

Derivate delle principali funzioni elementari

Nel seguito $\alpha \in \mathbb{R}$ è un numero reale, $q \in \mathbb{Q}$ è un numero razionale.

$$\begin{aligned}
 (e^{\alpha x})' &= \alpha e^{\alpha x} & (\log(\alpha x))' &= \frac{1}{x} & (\sinh(\alpha x))' &= \alpha \cosh(\alpha x) & (\cosh(\alpha x))' &= \alpha \sinh(\alpha x) \\
 (\sin(\alpha x))' &= \alpha \cos(\alpha x) & (\cos(\alpha x))' &= -\alpha \sin(\alpha x) & (\tan(\alpha x))' &= \alpha(1+\tan^2(\alpha x)) = \frac{\alpha}{\cos^2(\alpha x)} \\
 (\arcsin(\alpha x))' &= \frac{\alpha}{\sqrt{1-\alpha^2 x^2}} & (\arccos(\alpha x))' &= -\frac{\alpha}{\sqrt{1-\alpha^2 x^2}} & (\arctan(\alpha x))' &= \frac{\alpha}{1+\alpha^2 x^2} \\
 (x^q)' &= qx^{q-1} \text{ (per ogni } x \text{ se } q = \frac{m}{n} \text{ con } n \text{ dispari, o } m \text{ pari, solo per } x > 0 \text{ negli altri casi)} \\
 x^\alpha &= \alpha x^{\alpha-1} \text{ (solo per } x > 0 \text{)}.
 \end{aligned}$$

Primitive delle principali funzioni elementari

Nel seguito $\alpha, \beta \in \mathbb{R}$ (sia positivi che negativi), $c \in \mathbb{R}$, $a > 0, b > 0$ (a, b numeri reali positivi).

$$\begin{aligned}
 \int e^{\alpha x} dx &= \frac{1}{\alpha} e^{\alpha x} + c & \int \sinh(\alpha x) dx &= \frac{1}{\alpha} \cosh(\alpha x) + c & \int \cosh(\alpha x) dx &= \frac{1}{\alpha} \sinh(\alpha x) + c \\
 \int \sin(\alpha x) dx &= -\frac{1}{\alpha} \cos(\alpha x) + c & \int \cos(\alpha x) dx &= \frac{1}{\alpha} \sin(\alpha x) + c \\
 \int \frac{1}{\cos^2(\alpha x)} dx &= \frac{1}{\alpha} \tan(\alpha x) + c & \int \frac{1}{\sin^2(\alpha x)} dx &= -\frac{1}{\alpha} \frac{1}{\tan(\alpha x)} + c \\
 \int x^k dx &= \frac{1}{k+1} x^{k+1} \text{ (per } k \neq -1 \text{)} & \int x^{-1} dx &= \int \frac{1}{x} dx = \log|x| + c. \\
 \int (\alpha x + \beta)^k dx &= \frac{1}{\alpha(k+1)} (\alpha x + \beta)^{k+1} + c \text{ (per } k \neq -1 \text{)} & \int \frac{1}{\alpha x + \beta} dx &= \frac{1}{\alpha} \log|\alpha x + \beta| + c \\
 \int \frac{1}{ax^2 + b} dx &= \frac{1}{\sqrt{a}\sqrt{b}} \arctan\left(\frac{\sqrt{a}}{\sqrt{b}} x\right) + c = \frac{1}{\sqrt{ab}} \arctan\left(\sqrt{\frac{a}{b}} x\right) + c \\
 \int \frac{1}{\sqrt{b-ax^2}} dx &= \frac{1}{\sqrt{a}} \arcsin\left(\frac{\sqrt{a}}{\sqrt{b}} x\right) + c = \frac{1}{\sqrt{a}} \arcsin\left(\sqrt{\frac{a}{b}} x\right) + c.
 \end{aligned}$$

Primitive che si trovano con integrazione per parti o con sostituzioni.

$$\begin{aligned}
 \int \log(\alpha x) dx &= x \log(\alpha x) - x + c & \int \tan(\alpha x) dx &= -\frac{1}{\alpha} \log|\cos(\alpha x)| + c \\
 \int \arctan(\alpha x) dx &= x \arctan(\alpha x) - \frac{1}{2\alpha} \log(1 + \alpha^2 x^2) + c \\
 \int \arcsin(\alpha x) dx &= x \arcsin(\alpha x) + \frac{1}{\alpha} \sqrt{1 - \alpha^2 x^2} + c & \int \arccos(\alpha x) dx &= x \arccos(\alpha x) - \frac{1}{\alpha} \sqrt{1 - \alpha^2 x^2} + c. \\
 \int \sqrt{b-ax^2} dx &= \frac{b}{2\sqrt{a}} \arcsin\left(\sqrt{\frac{a}{b}} x\right) + \frac{1}{2} x \sqrt{b-ax^2} + c
 \end{aligned}$$