

$$1. \lim_{x \rightarrow +\infty} \frac{5x^4 + x^3 + 1}{3^{2x} + 5^x} = \lim_{x \rightarrow +\infty} \frac{x^4 \left(5 + \frac{1}{x} + \frac{1}{x^4} \right)}{9^x \left(1 + \left(\frac{5}{9} \right)^x \right)} = 0$$

$\nearrow 5$
 $\nearrow 1$

per confronto infiniti.

(x^4 è infinito di ordine minore di 9^x .)

$$2. \lim_{x \rightarrow -\infty} \frac{5x^4 + x^3 + 1}{3^{2x} + 5^x} = \lim_{x \rightarrow -\infty} \frac{x^4 \left(5 + \frac{1}{x} + \frac{1}{x^4} \right)}{\underbrace{3^{2x} + 5^x}_{\downarrow 0^+}} = +\infty$$

$\nearrow +\infty$
 $\nearrow 5$

$$3. \lim_{x \rightarrow 0^+} 4x^4 \lg(x^5 + x^2) = \lim_{x \rightarrow 0^+} 4x^4 \lg \left[\underbrace{x^2(1+x^3)}_{\downarrow 0} \right] =$$

$$= \lim_{x \rightarrow 0^+} \underbrace{4x^2}_{\downarrow 0} \underbrace{\frac{x^2(1+x^3)}{1+x^3}}_{\downarrow 1} \lg(x^2(1+x^3)) = 0$$

$\downarrow 0$ ($\lim_{y \rightarrow 0^+} y \cdot \lg y = 0$)

$$4. \lim_{x \rightarrow 0^+} x^{x \lg x} = \lim_{x \rightarrow 0^+} e^{x \lg x \lg x}$$

$$= \lim_{x \rightarrow 0^+} e^{\underbrace{x(\lg x)^2}_{\downarrow 0}} = 1$$

$$\boxed{\lim_{x \rightarrow 0^+} x(\lg x)^2 = 0}$$

$$\begin{aligned}
 \underline{5.} \quad \lim_{x \rightarrow 0^+} |\lg x|^{\frac{1}{x}} &= \lim_{x \rightarrow 0^+} \lg\left(\frac{1}{x}\right)^{\frac{1}{x}} \\
 &= \lim_{y \rightarrow +\infty} (\lg y)^y = \lim_{y \rightarrow +\infty} e^{y \lg(\lg y)} = +\infty
 \end{aligned}$$

$\left[\begin{array}{l} \text{per } x < 1 \\ \lg x < 0 \\ |\lg x| = -\lg x = \\ = \lg \frac{1}{x} \end{array} \right]$

(2)

$$\begin{aligned}
 \underline{6.} \quad \lim_{x \rightarrow +\infty} \frac{\lg x^5 + \sqrt{x} + 2x}{x + \arctg x + \sin x} &= \\
 &= \lim_{x \rightarrow +\infty} \frac{\cancel{x} \left[\underbrace{\frac{\lg x^5}{x}}_{\downarrow 0} + \underbrace{\frac{1}{\sqrt{x}}}_{\downarrow 0} + 2 \right]}{\cancel{x} \left[1 + \underbrace{\frac{\arctg x}{x}}_{\downarrow 0} + \underbrace{\frac{\sin x}{x}}_{\downarrow 1} \right]} = 2
 \end{aligned}$$

$$\begin{aligned}
 \underline{7.} \quad \lim_{x \rightarrow +\infty} (x + \sqrt{x}) \sin\left(\frac{5}{x}\right) & \quad \frac{5}{x} = y \quad x = \frac{5}{y} \\
 & \quad y = \frac{5}{x} \Rightarrow x = \frac{5}{y} \\
 &= \lim_{y \rightarrow 0^+} \left(\frac{5}{y} + \sqrt{\frac{5}{y}} \right) \sin y = \\
 &= \lim_{y \rightarrow 0^+} \underbrace{\left(5 + \sqrt{5\sqrt{y}} \right)}_{\downarrow 5} \underbrace{\frac{\sin y}{y}}_{\downarrow 1} = 5
 \end{aligned}$$

$$\underline{8.} \quad \lim_{x \rightarrow \pi^-} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{1 - \cos^2 x} = \quad 1 - \cos^2 x = \sin^2 x$$

$$= \lim_{x \rightarrow \pi^-} \frac{\sqrt{(1+\sin x)} - \sqrt{1-\sin x}}{\sin^2 x} \cdot \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}} = \quad (3)$$

$$= \lim_{x \rightarrow \pi^-} \frac{1+\sin x - 1 + \sin x}{\sin^2 x} \cdot \frac{1}{\sqrt{1+\sin x} + \sqrt{1-\sin x}} =$$

$$= \lim_{x \rightarrow \pi^-} \frac{2}{\sin x} \cdot \frac{1}{\sqrt{1+\sin x} + \sqrt{1-\sin x}} = +\infty$$

$\downarrow$ $\downarrow$
 0^+ 2

9. $\lim_{x \rightarrow -\infty} (\sqrt{3x^2-x} - \sqrt{3x^2+x+1}) \cdot \frac{\sqrt{3x^2-x} + \sqrt{3x^2+x+1}}{\sqrt{3x^2-x} + \sqrt{3x^2+x+1}} =$

$$= \lim_{x \rightarrow -\infty} \frac{3x^2-x - (3x^2+x+1)}{\sqrt{3x^2-x} + \sqrt{3x^2+x+1}} = \lim_{x \rightarrow -\infty} \frac{-2x-1}{\sqrt{3x^2-x} + \sqrt{3x^2+x+1}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x-1}{\sqrt{x^2} \sqrt{3-\frac{1}{x}} + \sqrt{x^2} \sqrt{3+\frac{1}{x}+\frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{-2x-1}{-x \left[\sqrt{3-\frac{1}{x}} + \sqrt{3+\frac{1}{x}+\frac{1}{x^2}} \right]}$$

$\sqrt{x^2} = -x \text{ dato che } x < 0!$

$$= \lim_{x \rightarrow -\infty} \frac{\cancel{-x} \left(+2 + \frac{1}{x} \right)^{\frac{1}{2}}}{\cancel{-x} \left[\underbrace{\sqrt{3-\frac{1}{x}}}_{\downarrow 3} + \underbrace{\sqrt{3+\frac{1}{x}+\frac{1}{x^2}}}_{\downarrow 3} \right]} = \frac{2}{6} = \frac{1}{3}$$

10. $\lim_{x \rightarrow +\infty} \frac{1+3\sin x - x \sin 2x}{x^2-1} =$

$$= \lim_{x \rightarrow +\infty} \frac{x \left(\frac{1}{x} + \frac{3 \sin x}{x} - \sin^2 x \right)}{x \left(1 - \frac{1}{x} \right)} = 0 \quad (4)$$

$\downarrow$ limitato
 $\downarrow 1$

11. $\lim_{x \rightarrow +\infty} \frac{\lg(\lg x)}{1 + \lg x} = \lim_{y \rightarrow +\infty} \frac{\lg(y)}{1 + y} = 0$ per gerarchie infinite

$y = \lg x$

12. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} =$

$$= \lim_{x \rightarrow 0} \frac{1+x - (1-x)}{x [\sqrt{1+x} + \sqrt{1-x}]} = \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{1+x} + \sqrt{1-x})} = 1$$

$\downarrow 2$

13. $\lim_{x \rightarrow 0} \frac{\sin(\lg x)}{\sin x} = \lim_{x \rightarrow 0} \frac{\sin(\lg x)}{(\lg x)} \cdot \frac{\lg x}{x} \cdot \frac{x}{\sin x}$

$\downarrow 1$ $\downarrow 1$ $\downarrow 1$
 $\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$
 $y = \lg x$

$= 1$

14. $\lim_{x \rightarrow \sqrt{2}} \frac{x - \sqrt{2}}{\sin(x^2 - 2)} = \lim_{x \rightarrow \sqrt{2}} \frac{x^2 - 2}{\sin(x^2 - 2)} \cdot \frac{1}{x + \sqrt{2}} = \frac{1}{2\sqrt{2}}$

$\downarrow 1$ $\downarrow 2\sqrt{2}$
 $\lim_{y \rightarrow 0} \frac{y}{\sin y} = 1$

$$15. \lim_{x \rightarrow -\infty} \underbrace{\sqrt{x^2+3x-2}}_{+\infty} + \underbrace{x}_{-\infty} = \lim_{x \rightarrow -\infty} \left(\sqrt{x^2+3x-2} + x \right) \frac{(\sqrt{x^2+3x-2} - x)}{(\sqrt{x^2+3x-2} - x)}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2+3x-2-x^2}{\sqrt{x^2+3x-2}-x} = \lim_{x \rightarrow -\infty} \frac{x(3-\frac{2}{x})}{\underbrace{\sqrt{x^2}}_{-x} \sqrt{1+\frac{3}{x}-\frac{2}{x^2}}-x} =$$

$$= \lim_{x \rightarrow -\infty} \frac{x(3-\frac{2}{x})}{-x \left[\sqrt{1+\frac{3}{x}-\frac{2}{x^2}} + 1 \right]} = -\frac{3}{2}$$

$$16. \lim_{x \rightarrow +\infty} 3^{x+1} - 3^{\sqrt{x^2+1}} = \lim_{x \rightarrow +\infty} 3^{x+1} \left[1 - 3^{\sqrt{x^2+1}-x-1} \right]$$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2+1}-x-1 = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+1}-x-1)(\sqrt{x^2+1}+x+1)}{\sqrt{x^2+1}+x+1} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2+1-(x+1)^2}{\sqrt{x^2} \sqrt{1+\frac{1}{x^2}}+x+1} = \lim_{x \rightarrow +\infty} \frac{-2x}{x \left[\sqrt{1+\frac{1}{x^2}} + 1 + \frac{1}{x} \right]} = -1$$

$\sqrt{x^2} = x!$

so Quindi $\lim_{x \rightarrow +\infty} (1 - 3^{\sqrt{x^2+1}-x-1}) = 1 - 3^{-1} =$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

$$\lim_{x \rightarrow +\infty} 3^{x+1} \left(1 - 3^{\sqrt{x^2+1}-x-1} \right) = +\infty.$$

17. ~~lim x → +∞~~ ~~1/3~~ ~~sin~~ $a > 0$

⑥

$$\lim_{x \rightarrow +\infty} \frac{a^x + 4^x + \sinh x}{7^x + 2^x \sin(e^x)} = \lim_{x \rightarrow +\infty} \frac{a^x + 4^x + \frac{e^x - e^{-x}}{2}}{7^x + 2^x \sin(e^x)} =$$

$$\frac{e^{-x}}{a^x} = (ea)^{-x} \rightarrow 0$$

① caso $a > 4$

$$= \lim_{x \rightarrow +\infty} \frac{a^x \left[1 + \left(\frac{4}{a}\right)^x + \frac{1}{2} \left(\frac{e}{a}\right)^x - \frac{1}{2} \left(\frac{e}{a}\right)^{-x} \right]}{7^x \left[1 + \left(\frac{2}{7}\right)^x \sin(e^x) \right]} =$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{a}{7} \right)^x \left[\frac{1 + \left(\frac{4}{a}\right)^x + \frac{1}{2} \left(\frac{e}{a}\right)^x - \frac{1}{2} \left(\frac{e}{a}\right)^{-x}}{1 + \left(\frac{2}{7}\right)^x \sin(e^x)} \right]$$

$$a > 7 \quad \left(\frac{a}{7}\right)^x \rightarrow +\infty \quad \frac{4}{a} < 1 \quad \frac{e}{a} < 1$$

DUNQUE $a > 7 \quad \lim_{x \rightarrow +\infty} () = +\infty$

$a = 7 \quad \lim_{x \rightarrow +\infty} () = 1$

$4 < a < 7 \quad \lim_{x \rightarrow +\infty} () = 0$ dato che $\left(\frac{a}{7}\right)^x \rightarrow 0$

② caso $a = 4$

$$= \lim_{x \rightarrow +\infty} \frac{4^x \left(2 + \left(\frac{e}{4}\right)^x \frac{1}{2} 4 - \left(\frac{e}{4}\right)^{-x} \frac{1}{2} \right)}{7^x \left(1 + \left(\frac{2}{7}\right)^x \sin(e^x) \right)} = 0$$

③ caso $a < 4$

$$= \lim_{x \rightarrow +\infty} \frac{4^x \left(\frac{a^x}{4^x} + 1 + \frac{e^x}{2a^x} - \frac{e^{-x}}{2a^x} \right)}{7^x \left(1 + \left(\frac{2}{7} \right)^2 \sin e^x \right)} = 0$$

②

Riassumendo

[	$a > 7$	limite $\bar{e} + \infty$
	$a = 7$	limite $\bar{e} 1$
	$a < 7$	limite $\bar{e} 0$

