ESERCIZI SU STUDIO DI FUNZIONE RISOLTI

(1) $\boxed{f(x) = \operatorname{arctg}\left(\dfrac{\sqrt[5]{x^4}}{1+x^4}\right)}$

Non è richiesto lo studio di f''.

1. $D(f) = \mathbb{R}$
2. $f(x) = f(-x)$ f è PARI (il grafico è simmetrico rispetto all'asse delle y)
3. $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \operatorname{arctg}\left(\dfrac{x^{4/5}}{1+x^4}\right) =$

$$= \lim_{x\to+\infty} \operatorname{arctg} \frac{x^{4/5}}{x^4\left(1+\frac{1}{x^4}\right)} = \lim_{x\to+\infty} \operatorname{arctg}\underbrace{\left(\frac{1}{x^{\frac{16}{5}}\left(1+\frac{1}{x^4}\right)}\right)}_{\downarrow \atop 0}$$

$$= 0$$

$y=0$ È ASINTOTO ORIZZONTALE ~~A~~ $+\infty$ (E ~~A~~ $-\infty$ PER SIMMETRIA)

4. f è continua

$$f'(x) = \frac{1}{1+\left(\frac{x^{4/5}}{1+x^4}\right)^2} \cdot \frac{\frac{4}{5}x^{-1/5}(1+x^4) - 4x^3x^{4/5}}{(1+x^4)^2} =$$

$$= \frac{4}{5}\,\frac{1-4x^4}{x^{1/5}\left[(1+x^4)^2 + x^{8/5}\right]}$$

f^* è derivabile in $x \neq 0$.

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \frac{4}{5} \frac{1-4x^4}{x^{1/5}\left[(1+x^4)+x^{8/5}\right]} = +\infty$$

$$\lim_{x \to 0^-} f'(x) = -\infty$$

$x=0$ è un punto di <u>non</u> derivabilità.
è una CUSPIDE.

5. $f'(x) \geq 0$. $\quad f'(x) = \frac{4}{5} \frac{(1-2x^2)(1+2x^2)}{x^{1/5}\left[(1+x^4)^2+x^{8/5}\right]}$

1) $1-2x^2 \geq 0 \Rightarrow 2x^2-1 \leq 0 \Rightarrow -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

2) $1+2x^2 \geq 0 \quad \forall x$

3) $x^{1/5} > 0 \Rightarrow x > 0$

4) $(1+x^4)^2 + x^{8/5} > 0 \quad \forall x$

Studio il prodotto dei segni:

		$-1/\sqrt{2}$		$1/\sqrt{2}$	
1)	$-$	$+$	$+$	$-$	
2)	$+$	$+$	$+$	$+$	
3)	$-$	$-$	0 $+$	$+$	
4)	$+$	$+$	$+$	$+$	
	$+$	$-$	$+$	$-$	

$f' \geq 0$ se

$x \leq -\frac{1}{\sqrt{2}}$ o

& $0 < x \leq \frac{1}{\sqrt{2}}$

Segno di f'

	$-1/\sqrt{2}$		0		$1/\sqrt{2}$	
$+$		$-$		$+$		$-$

Monotonia di f

	$-1/\sqrt{2}$		0		$1/\sqrt{2}$	
$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

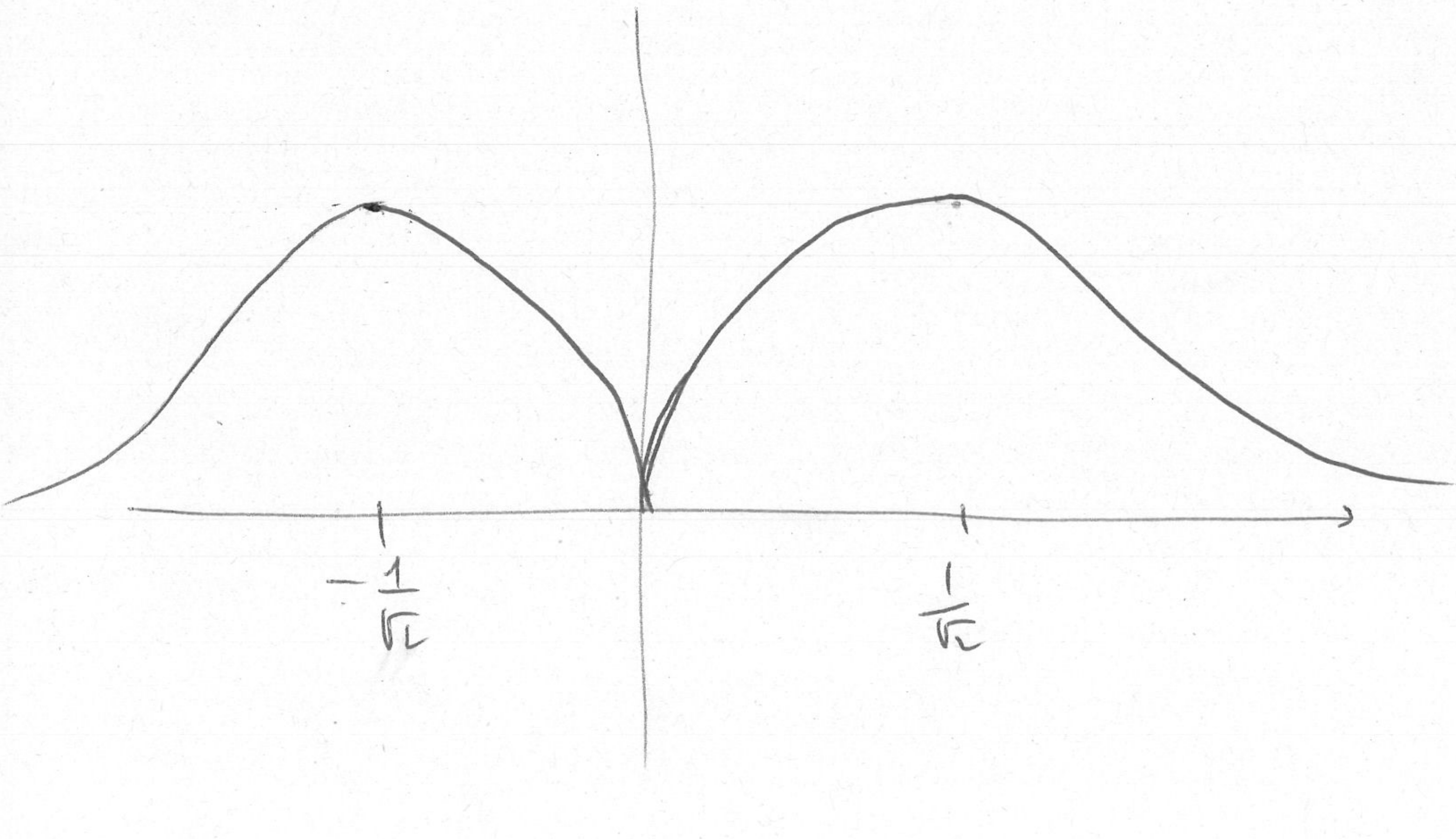
$x = \frac{1}{\sqrt{2}}$, $x = -\frac{1}{\sqrt{2}}$ PTI DI MAX LOCALE

Sono anche pti di max assoluto

$x = 0$ PTO DI MINIMO LOCALE

$f(0) = 0$ $\cancel{\Rightarrow}$ $x = 0$ è anche pto di min assoluto

6. Grafico



2) $$\boxed{f(x) = (1+x)\, e^{-\frac{1}{x-1}}}$$

1. $D(f) = \{x \in \mathbb{R} \mid x \neq 1\} = (-\infty, 1) \cup (1, +\infty)$

2. f non ha simmetrie né periodicità

3. $$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (1+x) \underbrace{e^{-\frac{1}{x-1}}}_{\downarrow\, 1} = +\infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (1+x) \underbrace{e^{-\frac{1}{x-1}}}_{\downarrow\, 1} = -\infty$$

<u>Nota</u>: f non ha max e min assoluto!

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} \underbrace{(1+x)}_{\downarrow\, 2} \underbrace{e^{-\frac{1}{x-1}}}_{\downarrow\, 0} = 0 \qquad -\frac{1}{x-1} \to -\infty$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} \underbrace{(1+x)}_{\downarrow\, 2} \underbrace{e^{-\frac{1}{x-1}}}_{\downarrow\, +\infty} = +\infty \qquad -\frac{1}{x-1} \to +\infty$$

$x = 1$ È ASINTOTO VERTICALE SINISTRO.

Cerco asintoti OBLIQUI

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\overbrace{(1+\frac{1}{x})}^{\to 1}}{\cancel{x}} \underbrace{e^{-\frac{1}{x-1}}}_{\downarrow\, 1} = 1 = m$$

$$\left(\lim_{x\to-\infty} \frac{f(x)}{x} = 1\right)$$

$$\lim_{x\to+\infty} f(x) - mx = \lim_{x\to+\infty} (x+1)\, e^{-\frac{1}{x-1}} - x =$$

$$= \lim_{x\to+\infty} x\left[\left(1+\frac{1}{x}\right) e^{-\frac{1}{x-1}} - 1\right] = \underset{+\infty\cdot 0}{\text{f.i.}}$$

$$= \lim_{x\to+\infty} \frac{\left(1+\frac{1}{x}\right) e^{-\frac{1}{x-1}} - 1}{\frac{1}{x}} = \text{f.i } \frac{0}{0}$$

$$= \underset{(H)}{\text{applico de l'Hopital}} = \lim_{x\to+\infty} \frac{-\frac{1}{x^2} e^{-\frac{1}{x-1}} + \left(1+\frac{1}{x}\right)\frac{1}{(x-1)^2} e^{-\frac{1}{x-1}}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to+\infty} (-x^2)\cdot\left[-\frac{1}{x^2} e^{-\frac{1}{x-1}} + \frac{1}{(x-1)^2}\left(1+\frac{1}{x}\right) e^{-\frac{1}{x-1}}\right] =$$

$$= \lim_{x\to+\infty} \underbrace{\left[1 - \underbrace{\frac{x^2}{(x-1)^2}\left(1+\frac{1}{x}\right)}_{\downarrow\, 1}\right]}_{\downarrow\, 0} \underbrace{e^{-\frac{1}{x-1}}}_{\downarrow\, 1} = 0$$

$y = x$ è ASINTOTO OBLIQUO A $+\infty$
(E A $-\infty$)

$$\lim_{x\to-\infty} f(x) - x = 0.$$

4. f è continua nel suo dominio

$$f'(x) = 1 \cdot e^{-\frac{1}{x-1}} + (1+x)\left(\frac{1}{(x-1)^2}\right) e^{-\frac{1}{x-1}} =$$

$$= \frac{x^2 - x + 2}{(x-1)^2} e^{-\frac{1}{x-1}}$$

f è derivabile nel suo dominio.

Calcolo se esiste $f'_+(1)$

$$\lim_{x \to 1^+} \frac{\overset{\nearrow 2}{x^2 - x + 2}}{\underset{\downarrow 0}{(x-1)^2}} e^{\overset{\to -\infty}{-\frac{1}{x-1}}} \quad \text{f.i } \infty \cdot 0$$

$e^{-\frac{1}{x-1}} \to 0$

CAMBIO VARIABILE $\quad t = \frac{1}{x-1}$

$$t = \frac{1}{x-1} \iff x - 1 = \frac{1}{t} \iff x = 1 + \frac{1}{t}$$

$$x \to 1^+ \iff t \to +\infty$$

$$\frac{x^2 - x + 2}{(x-1)^2} e^{-\frac{1}{x-1}} \longrightarrow \frac{\left(1+\frac{1}{t}\right)^2 - \left(1+\frac{1}{t}\right) + 2}{\left(\frac{1}{t}\right)^2} e^{-t} =$$

$$= \left[\left(1+\frac{1}{t}\right)^2 - \left(1+\frac{1}{t}\right) + 2\right] \frac{t^2}{e^t}$$

$$\lim_{x \to 1^+} f'(x) = \lim_{t \to +\infty} \underbrace{\left[\left(1+\frac{1}{t}\right)^2 - \left(1+\frac{1}{t}\right) - 2\right]}_{+2} \underbrace{\frac{t^2}{e^t}}_{\substack{0 \text{ per} \\ \text{CONFRONTO} \\ \text{INFINITI}}} = 0$$

$$\underline{f'_+(1) = 0}$$

5. Segno $f' \geq 0$

$x^2 - x + 2 \geq 0 \quad \forall x \qquad (\Delta < 0!)$

$(x-1)^2 > 0 \quad \forall x \neq 1$

$e^{-\frac{1}{x-1}} > 0 \quad \forall x \neq 1$

$f' > 0 \quad \forall x \implies f$ è MONOTONA CRESCENTE

6. $$f''(x) = \frac{(2x-1)(x-1)^2 - (x^2-x+2)\,2(x-1)}{(x-1)^4} e^{-\frac{1}{x-1}} + \frac{(x^2-x+2)}{(x-1)^2} \frac{1}{(x-1)^2} e^{-\frac{1}{x-1}}$$

$$= \left[\frac{2x^3 - x^2 - 4x^2 + 2x + 2x - 1 - 2x^3 + 2x^2 + 2x^2 \cancel{\;x\;} - 4x - 2x + 4 + x^2 - x + 2}{(x-1)^4}\right] e^{-\frac{1}{x-1}}$$

$$= \frac{-3x+5}{(x-1)^4} e^{-\frac{1}{x-1}}$$

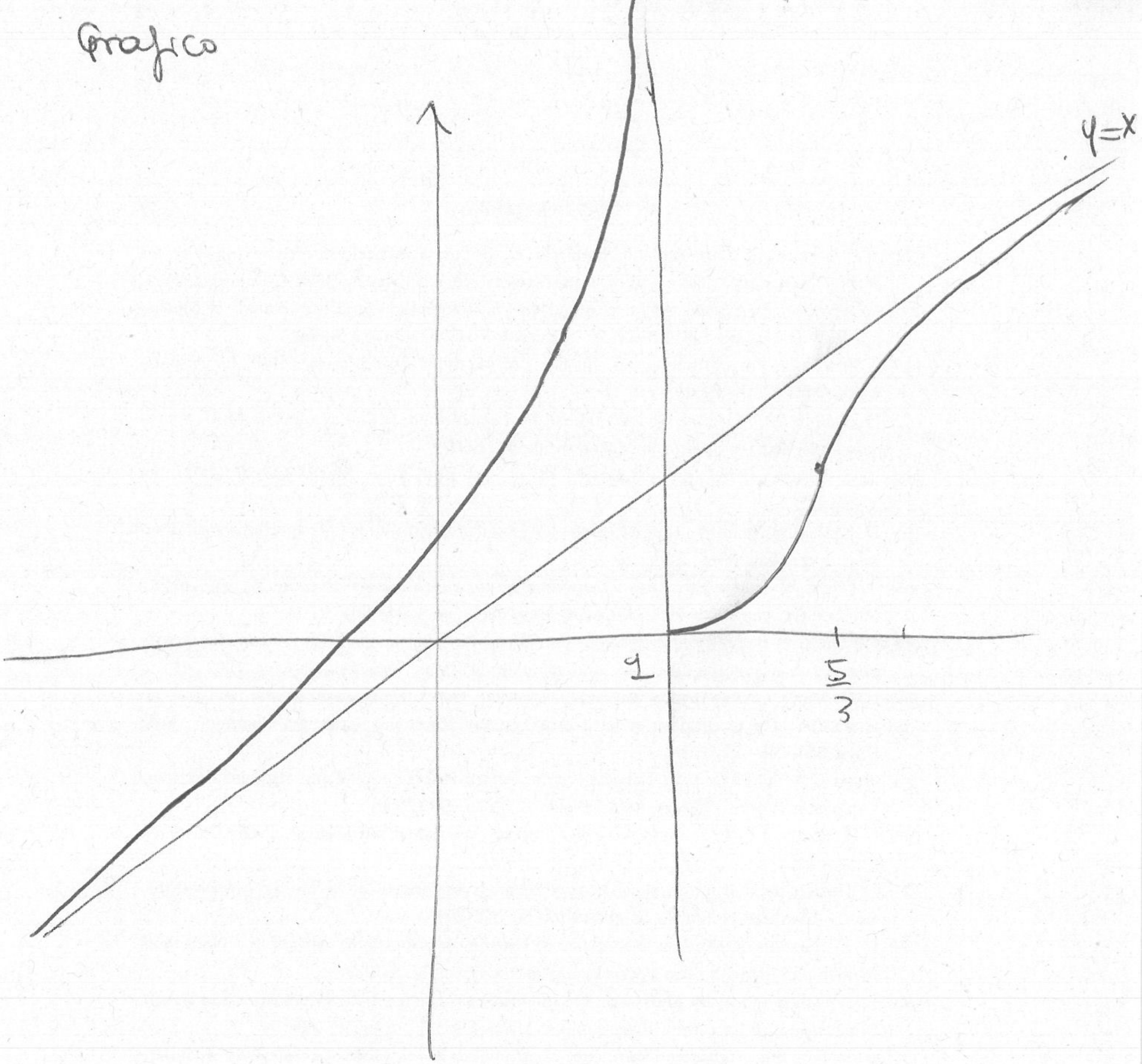
$f'' \geq 0 \iff x \leq \frac{5}{3}$

f è convessa per $x \leq \frac{5}{3}$

concava per $x \geq 5/3$

$x = \frac{5}{3}$ PTO DI FLESSO

7. Grafico



2) $f(x) = \log\left(\frac{x+4}{(x+1)^2}\right)$ (13/12/12)

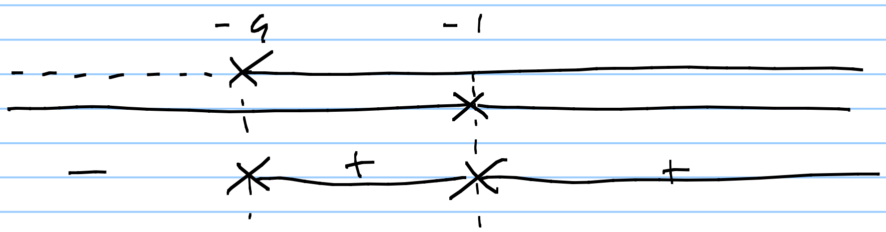
1) dom f

$\rightarrow \quad \frac{x+4}{(x+1)^2} > 0$

$\rightarrow \quad x+1 \neq 0$

$N \geq 0 \qquad x+4 > 0 \qquad x > -4$

$D > 0 \qquad (x+1)^2 > 0 \qquad x \neq -1$



$\text{dom} f = \left\{ x \in \mathbb{R} : \frac{x+4}{(x+1)^2} > 0 \text{ e } x \neq -1 \right\} =$

$= \{ x \in \mathbb{R} : x > -4 , x \neq -1 \} =$

$= (-4, -1) \cup (-1, +\infty)$

2) $A = \{$ pti di accumulazione del dom $f \}$

$\forall x_0 \in A , \lim_{x \to x_0} f(x)$

$A = [-4, -1] \cup [-1, +\infty) \cup \{+\infty\}$

$= [-4, +\infty) \cup \{+\infty\}$

$\lim_{x \to x_0} f(x) = ? \qquad \forall x_0 \in A$

$x_0 \in A \cap (\text{dom} f) = (-4,-1) \cup (-1,+\infty)$

$\underbrace{x_0 \in A \setminus (\text{dom} f)}$ $= \underbrace{\{-4\} \cup \{-1\} \cup \{+\infty\}}$

$\lim_{x \to x_0} f(x) = f(x_0) \qquad \forall x_0 \in \text{dom} f (\cap A)$

$$\lim_{x \to -4^+} \log\left(\frac{x+4}{(x+1)^2}\right) = -\infty$$

$$\lim_{x \to -1} \log\underbrace{\left(\frac{x+4}{(x+1)^2}\right)}_{+\infty} = +\infty$$

$$\lim_{x \to +\infty} \log\left(\underbrace{\frac{x+4}{(x+1)^2}}_{\to 0}\right) = -\infty$$

3) determinare eventuali asintoti

verticali : $x = -4$, $x = -1$ sono asintoti verticali

orizzontali : non ce ne sono

obliqui : $\lim_{x \to +\infty} \dfrac{\log\left(\frac{x+4}{(x+1)^2}\right)}{x} =$

$$= \lim_{x \to +\infty} \underbrace{\underbrace{\frac{1}{x}}_{0} \log\left(\underbrace{\frac{x+4}{(x+1)^2}}_{0}\right)}_{\to -\infty} =$$

$$= \lim_{x \to 0^+} \boxed{\frac{(x+1)^2}{x(x+4)}} \boxed{\frac{x+4}{(x+1)^2} \log\left(\frac{x+4}{(x+1)^2}\right)} = 0$$

$\frac{x+4}{(x+1)^2} = t$; $t \to 0$ se $x \to +\infty$

$$\lim_{t \to 0} t \log t = 0$$

non ci sono asintoti obliqui!

4) Determinare l'insieme B dei punti nei quali f è derivabile e calcolare $f'(x)$, $\forall x \in B$.

Per il teorema sull'algebra delle derivate e il teorema sulla composizione di funzioni derivabili

f è derivabile in x, $\forall x \in \operatorname{dom} f$

$\operatorname{dom} f = B = (-4,-1) \cup (-1,+\infty)$

$$f'(x) = \frac{\cancel{(x+1)^2}}{x+4} \cdot \frac{(x+1)^{\cancel{2}} - 2(x+4)\cancel{(x+1)}}{(x+1)^{\cancel{4}\,\cancel{3}}} =$$

$$= \frac{(x+1) - 2x - 8}{(x+4)(x+1)}$$

$$= \frac{-x-7}{(x+4)(x+1)}$$

Calcolare $\lim_{x \to -1^{\pm}} f'(x) = \lim_{x \to -1^{\pm}} \frac{-x-7}{(x+4)(x+1)} = \mp\infty$

$-x-7$: <0, <0

$(x+4)$: >0, >0

$(x+1)$: >0, <0

5) Determinare gli intervalli di monotonia.

$$f'(x) = -\frac{(x+7)}{(x+4)(x+1)} > 0$$

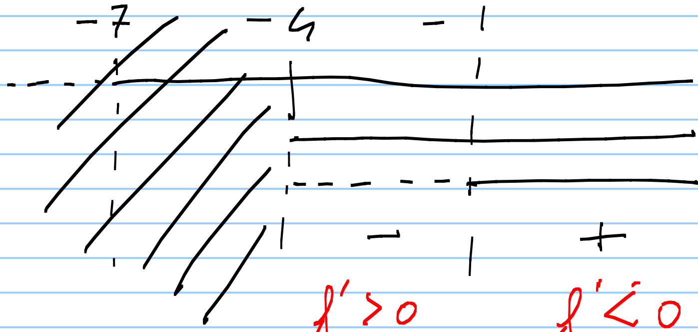
$\Updownarrow$

$$\frac{(x+7)}{(x+4)(x+1)} < 0$$

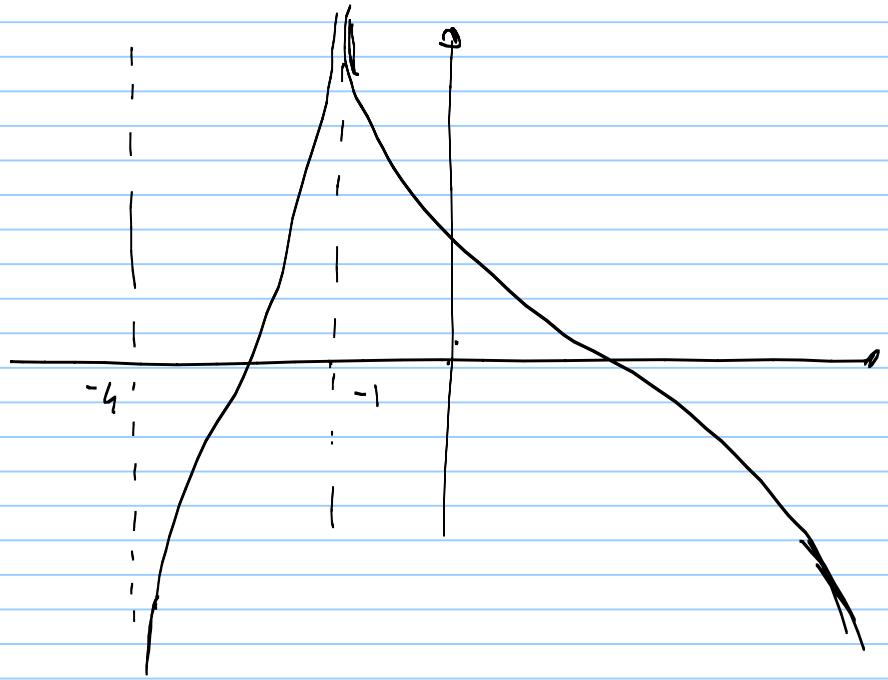
$(x+7) > 0 \iff x > -7$

$(x+4) > 0 \iff x > -4$

$(x+1) > 0 \iff x > -1$



$f' > 0$ ↗ $\qquad$ $f' < 0$ ↘



f è strettamente crescente $(-4, -1)$

f è strettamente decrescente $(-1, +\infty)$

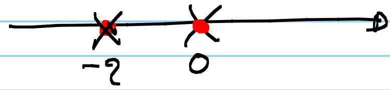
6) Determinare eventuali punti di max/min rel e/o assoluto.

non ci sono max/min rel né assoluti

3) $f(x) = \dfrac{(x+2)}{x}\, e^{-\frac{1}{(x+2)}}$ (19/12/11)

1) $\operatorname{dom} f = \{x \in \mathbb{R},\ x \neq 0,\ x+2 \neq 0\}$

$= \{x \in \mathbb{R} :\ x \neq 0,\ x \neq -2\}$



$= (-\infty, -2) \cup (-2, 0) \cup (0, +\infty)$

2) $A = \{\text{pti di acc. di } \operatorname{dom} f\} =$

$= \{-\infty\} \cup (-\infty, -2] \cup [-2, 0] \cup [0, +\infty) \cup \{+\infty\}$

$= \mathbb{R} \cup \{-\infty\} \cup \{+\infty\}$

$\lim\limits_{x \to x_0} f(x) \qquad \forall x_0 \in A$

•) $x_0 \in A \cap (\operatorname{dom} f) = \operatorname{dom} f = (-\infty, -2) \cup (-2, 0) \cup (0, +\infty)$

f è continua in $x_0 \Rightarrow \lim\limits_{x \to x_0} f(x) = f(x_0)$

•) $x_0 \in A \setminus (\operatorname{dom} f) = \{+\infty\} \cup \{-\infty\} \cup \{-2\} \cup \{0\}$

$$\lim_{\substack{x \to +\infty \\ -\infty}} \underbrace{\frac{(x+2)}{x}}_{\to 1}\ \underbrace{e^{\overbrace{-\frac{1}{(x+2)}}^{\to 0}}}_{\to 1} = 1$$

$$\lim_{x \to -2^+} \underbrace{\frac{(x+2)}{x}}_{\to 0}\ \underbrace{e^{-\frac{1}{(x+2)}}}_{\to 0} = 0$$

$$\lim_{x \to -2^-} \underbrace{\frac{(x+2)}{x}}_{\to 0} \cdot \underbrace{e^{\overbrace{-\frac{1}{(x+2)}}^{t}}}_{\to +\infty} =$$

$t \to +\infty$

$\lim\limits_{t \to +\infty} \dfrac{e^t}{t} = +\infty$

$$\frac{\cancel{x+2}}{x}\left(-\frac{1}{\cancel{(x+2)}}\right)\frac{e^{-\frac{1}{(x+2)}}}{-\frac{1}{(x+2)}} = -\frac{1}{x}\;\frac{e^{-\frac{1}{(x+2)}}}{-\frac{1}{(x+2)}} \to +\infty$$

$$= +\infty$$

$$\lim_{x \to 0^{\pm}} \frac{x+2}{x}\, e^{-\frac{1}{x+2}} = \begin{matrix} +\infty \\ -\infty \end{matrix}$$

3) Asintoti

orizzontali: $y = 1$ sia a $+\infty$ che a $-\infty$

verticali: $x = 0$, $x = -2$

obliqui non ce ne sono (ci sono gli asintoti orizzontali)

4) Determinare $B = \{$punti dove f è derivabile$\}$ e calcolare $f'(x)$, $\forall x \in B$.

$$f(x) = \frac{x+2}{x}\, e^{-\frac{1}{x+2}}$$

per il teo sull'algebra delle derivate e sulla composizione di funzioni derivabili $B = \text{dom } f$ e

$$f'(x) = \left[\frac{\cancel{x} - (\cancel{x}+2)}{x^2}\right] e^{-\frac{1}{x+2}}$$

$$+ \left[\frac{\cancel{x+2}}{x}\right] e^{-\frac{1}{x+2}} \left(\frac{1}{(x+2)^{\cancel{2}}}\right)$$

$$= e^{-\frac{1}{x+2}} \left[-\frac{2}{x^2} + \frac{1}{x(x+2)} \right]$$

$$= e^{-\frac{1}{x+2}} \frac{-2x-4+x}{x^2(x+2)} =$$

$$= - e^{-\frac{1}{x+2}} \frac{4+x}{x^2(x+2)}$$

5) Intervalli di monotonia

$$f'(x) = - e^{-\frac{1}{x+2}} \frac{4+x}{x^2(x+2)} > 0$$

$$\Updownarrow$$

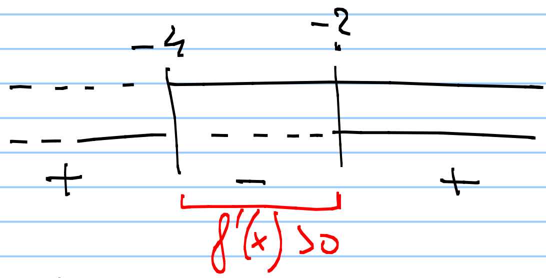
$$\underbrace{\boxed{e^{-\frac{1}{x+2}}}}_{>0} \frac{4+x}{\underbrace{x^2}_{>0}(x+2)} < 0$$

$$\Updownarrow$$

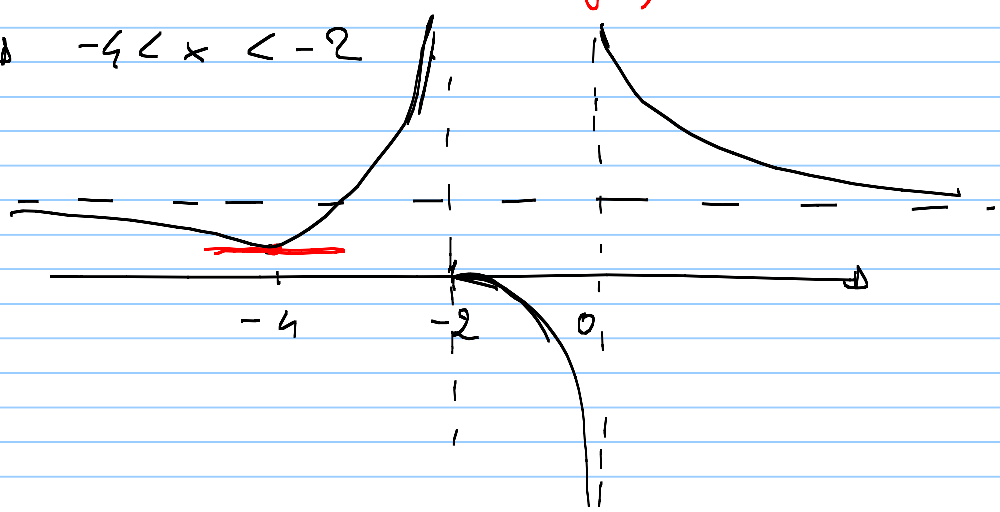
$$\frac{4+x}{(x+2)} < 0$$

$(4+x) > 0 \iff x > -4$

$(x+2) > 0 \iff x > -2$



$f'(x) > 0 \iff -4 < x < -2$



f è decrescente in $(-\infty, -4)$

in $(-2, 0)$

in $(0, +\infty)$

f è crescente in $(-4, -2)$

6) max / min.

$f'(x) = 0 \iff x = -4$

$x = -4$ è di min relativo

non assoluto $\left(\lim_{x \to 0^-} f(x) = -\infty \right.$

$$\lim_{x \to -2^+} f'(x) = \lim_{x \to -2^+} - \underbrace{e^{-\frac{1}{x+2}}}_{\downarrow \atop 0} \underbrace{\boxed{\frac{4+x}{x^2(x+2)}}}_{\downarrow \atop +\infty} =$$

$$= \lim_{x \to -2^+} - \frac{\frac{4+x}{x^2(x+2)}}{e^{\frac{1}{x+2}}} =$$

$$= \lim_{x \to -2^+} - \underbrace{\boxed{\frac{4+x}{x^2}}}_{\downarrow \atop \frac{1}{2}} \boxed{\frac{\frac{1}{(x+2)}}{e^{\frac{1}{(x+2)}}}} = 0$$

$t = \frac{1}{x+2}$

$x \to -2^-$

$t \to +\infty$

$$\lim_{t \to +\infty} \frac{t}{e^t} = 0$$

Esercizio 5

Studiare la funzione $f(x) = \frac{|x-1|}{\lg|x-1|}$

1. $D(f) = \{x \in \mathbb{R} \mid x \neq 1 \quad |x-1| \neq 1\} = \{x \in \mathbb{R}, x \neq 0, 1, 2\}$

$= (-\infty, 0) \cup (0,1) \cup (1,2)$

2. Limiti agli estremi

$$\lim_{x \to 1} \frac{|x-1|}{\lg|x-1|} = 0$$

$$\lim_{x \to 0^+} \frac{|x-1|}{\lg|x-1|} = \lim_{x \to 0^+} \frac{1-x}{\underbrace{\lg(\underbrace{1-x}_{1^-})}_{\to 0^-}} = -\infty$$

$$\lim_{x \to 0^-} \frac{1-x}{\underbrace{\lg(1-x)}_{\to 0^+}} = +\infty$$

$\boxed{x=0 \text{ è ASINTOTO VERTICALE}}$

$$\lim_{x \to 2^-} \frac{|x-1|}{\lg|x-1|} = \lim_{x \to 2^-} \frac{x-1}{\underbrace{\lg(\underbrace{x-1}_{1^-})}_{\to 0^-}} = -\infty$$

$\boxed{x=2 \text{ è ASINTOTO VERTICALE}}$

$$\lim_{x \to 2^+} \frac{x-1}{\underbrace{\lg(x-1)}_{\to 0^+}} = +\infty$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x-1}{\lg(x-1)} = +\infty$$ (per confronto infiniti)

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1-x}{\lg(1-x)} = +\infty$$

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \underbrace{\frac{x-1}{x}}_{1} \frac{1}{\lg(x-1)} = 0$$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = 0$$

] no asintoti obliqui

3. f è continua in $\mathbb{R} \setminus \{0, 1, 2\}$.
Posso estendere f per continuità in $x=1$ ponendo $f(1) = 0$ ($x=1$ è discontinuità eliminabile)

Calcolo f'.

$\underline{x>1}$ $\quad f(x) = \dfrac{x-1}{\lg(x-1)}$

$$f'(x) = \frac{\lg(x-1) - (x-1)\frac{1}{x-1}}{(\lg(x-1))^2} = \frac{\lg(x-1) - 1}{[\lg(x-1)]^2}$$

$\underline{x<1}$ $\quad f(x) = \dfrac{1-x}{\lg(1-x)}$

$$f'(x) = -\lg(1-x) - (1-x)\frac{1}{1-x}(-1) = \frac{1-\lg(1-x)}{[\lg(1-x)]^2}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^+} f'(x) = \lim\limits_{x\to 1^+} \dfrac{\lg(x-1)-1}{(\lg(x-1))^2} = 0 \\ \lim\limits_{x\to 1^-} f'(x) = \lim\limits_{x\to 1^-} \dfrac{1-\lg(1-x)}{(\lg(1-x))^2} = 0 \end{array}\right\} f'(1) = 0$$

f è derivabile in $\mathbb{R} \setminus \{0, 2\}$.

4. Segno derivata

$x>1 \quad \underline{f'(x) \geq 0} \Rightarrow \lg(x-1) \geq 1 \quad (x-1) \geq e$
$x \geq e+1$

$x<1 \quad f'(x) \geq 0 \Rightarrow 1-\lg(1-x) \geq 0 \quad \lg(1-x) \leq 1 \Rightarrow$
$1-x \leq e \Rightarrow x \geq 1-e.$

$\boxed{\text{Segno } f'}$

$\boxed{\text{Monotonia di } f}$

	$1-e$		0		1		2		$1+e$	
$-$		$+$				$-$				$+$
$\searrow$		$\nearrow$				$\searrow$				$\nearrow$

$x = 1-e$, $x = 1+e$ sono pti di MINIMO LOCALE

$$f(1-e) = \frac{e}{\lg e} = e = f(1+e)$$

$x = 1$ pto di MAX LOCALE

$$f(1) = 0$$

Non ci sono max e min assoluti.

5. Derivata seconda.

$x > 1 \quad f'(x) = \dfrac{\lg(x-1) - 1}{(\lg(x-1))^2}$

$$f''(x) = \frac{\frac{1}{x-1}(\lg(x-1))^2 - (\lg(x-1) - 1) \cdot 2 \lg(x-1) \frac{1}{x-1}}{(\lg(x-1))^4} =$$

$$= \frac{\lg(x-1) - 2\lg(x-1) + 2}{(x-1)(\lg(x-1))^3} = \frac{2 - \lg(x-1)}{(x-1)(\lg(x-1))^3}$$

$x < 1 \quad f'(x) = \dfrac{1 - \lg(1-x)}{(\lg(1-x))^2}$

$$f''(x) = \frac{\frac{1}{1-x}(\lg(1-x))^2 - (1 - \lg(1-x))\, 2\lg(1-x) \frac{(-1)}{1-x}}{(\lg(1-x))^4} =$$

$$= \frac{2 - \lg(1-x)}{(1-x)(\lg(1-x))^3}$$

segno f''

$x>1$ $\quad f''\geq 0 \iff$ ① $2-\lg(x-1)\geq 0$

$\lg(x-1)\leq 2 \qquad x\leq 1+e^2$

② $x-1>0 \qquad x>1$

③ $\lg(x-1)>0 \Rightarrow x>2$

	1 — 2	2 — $1+e^2$	$1+e^2$ —
①	+	+	−
②	+	+	+
③	−	+	+
	−	+	−

$x<1$ $\quad f''\geq 0 \iff$ ① $2-\lg(1-x)\geq 0$

$\lg(1-x)\leq 2 \Rightarrow 1-x\leq e^2$

$\Rightarrow \cancel{x\geq} \; x\geq 1-e^2$

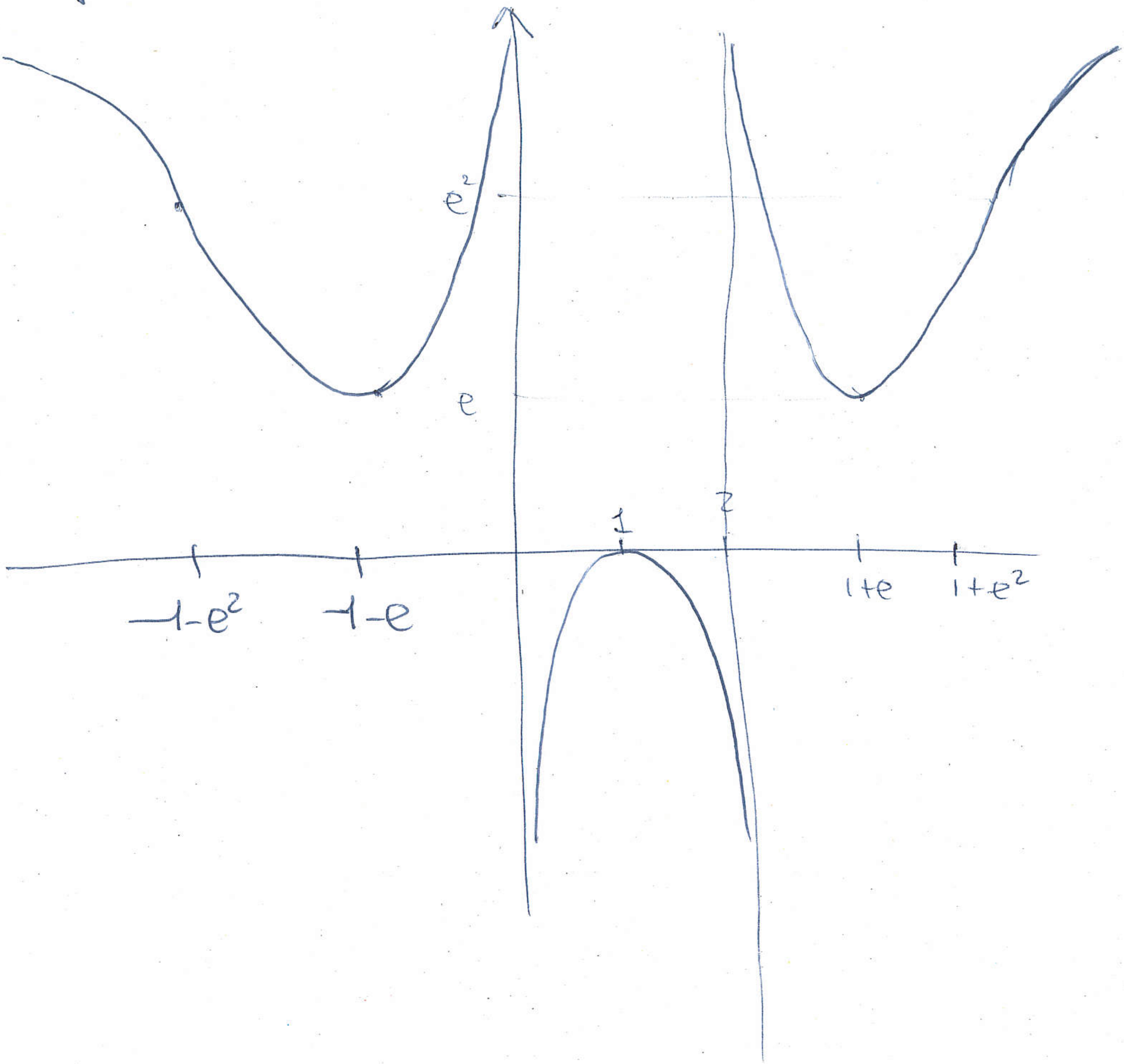
② $1-x>0 \qquad x<1$

③ $\lg(1-x)>0 \Rightarrow 1-x>1 \Rightarrow \underline{x<0}$

	— $1-e^2$	$1-e^2$ — 0	0 — 1
①	−	+	+
②	+	+	+
③	+	+	−
	−	+	−

	— $1-e^2$	$1-e^2$ — 0	0 — 1	1 — 2	2 — $1+e^2$	$1+e^2$ —
segno f''	−	+	−	−	+	−
concavità di f	$\cap$	$\cup$	$\cap$	$\cap$	$\cup$	$\cap$

Grafico di f



Es 6

$$f(x) = \frac{2 \sin x}{\sin^2 x + 2\cos^2 x}.$$

$D = \mathbb{R}$ (dato che $\sin^2 x + 2\cos^2 x > 0 \ \forall x$)

$f(x+2\pi) = f(x)$ perché seno e coseno sono periodiche di periodo 2π

$\Downarrow$

f è periodica di periodo 2π

$$f(-x) = \frac{2\sin(-x)}{\sin^2(-x) + 2\cos^2(-x)} = \frac{-2\sin x}{\sin^2 x + 2\cos^2 x} = -f(x)$$

f è dispari.

Studio f in un intervallo di lunghezza 2π.

Per es $\boxed{[-\pi, \pi]}$

Segno $f(x) \geq 0 \iff \operatorname{sen} x \geq 0 \iff 0 \leq x \leq \pi$.

non calcolo i limiti (è una funzione periodica!)

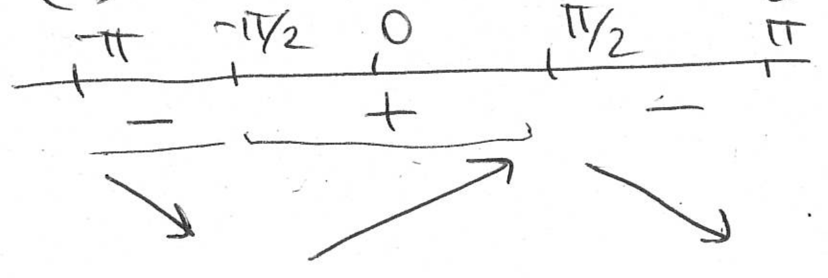
f è continua e derivabile in ogni pto $x \in \mathbb{R}$.

$$f'(x) = \frac{2\cos x(\sin^2 x + 2\cos^2 x) - 2\sin x(2\sin x\cos x - 4\sin x\cos x)}{(\sin^2 x + 2\cos^2 x)^2} =$$

$$= \frac{2\cos x \sin^2 x + 4\cos^3 x + 4\sin^2 x \cos x}{(\sin^2 x + 2\cos^2 x)^2} =$$

$$= \frac{2\cos x\,(3\sin^2 x + 2\cos^2 x)}{(\sin^2 x + 2\cos^2 x)^2}$$

$f'(x) \geq 0 \iff \cos x \geq 0 \iff -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

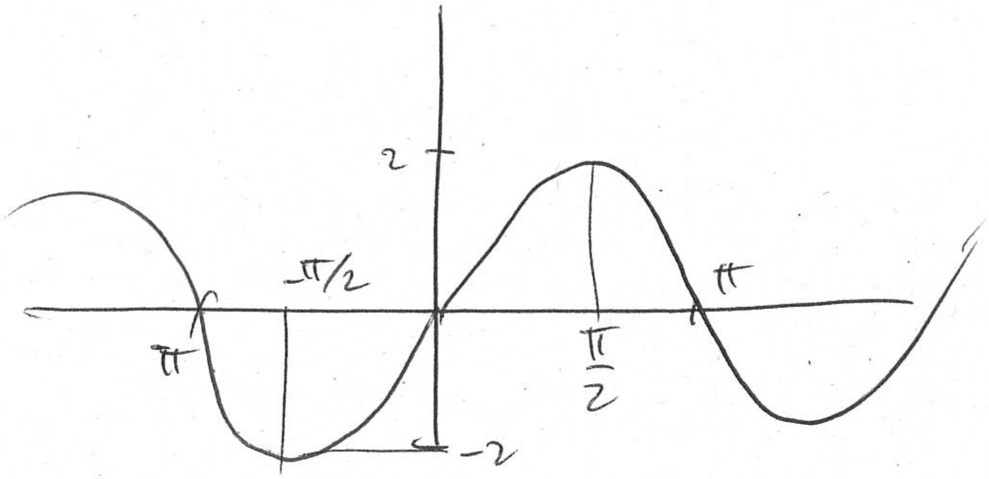


segno f'

monotonia f

$x=\frac{\pi}{2}$ pto di max locale e assoluto

$x=-\frac{\pi}{2}$ pto di min locale e assoluto



~ o ~

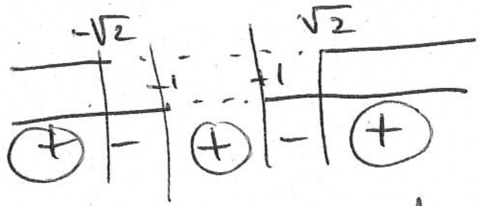
Es 7 $f(x)=\arcsin\left(\frac{1}{x^2-1}\right)$

Dominio $-1\le\frac{1}{x^2-1}\le 1$ (DOMINIO ARCOSENO)

$$\begin{cases}\frac{1}{x^2-1}\le 1\\ \frac{1}{x^2-1}\ge -1\end{cases}\Rightarrow\begin{cases}\frac{1-x^2+1}{x^2-1}\le 0\\ \frac{1+x^2-1}{x^2-1}\ge 0\end{cases}\Rightarrow\begin{cases}\frac{x^2-2}{x^2-1}\ge 0\\ \frac{x^2}{x^2-1}\ge 0\end{cases}$$

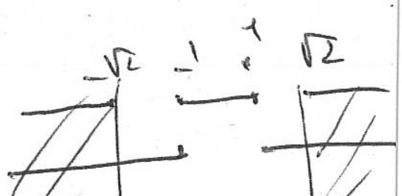
Studio $\frac{x^2-2}{x^2-1}\ge 0 \Rightarrow \begin{matrix}x^2-2\ge 0\\ x^2-1>0\end{matrix}$ $\quad\begin{matrix}x\ge\sqrt{2},\ x\le-\sqrt{2}\\ x>1\quad x<-1\end{matrix}$

$x\le-\sqrt{2},\quad -1<x<1,\quad x\ge\sqrt{2}$



Studio $\frac{x^2}{x^2-1}\ge 0 \Rightarrow \begin{matrix}x^2\ge 0\ \forall x\\ x^2-1>0\end{matrix}$ $\quad x<-1\quad x>+1 \Rightarrow \begin{matrix}1<x\\ x<-1\end{matrix}$

Metto a sistema $\begin{cases}x\le-\sqrt{2},\ -1<x<1,\ x\ge\sqrt{2}\\ x<-1\quad x>-1\end{cases}$



D: $x\le-\sqrt{2},\ x\ge\sqrt{2}\Rightarrow(-\infty,-\sqrt{2}]\cup[\sqrt{2},+\infty)$.

$f(-x) = \arcsin\left(\frac{1}{(-x)^2-1}\right) = \arcsin \frac{1}{x^2-1} = f(x)$

f è pari

f è continua in D.

$f(x) \geq 0 \iff \frac{1}{x^2-1} \geq 0 \iff x^2-1>0 \iff \boxed{\begin{array}{l} x>1 \\ x<-1 \end{array}}$

interseco con il dominio

$f(\sqrt{2}) = \arcsin\left(\frac{1}{2-1}\right) = \frac{\pi}{2} = f(-\sqrt{2})$

$f(x) \geq 0 \quad \forall x \in D.$

$$\lim_{\substack{x\to+\infty \\ -\infty}} f(x) = \lim_{\substack{x\to+\infty \\ -\infty}} \arcsin\Big(\underbrace{\frac{1}{x^2-1}}_{0}\Big) = 0$$

$y=0$ asintoto orizzontale a $+\infty$ e a $-\infty$.

<u>Derivata</u>

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{1}{x^2-1}\right)^2}} \cdot \frac{(-2x)}{(x^2-1)^2} = \frac{1}{\sqrt{\frac{x^4-2x^2+1-1}{(x^2-1)^2}}} \cdot \frac{(-2x)}{(x^2-1)^2} =$$

$$= \frac{\sqrt{(x^2-1)^2}}{\sqrt{x^4-2x^2}} \cdot \frac{(-2x)}{(x^2-1)^2} = \frac{\cancel{(x^2-1)}\,(-2x)}{\sqrt{x^2(x^2-2)}\,(x^2-1)^{\cancel{2}}}$$

$\underline{x^2-1>0 \quad \forall x \in D}$

f è derivabile $\forall x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$.

Studio la derivabilità in $\pm\sqrt{2}$

$$\lim_{x\to+\sqrt{2}^+} \frac{-2x^{\to+\sqrt{2}}}{\underbrace{\sqrt{x^2(x^2-2)}}_{0^+}\underbrace{(x^2-1)}_{1}} = -\infty$$

in $x=\sqrt{2}$ ho un attacco verticale (tangente verticale)

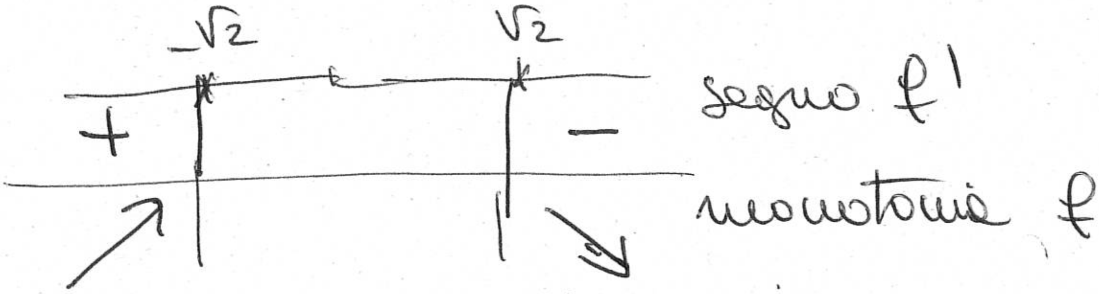
$$\lim_{x\to-\sqrt{2}^-} \frac{-2x^{\to+\sqrt{2}}}{\underbrace{\sqrt{x^2(x^2-2)}}_{0^+}} \frac{1}{(x^2-1)} = +\infty$$

in $x=-\sqrt{2}$ ho tangente verticale.

segno $f'(x) = \dfrac{-2x}{\sqrt{x^2(x^2-2)}\,(x^2-1)}$

$f'(x) \geq 0 \iff -x \geq 0$ (perché $x^2-1>0$ nel dominio).

$x \leq 0$



$x=-\sqrt{2}$ pto di massimo locale
$x=+\sqrt{2}$ pto di massimo locale
Inoltre $f(x)\geq 0$ $\forall x$ e $f(x) \leq \frac{\pi}{2}$ $\forall x \Rightarrow$
$x=\pm\sqrt{2}$ pti di massimo ASSOLUTO