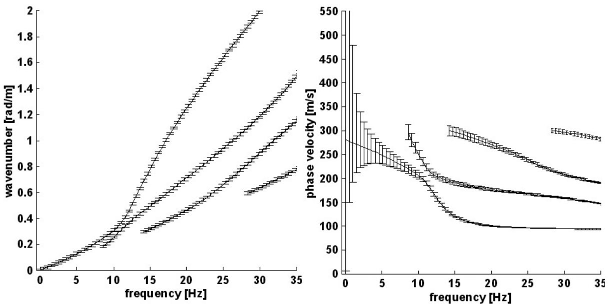

FIGURE 11
Uncertainty distribution in *f–k* k and *f–v*: considering the modal curves, a constant uncertainty in the wavenumber is mapped to a variable uncertainty in the phase velocity. Because of the higher wavenumber, higher modes have a lower uncertainty in the high velocities.

higher modes, then have a lower uncertainty amplification with respect to those related to the fundamental mode. Figure 11 shows (for the soil model shown in Figs 2–6) the difference of uncertainty amplification in the different modes: the same uncertainty in the wavenumber produces very different uncertainties in the velocity. Even if it is not possible to recognize unambiguously the different modes, and it is not possible to assign *a priori* a mode number to the relative maxima, they are useful because they can reinforce the information: they can confirm, for instance, the high velocity that the first mode reaches at low frequencies where the uncertainty is strongly amplified and the energy is usually low. Moreover, the increase in sensitivity and resolution obtainable, including in the higher modes, makes the effort of extracting higher modes in the processing worthwhile.

Several modes can be superimposed and the identification of the position of modes may not be possible by simply searching for the maxima: the comparison of the results of different spectral estimates, having different widths of the main lobe, can be useful in this respect.

Problems of the global estimates

The *f–k* approach is stable and robust supplying a global estimate of the propagation properties and averaging all the redundant information. The problem is that a single estimate is obtained: in practice, data are stacked in x and the information regarding the horizontal direction is therefore lost. It is then important to find a way of evaluating both the variation of the site properties and of the uncertainty as a function of the offset.

The uncertainty is not constant in the data, but is strongly dependent on the frequency and on the offset: this implies that data weighting should be introduced to take into account their differing quality. Furthermore, the presence of lateral variations beneath the geophone array can be important. A global estimate does not detect the lateral variations but is influenced by them: this problem in the inversion will produce model errors when fitting a data set that has 2D or 3D effects with a 1D simulation. The processing should then also be able to recognize the presence of lateral variations in order to verify the hypotheses of the method.

Simple ways to detect lateral variations are the acquisition of shots at the opposite side of the measuring array and the comparison of data, or the use of refracted body waves to assess the 1D geometry of the site. Another possible approach, described in the following, is the use of a local analysis of the propagation properties: the array can be split into overlapping subsets that are analysed separately to verify the stationarity of the propagation properties.

Multi-offset phase analysis (MOPA) (Foti and Strobbia 2003, 2004) is an example of this kind of approach: to extract the local properties of the propagation, the wavefield is analysed by considering the variation of the phase and amplitude versus the frequency and the offset. The wavefield associated with surface waves in a dissipative medium can be written, in the frequency domain, as the sum of the displacements due to the different modes (Aki and Richards 1980), as

$$s(\omega,x)=\sum_m A_m(\omega,x)\mathrm{e}^{\mathrm{i}(\omega t-k_m(\omega)x)},$$

where $k_m(\omega)$ is the wavenumber of the mth mode as a function of the frequency, A_m is the amplitude, with respect to the source and the path.

The expression can be rearranged, separating the space-dependent and frequency-dependent parts of the amplitude, as

$$s(\omega,x)=\sum_m I(\omega)R_m(\omega)\frac{\mathrm{e}^{-\alpha_m(\omega)x}}{\sqrt{x}}\mathrm{e}^{\mathrm{i}(\omega t-k_m(\omega)x+\varphi_I(\omega))}.$$

The amplitude depends on the input source I, the response of the site R, the geometric attenuation and the intrinsic attenuation; the phase has a simple linear dependence on the offset if a single mode is considered.

This complex expression, for each mode, has amplitude

$$I(\omega)R(\omega)\frac{\mathrm{e}^{-\alpha(\omega)x}}{\sqrt{x}}$$

and phase

$$-k(\omega)\cdot x+\varphi_I(\omega),$$

and these two expressions can be used to estimate the wavenumber k and the attenuation coefficient α from experimental data.

The experimental phases as a function of the frequency and of the offset are extracted with a single Fourier transform: a statistical approach allows, with a population of seismograms, the probability distribution of the phase to be estimated (Strobbia 2002a,b). Then a linear weighted inversion of the phase gives the estimate of the wavenumber probability density, which is statistically estimated. In addition, the approach allows the assumed model to be tested, analysing the linearity of the phases taking into account their experimental uncertainty. Figure 12 shows an example of this pro-

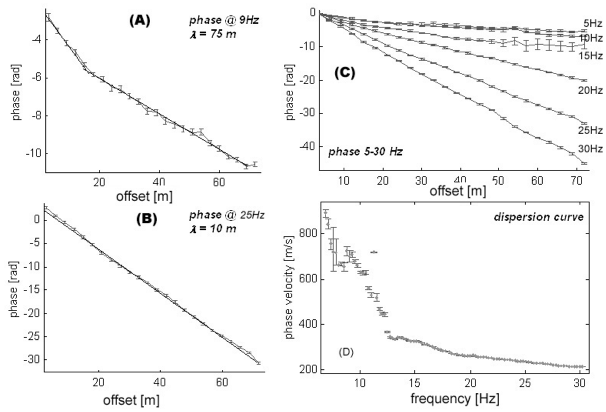

FIGURE 12

Application of MOPA to an experimental data set: two phase curves, (a) and (b), show respectively the near-field effects at low frequencies and a good reariy at higher frequencies. A set of phase curves, with increasing wavenumber (slope), is shown in (c). The final dispersion curve, with the estimated uncertainties, is depicted in (d).

cessing: in Fig. 12(a) the near-field effects at 9 Hz cause the phase-versus-offset curve to deviate from linearity, while a good fit is obtained for instance at 25 Hz (Fig. 12b). Figure 12(c) depicts the curves from 5 to 30 Hz, which are fitted, after filtering the near-field, in order to estimate the wavenumbers. The final dispersion curve with its uncertainty is shown in Fig. 12(d).

The analysis of the intrinsic attenuation can be performed by analysing the decay of the spectral amplitudes with the offset: different approaches can be used in order to achieve this aim (Foti, this issue). In general the control of the amplitude in the acquisition phase is not straightforward and needs some special care. Controlled sources or deconvolution of the traces with a reference trace can be used to extract an attenuation curve.

INVERSION

In geophysics the term inversion means the "estimation of the parameters of a postulated earth model from a set of observations" (Lines and Treitel 1984). In the case of the SWM, the inversion supplies the estimated velocity and/or attenuation profile from the dispersion characteristics furnished by data processing, and thus it represents the last 'chapter of our story'. The requirements for the inversion are derived directly from the properties of the final result that can be summarized in the following statement: the result should be a unique subsoil model with adequate reliability and resolution down to the depth of interest and should be presented with the associated uncertainties.

It is important to stress that the SWM inverse problem is non-linear and mix-determined; in addition, as shown in the previous paragraphs, the object that is usually interpreted (the dispersion curve) is often not continuous and therefore, automatic inversion procedures can be successfully applied only in the cases in which branches of modal curves are selected within a proper frequency range.

Different approaches have been proposed for the inversion procedure. The first simplified technique that was used for the interpretation of dispersion data is the λ/3 or λ/2 interpretation, which assumes that the shear-wave velocity is equal to 110% of the Rayleigh phase velocity, and refers to a depth equal to one-third or one-half of the wavelength (see e.g. Abbiss 1981). The inversion of dispersion data using the trial and error method was performed and described by Stokoe *et al*. (1994). The number of parameters to be found with this technique, based on intuition of the physics of the problem by the analyst, is limited.

The more widely used approach is the linearized iterative least-squares approach. This has been used by many authors, with some differences in the data concerned, the model parameters, the computation of the partial derivatives, the inversion strategies, the use of smoothness constraints, etc. (Horike 1985; Gabriels 1987; Herrmann and Al-Eqabi 1991; Herrmann 1994; Lai 1998; Mokhart *et al.* 1998; Tselentis and Delis 1998; Shtivelman 1999; Xia *et al*. 1999; Foti 2000, among the others). Among the direct search methods, the Monte-Carlo method, on the planetary scale, was used by Press (1968). This generated more than five million models to fit the data of long-period oscillation of the earth; more recently at the engineering scale, the neighbourhood algorithm has been used on passive measurement results by Wathelet *et al.* (2004, this issue). Genetic algorithms (Al-Hunaidi 1998) and the simulated annealing procedure (Martinez *et al*. 2000) have been applied. Joint inversion with other techniques, especially electric and electromagnetic techniques (Hering *et al*. 1995; Comina *et al*. 2002), and coupled inversion of phase velocity and attenuation (Lai 1998) have been used. Often only the first mode is considered, but interesting examples of inversion of several modes (Gabriels *et al*. 1987) and apparent curves have been discussed. Recently the inversion of the full waveform has been proposed (Fobriger 2003a,b) in order to overcome the limitations of the usual approach based on the inversion of the dispersion curve.

In the following, the inversion is analysed focusing mainly on the properties of the solution, the quality and the amount of information, the effects of the possible misinterpretation of some data features, the model parametrization and the adopted constitutive law: all these aspects strongly influence the final result but are slightly influenced by the chosen inversion algorithm. For this reason, we will not concentrate our attention on algorithms but we will focus much more on the physical relationship between the data and the model parameters, with the aim of pointing out some pitfalls and rules of good practice that are independent of the inversion algorithm that is chosen.

The properties of the solution are considered starting from the analysis of the data to be inverted. These data have to be analysed with the aim of increasing the amount of information produced, reducing the amount of useless data that duplicate the same information and selecting other properties of the propagation, besides the velocity, that can add useful independent information about the model.

The other aspect to be examined is the model in terms of its parametrization, the reliability of the estimated parameters and the depth of investigation.

The data to be inverted

The data to be inverted usually consist of the Rayleigh-wave velocity as a function of the frequency (dispersion curve), but could also include other propagation properties, such as the power spectra or the distribution of the energy as a function of the frequency along a mode. The attenuation and the velocity of the layers are physically coupled, and may be uncoupled in weakly dissipative media (Lai and Rix 2002; Foti 2004, this issue). Usually only the velocity is considered while the energy distribution is disregarded: both the phase velocity and group velocity can be inverted.

Due to the nature of the data that are inverted, the SWM inverse problem is typically mix-determined. Therefore, the high-frequency data can be used to estimate the properties at shallow depth, but, on the other hand, the properties at shallow depth influence the data of all the frequencies, making the inversion of data that are acquired, for instance on pavements, particularly complex (Ryden *et al.* 2004, this issue).

The modal curves

To estimate the information content of the data, the sensitivity of modal curves to the variation in model parameters ($\frac{\partial V_R}{\partial V_S}$) can be considered (see Fig. 5). The sensitivity shows how the variation of a model parameter can influence the data and it is therefore a measure of the information present in the data about that variation; it is thus an estimate of the detectability of the parameter variation. This approach could be used for the assessment of the properties of the solution and for the design of experiments; however, it must be remembered that this simple approach has a limit due to the energy distribution on modes, which can make some modes undetectable, and can influence the modal superposition. The modal curves are only possible solutions: any considerations about the information in modal curves cannot disregard the real possibility of detecting them. The modal superposition can play a strong role both in normal dispersive and inversely dispersive sites (see Fig. 6). The array length influences the modal superposition in experimental data, and has to be considered in the inversion.

The apparent-dispersion curve

The apparent-dispersion curve can be the result of the superposition of different modes in an experimental configuration. Of course all the influencing factors have to be considered in the inversion, and therefore modelling that simulates modal superposition has to be used. The inversion of such a function poses a series of problems: the jumps between different modes are discontinuities, producing large variations in the data even for small variations in the model parameters. When different modal curves are obtained in experimental data, a joint inversion of different modal curves can be performed. However the identification of mode number is not straightforward and, as will be shown below, large errors can be generated by the assumption of the wrong mode number for a branch of an experimental curve. Besides this, the sensitivity of the apparent-dispersion curve to model parameters, which could supply information about resolution and investigation depth, is not easily assessed and extensive modelling must be employed (Socco and Strobbia 2003). In many cases, a trial and error inversion is the only way to obtain a stable result when inverting apparent curves: it can be used, of course, as a first estimate for a local-search refinement.

The energy distribution

Along a single modal curve, even with a flat frequency band source and not considering the attenuation, the energy distribution can be very inhomogeneous. When strong acoustic impedance changes occur at the site within the depth of investigation, resonant frequencies in the displacements at the free surface are easily observed and identified; in addition, since the modelling allows for amplitude simulation, the energy distribution could be introduced within the inversion procedure. But it must then be considered that a quantitative analysis of the amplitude implies knowledge and control of the frequency response of all the acquisition and processing steps that influence the data and this is not an easy task to be achieved. Also, dissipative characteristics of the subsoil have to be accounted for in the inversion if the amplitude is considered, for instance with a joint inversion.

The data distribution

The sampling of the dispersive characteristics in the frequency domain is crucial for the inversion since it heavily influences the computation time and the uncertainty of the final model. The information density differs in the different frequency bands, and so different sampling strategies may be adopted. One method of assessing the independence of data is the analysis of the data resolution matrix (Menke 1989), which can be obtained from the Jacobian matrix of the final model achieved by inversion (con-

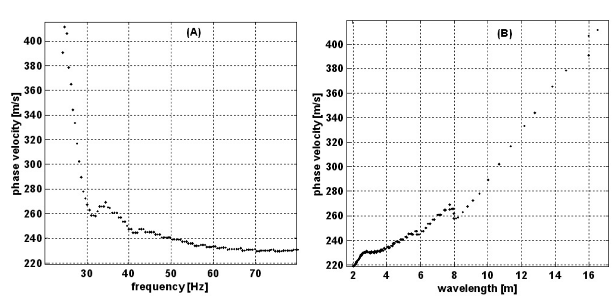

FIGURE 13

Comparison between an experimental dispersion curve represented as velocity versus frequency (a) and velocity versus wavelength (b). A uniform sampling in the frequency domain corresponds to a coarser sampling of long wavelengths.

taining the partial derivative of the Rayleigh-wave velocity at the different frequencies with respect to the shear-wave velocity of the different layers).

It is easy to show that a uniform sampling in the frequency domain corresponds to a coarser sampling of the wavelength at low frequency. Figure 13 shows an experimental dispersion curve: phase velocity versus both frequency and wavenumber.

Even if the description of the non-linear relationships between data space and model space is not straightforward, in order to sample the different depths in the model space adequately, it is necessary to sample the different wavelengths in the data space adequately.

The model parameters

As mentioned above, the model parameters are in general the physical and geometric properties of the model, i.e. the shear-wave velocity, Poisson's ratio, the damping ratios, the bulk density and the thickness of the layers. The choice of the model parametrization depends on the available forward modelling and on the need to balance the number of unknowns with the information content and the uncertainty of the data. In the layered model illustrated, the model parameters are the properties of each layer and the characteristics of the acquisition layout. The latter is known and has to be introduced into the modelling to compare synthetic data with experimental data coherently. Conversely, the layer properties are the unknowns of the inversion process and have to be selected according to the sensitivity of the propagation of surface waves to their variations. Parametric studies (Nazarian and Stokoe 1984; Xia *et al*. 1999) indicate that the shear-wave velocity and the thickness of the layers are the parameters to which the propagation is more sensitive, while the mass density, which has small ranges and influence, can be estimated *a priori*. For the same reason, Poisson's ratio is also usually assumed *a priori* or neglected, but it can be shown (Foti and Strobbia 2002) that the effect of the saturating water on the P-wave velocity, and therefore on Poisson's ratio, is high, and the abrupt variation of the compressibility of the medium can produce effects that should be accounted for. Therefore information on the water-table position should be used to make a proper *a priori* choice of Poisson's ratio values and to avoid possible overestimation of the velocities in the inversion.

Number of layers and resolution

The layered model can be viewed either as the description of the geological features (when significant property contrasts between the materials are present) or as the discretization of the investigated domain. In the first case, both velocity and layer thickness are unknown, in the second case, only velocity is considered unknown and the discretization has to be chosen in accordance with the loss of resolution with depth. In both cases, the parametrization, i.e. the number of layers and the maximum depth, has to be carefully designed for evaluating the investigation depth and the resolution.

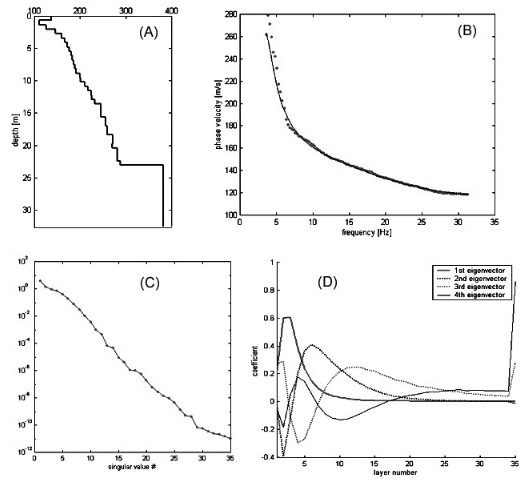

FIGURE 14

Resolution assessment with the singular value decomposition. An over-parametrized model (a) is assumed to invert an experimental dispersion curve. The fit of the experimental curve (dots) to the synthetic (solid line) is shown in (b). The SVD of the final linearized model gives the series of singular values sorted and shown in (c): the first four eigenvectors are depicted in (d), by plotting their 35 coefficients. The eigenvector associated with the first singular value indicates an average of the first five layers.

The number of layers has to be limited according to the amount of information present in the data and avoiding over-parametrization or under-parametrization of the model, which could lead to non-significant results and to inversion artefacts. In view of this, the resolution in the different parts of the model can be estimated (Menke 1989) with the model resolution matrix, or with the singular value decomposition (SVD) method. The SVD (Lanczos 1961) enables assessment of the information associated with a model parameter: each eigenvector gives a combination of model parameters, and is associated with a singular value, indicating the corresponding amount of information. Figure 14 shows this procedure for an experimental dispersion curve fitted using an over-parametrized model (Figs 14a and b): the unknowns are the shear-wave velocity of the 35 layers, and the SVD analysis of the final model shows their mutual dependence. In Figs 14(c) and 14(d) respectively, the sorted singular values and the first four eigenvectors are plotted, with the coefficients giving the contribution of the different layers. The first singular value corresponds to a weighted average of the first five layers, with the highest coefficients for the second and the third layers. The resolution decreases with depth, and the information on the semi-infinite bedrock is shown by the weight of the 35th layer.

The singular value decomposition stresses the problem of the equivalence of different final models, stating a kind of 'suppression principle': if the singular value is associated with an eigenvector that averages several layers, they may be substituted by a single average layer. A complicated stratigraphy can then be simplified with an equivalent averaging stratigraphy, which is more significant with

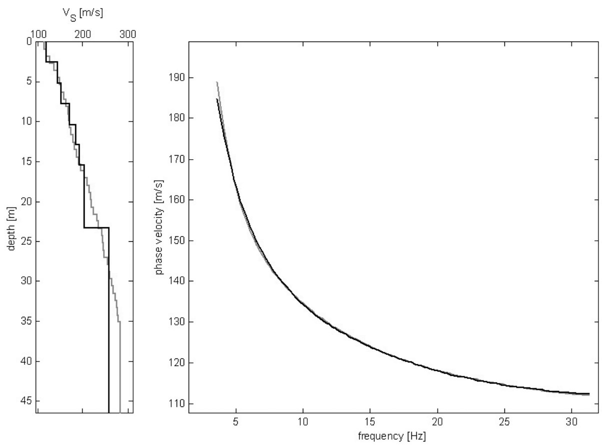

FIGURE 15
Equivalence in surface-wave inversion due to over-parametrization: two different models are shown on the left, and the corresponding first-mode dispersion curves are shown on the right. It can be said that they are equivalent, in the sense that there is not enough resolution to distinguish the rough model.

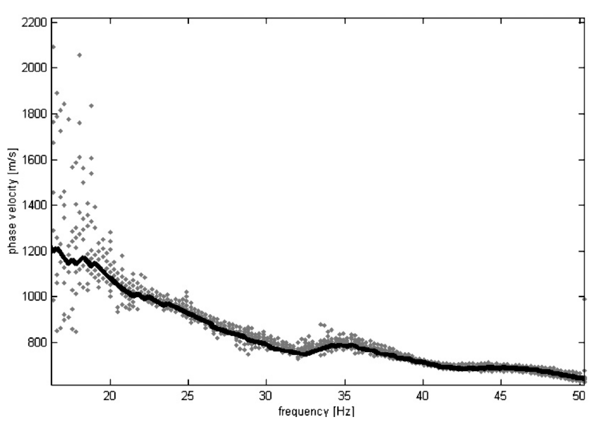

FIGURE 16
The amplification of the uncertainty can give an explosion at low frequencies: the black dots show the curve obtained by processing the averaged seismogram, and the scattered points, single realizations of the test, are derived from repetitions processed singly.

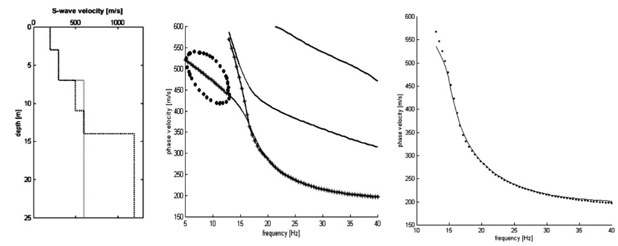

FIGURE 17
A possible pitfall of surface-wave inversion related to higher mode misinterpretation. For a given model (a, solid grey), the apparent curve is computed simulating the modal superposition of a 48 m array (b, modal curves and apparent curve). The apparent curve that would be obtained in an experiment is inverted assuming fundamental-mode propagation: the fitting is shown in (c), and the model obtained (dotted black) is compared to the true one in (a).

respect to resolution and data information content. Figure 15 shows an example of two equivalent models with completely different parametrization: the large number of layers is not resolvable.

We also stress that the analysis of the resolution shows the need to use higher modes when possible.

Investigation depth
The range of the investigation depth depends on the range of the propagating wavelengths and it is therefore related to the acquisition layout and to the site effects. The model parametrization should be performed taking into account the wavelength range within which good quality data are acquired and also considering the limits of the resolution for deeper layers. In general, the minimum depth depends on the minimum sampled wavelength. If no information can be recorded at high frequencies, the properties of the shallower layers of the model can be influenced by a thin layer that is not properly sampled (for instance a pavement): in this case it is necessary to incorporate *a priori* information about this layer. The maximum depth depends on the maximum reliably estimated wavelength: different authors suggest, as a rule of thumb, limiting the maximum depth to one-half of the maximum wavelength (Shtivelman 1999), but sometimes it is possible to go deeper, down to one wavelength (Herrmann and Al-Eqabi 1991). Large wavelengths correspond to small wavenumbers, for which the amplification of the uncertainty is greater: Figure 16 shows an experimental dispersion curve with the estimated uncertainties increasing at low frequencies. These uncertainties can be further amplified by the inversion, thus reducing the reliability of the deep layers.

Possible pitfalls and errors
One of the main sources of errors in the interpretation procedure is the misinterpretation of some data features, in particular related to disregarding the multimodal nature of the surface-wave propagation. Often a basic assumption is made: 'if the site is normally dispersive and has no strong velocity contrasts the fundamental mode is dominant'. This basic hypothesis deserves some comments: the possibility of acquiring information related to modal curves is implicit in this assumption while, as stated previously, only apparent dispersion characteristics can be acquired; one of the main advantages of the SWM is the possibility of also applying it in inversely dispersive sites, for example, urban sites characterized by noise and by the presence of pavements or soil deposits with slow clayey layers are typical targets for SWM but are not included in the previous basic hypothesis. Hence, applying the method only at normally dispersive sites means imposing

a strong and pointless limitation on the method's potentialities. Furthermore, it is not easy to predict *a priori* that a site is normally dispersive. Moreover, very often, the velocity contrast that exists between a soil deposit and the bedrock or even between a clayey layer and gravel or a sand layer is strong enough to cause the 'jump' of the apparent dispersion curve on higher modes at low frequencies (see e.g. Fig. 6b). When disregarded, this can produce a high degree of error in the inversion process as is easily shown by a simple example (Socco and Strobbia 2002). Considering a three-layer normally dispersive model (Fig. 17a), it is easy to demonstrate by modelling that the apparent dispersion curve obtained with a common 48-channel array is the effect of the superposition of the first mode (dominant at higher frequencies) and of the second mode (dominant at lower frequencies) (Fig. 17b). By considering the inversion of the experimental curve as due only to the first mode, it is possible to obtain a final model with a very good fit to the experimental curve (Fig. 17c) but affected by a high degree of error. On analysing the true model and the final model obtained by inversion, it can be seen that the upper parts of the models, related to the curve branch that does in fact follow the first mode, are in good agreement, while the velocity of the deeper layers, corresponding to the misinterpreted branch of the curve, has been significantly overestimated.

It must be stressed that it is not possible, from the experimental data alone, to recognize if the dispersion pattern obtained is due to a modal superposition or to a single mode and, moreover, if a single mode is the first one or a higher mode. A possible procedure to check the result is to use the final model as input to a multimodal forward modelling, to assess the relative importance of the different modes.

Other errors due to the modal superposition can be caused by the presence of a higher mode which is close to the first mode in a central frequency band. In the case of lack of spectral resolution, the two modes cannot be distinguished and the effect of the dominance of the higher mode in a limited frequency band can appear as an increase in the phase velocity of the first mode within a certain frequency band. The inversion of such types of dispersion curve can lead to a final model characterized by velocity inversion, which is in fact an interpretation artefact.

Another aspect to be considered is the effect of lateral variations. If their presence is not properly recognized, the inversion will produce incorrect final models.

As pointed out in these paragraphs, one of the main aspects of a good quality and reliable inversion procedure is to determine which information contained in the data is to be inverted. After this evaluation, the model parametrization has to follow a rigorous approach in order not to let the inversion algorithm lead the process completely. Finally, the results obtained have to be critically checked, considering the overall reliability and the uncertainties.

A CONCLUSIVE EXAMPLE

The case history described below refers to some measurements performed at a site at Villa Collemandina during the VEL project (Valutazione Effetti Locali – Site Effects Evaluation) promoted by the Seismic Service of the Regione Toscana (Tuscany Regional Administration). The VEL project is a wide-ranging project concerning many sites in different zones of Tuscany. In particular, surveying has been planned in the vicinity of the majority of schools and public buildings with a large investment of financial and technical resources. Seismic refraction (P and S) and downhole tests are the techniques most widely used (about 350 km of P- and S-wave seismic refraction have been acquired and interpreted up to now) but surface waves are starting to be used at some sites. Villa Collemandina is a quiet village in the Garfagnana Hills (the place in which every geophysicist would like to spend his time in the field) and the measurements that will be described were carried out in the vicinity of the primary school, just outside the centre of the village.

The aim of the survey is to reconstruct a subsoil model to be used in the modelling of the seismic response of the site. The site is characterized by fairly soft sediments down to about ten metres, followed by a compact and heterogeneous clayey layer that extends downwards some tens of metres: the top of the bedrock, compact sandstone, is expected to be from 50 to 100 m deep. The main target of the seismic survey is to detect the bedrock depth and to estimate the characteristics (V_S) of the sediments overlying it.

The results of a seismic refraction tomography investigation have confirmed the geological information. In particular, the P-wave survey was able to reach the depth of the bedrock (about 65 m), while the SH-wave survey supplied information about only the uppermost layers, presenting good ray coverage in only some parts of the profile. The pattern of the layers is regular, allowing surface-wave investigation with the opportunity for extending the S-wave information down to greater depths.

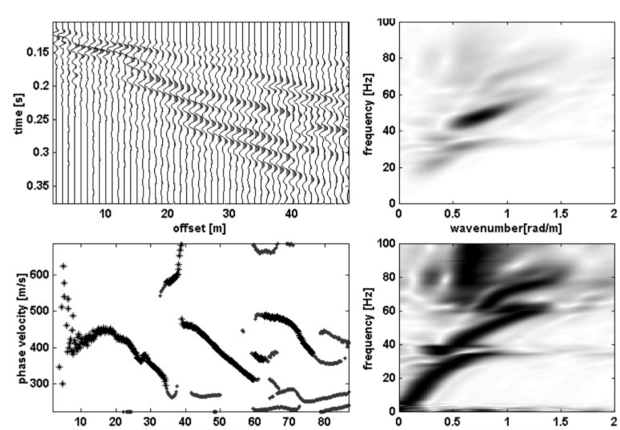

FIGURE 18

Data from Villa Collemandina: the seismogram, the *f–k* spectrum (true amplitude and frequency normalized) and the dispersion curve of one of the acquired shots. In the frequency–phase velocity graph, dots denote relative maxima, while the asterisks denote the absolute maximum at each frequency.

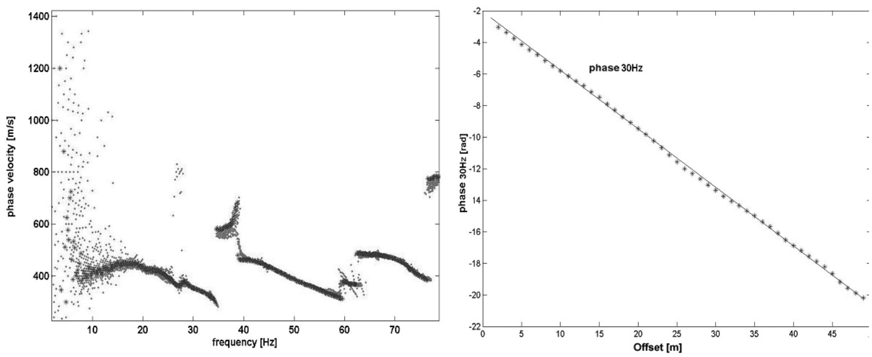

FIGURE 19
Example of preprocessing (same data as Fig. 18). After processing the stacked seismogram, the 30 repetitions of the shot are processed separately to estimate the uncertainty in the different frequency bands: (a) shows the average dispersion curve (asterisks) and the single repetition curves (dots); (b) shows a phase curve obtained by applying the MOPA approach.

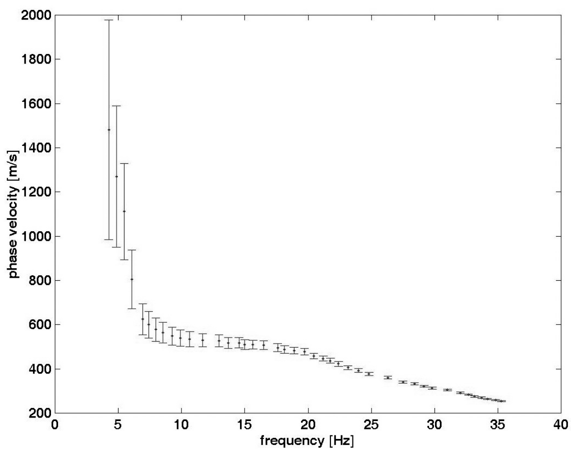

FIGURE 20
The final Rayleigh wave dispersion curve obtained, combining the information from the different arrays.

Acquisition
Acquisition of active data of Rayleigh and Love waves was carried out along three different profiles. Acquisition was performed by a 48-channel GEODE (Geometrics) using 4.5 Hz vertical geophones (Sensor) for Rayleigh waves and passive measurements, and 20 Hz horizontal geophones (Swyphone prototypes) (Sambuelli *et al*. 2002) for Love waves. The geophone spacing was set between 1 m and 2.5m, depending on the profile, and the source position was end-off, shot with a vertical sledgehammer for Rayleigh waves and a horizontal sledgehammer on a sleeper for Love waves. The acquisition parameters for active measurements were set, in order to acquire the full waveform in an adequate frequency band (2–3 s long signals with 1 ms sampling rate), several shots (minimum 10) were recorded separately for each shotpoint for statistic analysis of data and uncertainty estimation.

Processing
The estimate of the dispersion curves (Rayleigh and Love waves) was performed by two main different steps: the first step, the statistical preprocessing, enables checking of the 1D characteristics of the site with respect to the propagation of recorded signals and of the uncertainty estimate; the second step is the evaluation of the dispersion curve searching for absolute and relative maxima in the *f–k* domain and transforming the *k*-values in the phase velocity values. The processing was performed by the POLISURF code, expressly implemented in Matlab at the Applied Geophysics Laboratory of the Politecnico di Torino (Strobbia 2002a,b).

In the following some intermediate results of the processing of the acquired data are shown and discussed. Figure 18 shows the stacked seismograms, the *f–k* spectrum and the corresponding dispersion curve for one of the acquired shots. The spectrum in Fig. 18 shows a strong energy concentration, between 45 and 55 Hz, probably related to shallow soft sediments. In spite of this energy concentration, when searching for absolute and relative maxima, it is possible to select several branches of the dispersion curve in an adequate frequency range. Analysing the dispersion curve obtained, it can be seen that the apparent curve (absolute maxima, denoted by asterisks) appears to follow three different modes up to a frequency of about 70 Hz. These dispersion curve branches show, in agreement with the geological information, an increase in the phase velocity at low frequencies. The cut-off frequencies of higher modes indicate the presence of a layer of considerable thickness characterized by a Rayleigh-wave velocity greater than 450–500 m/s below a soft shallow layer. For the estimate of the velocity at greater depths, it will be shown that at low frequencies (below 10–15 Hz) the dispersion curve increases steeply up to a velocities higher than 1000 m/s (Fig. 20). The possibility of reaching such low frequencies with such high velocity values allows greater investigation depths to be reached at the field site. The relative maxima (blue dots) enable the enlargement of the frequency range within which the modal curves are obtained and some secondary but still coherent events due to noise or higher modes, that become important at high frequencies, to be located.

Figure 19 shows some steps of preprocessing: in Fig. 19(a), for the short array, the dispersion computed from the averaged seismogram (asterisks) is compared with the single dispersion curves of the single shots (dots). This clearly shows the frequency bands of increase in uncertainty. An example of MOPA analysis to assess the presence of lateral variations is shown in

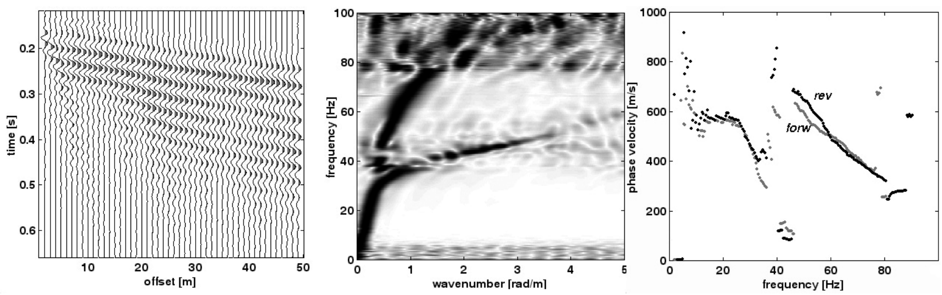

FIGURE 21
Love-wave data at Villa Collemandina. The seismogram (a), the *f–k* spectrum (b), and the dispersion curves for two opposite shots (c) are plotted.

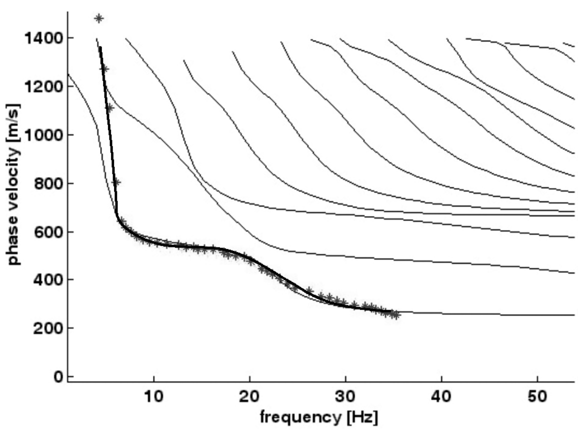

FIGURE 22
The final experimental Rayleigh-wave dispersion curve obtained, combining all the records (asterisks), is plotted with the synthetic best-fitting curve (thick solid line) and the modal curves (thin solid lines).

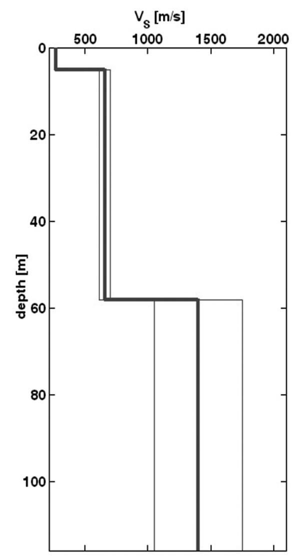

FIGURE 23
The final model with minimum parametrization.

Fig. 19(b): at 30 Hz, good linearity of the phase indicates good lateral homogeneity for the corresponding wavelength. Higher frequencies are shown to be more problematic from the point of view of lateral variations, but the loss of information relative to shallow layers does not affect the main objective of the survey.

All the acquired data were processed as described above, showing substantial agreement among them. From the combined information obtained by different shots, an experimental dispersion curve to be interpreted was selected (Fig. 20, dots).

Besides Rayleigh-wave data, Love waves were also acquired and in Fig. 21(a) an example of a seismogram acquired with horizontally polarized source and receivers is shown. The existence of high-amplitude Love waves depends on the site: in our case the acquired signal shows a strong preponderance of Love waves over a wide frequency band. These were processed using the same procedure as described for Rayleigh waves. By comparing the curves relating to different shots, i.e. forward and reverse shots, another confirmation of the 1D nature of the site was made; the curves obtained are, in fact, very similar (Fig. 21c) and also corroborate the information deduced by the preliminary analysis of the Rayleigh-wave data.

In particular, the first mode of Love waves (the pattern is confirmed by a clear higher mode) presents the typical shape of a three-layer model, characterized by a relatively slow (about 300 m/s) shallow layer, lying on a layer with higher velocity (around 600 m/s), and thick enough to establish the value of the dispersion curve over a fairly wide frequency band; a high-velocity layer is then revealed at a greater depth by the steep increase in the phase velocity at low frequencies. It is also worth noting that, in this case, it has been possible to obtain good quality data for frequencies below the proper frequency of the transducers used.

Inversion

The inversion of the experimental dispersion curve was carried out in three steps. The first step consists of the definition of an initial model, based on the representation of the data in the $\lambda/3$ versus apparent shear velocity domain. This smoothed profile is then transformed by the operator into a simple layered model with a limited number of layers. The second step is the inversion of the data assuming that the experimental curve is the first Rayleigh-wave mode of propagation. The inversion is carried out starting with the inferred initial model and using the w.d.l.s. inversion algorithm, SURF, implemented by Herrmann (1994). The last step consists of checking the obtained model in terms of reliability.

The final model obtained by the first-mode inversion is used as input for a multimodal forward-modelling algorithm, POLISURF (Strobbia 2003), to check the possible effect of modal superposition. As shown in previously, the good fit obtained with the first-mode inversion is not necessarily a proof of the fact that the experimental dispersion curve is the first Rayleigh mode. The multimodal modelling (Fig. 22) has in fact shown that at low frequencies, the influence of the second mode becomes predominent and therefore the experimental point should be attributed partially to the first mode and partially to the second mode, which are almost joined. The velocity and the depth of the deeper layers (overestimated by the first-mode inversion) have been then readjusted with a trial and error procedure, based on multimodal forward modelling.

The final result with minimum parametrization (Fig. 23) shows the presence of three layers: the uppermost one consists of soft sediments (about 300 m/s) and has a thickness of few metres, the second one is a thick layer of compacted clayey sediments (about 600 m/s), and the bedrock (about 1400 m/s) has been found to be at about 55 m depth. These results are in quite good agreement with the P-wave seismic tomography that was acquired along a profile located about 10 m above the ground level of the surface-wave array.

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